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A paper presented at the 233d meeting of the American Institute of Electrical Engineers, New York, January 8, 1909.

CONDITIONS AFFECTING STABILITY IN ELECTRIC LIGHTING CIRCUITS

BY ELIHU THOMSON

The term stability in this paper refers to that condition in which the current or potential, or both, remain at a steady value. Instability implies fluctuation which may arise from internal causes or be provoked by slight changes; and the fluctuations so arising may tend to be perpetuated automatically. An example of such action is the well-known " surging " in series arc-light circuits, which in the early days of development often became a difficulty.

The chief part of the present discussion will naturally have to do with the constant-current series circuits containing arc lamps operated by direct-current dynamos, by alternating-current transformers, by reactive devices in alternating-current circuits, or by rectified alternating currents from transformers. Much that will be touched upon is already well known; but there may be some matters which are not so well known, the bearing of which may be rendered more clear. The instability due to so-called " hunting " will be left out of present consideration, which will be confined chiefly to cases in which the load conditions themselves tend to instability.

The art of electric distribution began about 30 years ago with the direct-current series arc circuit. The load was inherently unstable owing to the peculiarity of the arc resistance falling with increase of current. This form of distribution seems destined to survive for a long period, especially for street service with the later luminous flame arcs of high economy. It will also hold its place in similar service with series incandescent lamps with metallic or other high economy filaments, the

nature of the resistance of which results in stability of current. Such a circuit as the latter is therefore outside of present consideration. Where in fact the load in a circuit is of the nature of "dead" resistance, stability is assured, provided no other disturbing causes exist, such as speed fluctuations in driving the generators. This is of course the case in the incandescent lamp circuit, whether in series, in parallel, or in a combination of the two. In early continuous current plants of small capacity, it was, however, generally easy to note the disturbances introduced by speed irregularities, or even to count the revolutions of the driving engine, or to watch the intermittent action of the engine governor.

The coupling of machines in parallel, and the later introduction of turbine driving, have resulted in the almost complete elimination of such fluctuations and they may, therefore, be left out of consideration as having nothing to do with inherent lack of stability or with such instability inherent in the arrangement of the circuit itself. In passing, however, it may be remarked that sometimes in self-excited direct-current dynamos of constant-potential type, shunt or compound, the degree of sensitiveness to speed variations may be abnormal. This sensitiveness has occasionally been noted by designers in high-speed, high-efficiency machines of moderate capacity. In such cases each slight increment of speed raises the potential and immediately reacts to increase the field strength, in turn causing a further increase, and so on. With a slight fall of speed there is of course the opposite effect. The difficulty is exaggerated if the engine governor is a little late in making its compensations, and they are at their worst when the iron of the machine is worked at the steep part of the curve of magnetization much below incipient saturation. The remedy for this form of instability is an alteration of proportions in the design and the removal of mechanical accessory causes. By working higher up on the saturation curve the tendency to magnification of the effects above alluded to is much reduced. The greater stability secured is, of course, purchased at the cost of more copper or a less efficient field circuit, and in a compounded machine the curve of potential regulation may not be so flat.

It is fortunate that the great fall of resistance in carbon filaments occurs only at ranges of temperature much below normal incandescence, or such lamps might become a

load tending to instability, as in the case of the Nernst glower, which requires the addition of devices for securing stability of current.

The metallic filament and metallized filament, having positive temperature coefficients, tend, if anything, to minimize all fluctuations, but the effect is naturally very small. Some tests made with rods of fairly pure silicon seem to show that, unlike carbon, it has a positive coefficient up to about 600° or 700° cent., beyond which the resistance falls very rapidly. Had, therefore, silicon possessed the refractory character of carbon, it would probably have been an unstable load, if otherwise suitable for filaments and tried in lamps.

The Nernst lamp glower is like the arc in being an unstable load. An arc started between electrodes held at a fixed distance apart and maintained at a constant potential difference would exemplify the most unstable condition possible, but the requirement is manifestly impracticable. It would result in infinite current passing. The current would grow in value until the assumed condition of constant potential difference ceased to exist. The condition reached would virtually be that of short-circuit. While the conditions assumed above are unrealizable, they are sufficiently approximated in practice, and they emphasize the fact of the need of some current-limiting or steadying device in circuit with the arc. Several arcs in series evidently do not change the nature of the problem. By an inspection of Fig. 1, in which the curves represent the relation of current and resistance and of current and potential in an open arc with fixed separation, these curves being properly regarded as "characteristics," the unstable condition is manifest. An enclosed arc gives substantially the same result.

It has been said with some truth that to design and construct a constant-potential dynamo that would work fairly well, was, even in the early years of the art, an easy undertaking compared with the production of a dynamo to operate a series of arc lights, involving a peculiar and little-known proportioning to secure stability of current in working a load so inherently unstable. The problem was not only this, but the stable current value had to be such as was proper to the machine, one which did not involve a destructive rise of temperature in the windings; and, on the other hand, one which was not so small in value as to involve such low temperatures in the windings as to indicate that the material was not being used at or near the maximum effectiveness.

It may be instructive to trace briefly the history of the development which preceded that of the constant-potential dynamo. Early in 1879 there existed arc machines working at about 10 amperes with a dozen or more arc lamps in series, the arc pressure being about 45 to 50 volts per lamp. The carbon-filament incandescent lamp was not brought out until about the beginning of 1880. It is well known that the original dynamos in service for arc lighting, excepting a small number of lighthouse plants, operated with the permanent magnet field (Holmes and Alliance machines), were constructed to work only a single arc with commutated direct current. Such were the Gramme machines of 1876, the Wallace of the same date, and the Siemens

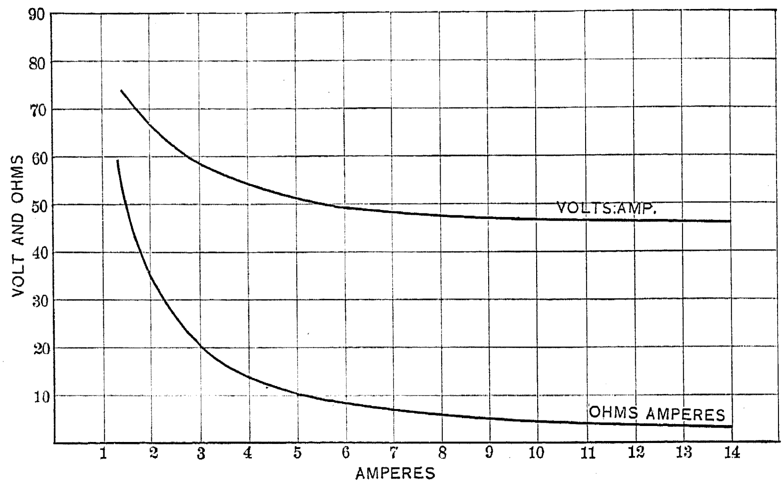


Fig.1

and Brush machines of a year or so later. The arc voltage was the same or nearly the same in these, but the current, according to the size of the machine, varied widely, being in some as low as 10 amperes and in others as high as 30 or 40 or more amperes. While the current values varied somewhat as the single arc burned with varying separation of its carbon, the current at any time was stable, without surging or fluctuations, such as might have been expected from the nature of the arc resistance, which was substantially the only resistance in circuit. But there were elements of compensatory character present in the circuit. As the current in an arc increases, the section of its hot gaseous conducting flame likewise increases; the resist-

ance falls as more material is volatilized, and more ions are furnished to conduct the current, unless at the same time the length of the arc itself be increased to compensate therefor. Most of the early arc lamps used with these machines possessed the feature of automatically lengthening and shortening the arc following the current changes in the arc-separating lamp magnets, this tending to regulate and keep constant the current strengths. The lamps were in fact called "regulators". But while this "floating arc length", as it may be called, assisted in securing stability, it would not have been of itself sufficient in the absence of other causes. It would have been too late in its action and of insufficient range.

When we examine the conditions of the circuit in these early machines, we find a large relative resistance in the circuit with the arc. As an example, a small Gramme machine of 1877 had a total circuit resistance including its arc, of 3.66 ohms, only 1.87 ohms being in the arc; the internal and external resistances were nearly equal. A Wallace machine had a total resistance of 7.83 ohms with only 2.77 ohms in its arc. An early Brush single-arc machine had 2.95 ohms total with 1.67 ohms in the arc. This relation of resistances would of itself be ample for securing stability of current, except that the dynamos being series-field excited, were of a nature to enhance current variations in the circuit by such variations reacting at full value through the series field. The machines would on this account be more likely to have unstable current. We now know that in such cases the high armature reaction allowed, and the amount and kind of iron and its nearness to saturation, were potent factors in the stability which existed. At the time these machines were designed very little was known of these matters, and the success obtained must have been largely the result of "cut and try", the experimental method. It was certainly easy to make the mistake of using too heavy iron sections, less armature reaction, less saturated field, with the result of using much less copper and obtaining even a more efficient machine for an equal energy output, and a better design if it had to work over dead resistance, but one which with an arc for its external work or load would not attain stability in current until the value was so large as to overheat and destroy the structure in a short time. The problem was the more difficult when later it was to be designed for a series of ten or more arcs, for in that case the steadying effect of internal re-

sistance, so potent in the case of the single-arc machines, could no longer be counted upon, such resistance falling more and more in relation to the external resistance with the size and increased capacity. In machines of a capacity of thirty to forty arc lights in series, the external resistance might bear to the internal a relation of approximately 10 to 1. There still remained the regulating effect of the lamp magnets giving a "floating arc length", and this action of the so-called regulators or lamps was the more important, as in the larger machines the time-constant of the circuit increased, giving more time in which the lamps could act to extend and shorten the arc lengths. When surging or instability took place it was marked by a lower rate of variation in the large machines than had been the case in the smaller ones, and consequently the period in which the lamps could act was more favorable. In series dynamos of large size the field-magnet reactance is naturally large, and mechanical adjustments of arc length were possible on account of the resultant sluggishness of the current variations indicating instability. There was, however, a more important property, the full significance of which was only realized at a later date.

Dr. John Hopkinson in 1879 called attention to the method of plotting curves, afterward called "characteristic curves" by Marcel Deprez. It was shown that by a study of the characteristic of a series dynamo the condition of stability or instability of current when working an arc in its circuit could be predicted. It is not necessary to repeat the work done by various investigators in the study of these curves. Suffice it to say that the conclusions as to instability are evidently modified somewhat under the conditions above pointed out, where the circuit has high reactance and there is time for the lamp mechanism to act in changing the resistance in circuit—increasing it when owing to an increase of current it tends to fall in the arc, and decreasing it when the current tends to die out, owing to the higher arc resistance which follows a diminished arc current.

At one time series direct-current circuits, made up of what were known as "short arcs" as distinguished from "long arcs", were employed in lighting to a considerable extent. In these short arcs the carbon electrodes were run so close together that there was but little appearance of any arc-stream between them. The arcs were hissing arcs, giving out continually a frying sound and frequently dropping out mush-

rooms of graphitic carbon which accumulated on the negative or lower carbon. The light itself was yellowish and came largely from the highly heated ends of the carbons next the slight separation or imperfect arc. In practice the current was about double that of the normal long arc; the voltage at the arc was about one-half. With the long arc the current was about 10 amperes and the voltage 45 to 50. In the short arc these amounts were approximately 20 amperes and 25 to 30 volts. Comparatively, the light production for a given energy expended was lower in the short arc. Reasonable life of carbons was only obtained by using extra hard sticks heavily copper-plated. The line losses formed a much higher proportion of the energy supplied. Unless the copper section were quadrupled in the case of the short arcs as compared with that used with the long arcs, the same loss per mile could not be attained.

This matter is, however, aside from our subject and is mentioned only as indicating one of the reasons for the non-survival of the type. So far as stability is concerned, however, since the lights were in considerable measure like dead resistances, it was far more readily obtained; it did not depend to such a large degree on the characteristic of the dynamo. So far as the writer is aware, these short arcs or semi-incandescent arcs were the only ones ever successfully run in series on shunt-excited arc dynamos, machines in which the proportioning was much more easy and more akin to that of constant-potential types. Despite the fact that a greater efficiency in the dynamo itself could be reached, the system gradually went out of use. It is readily seen that where the arcs were so short as to be only incipient arcs, the regulating effect of the series magnets of the lamp mechanism was quite important in determining the strength of current used. Slight increase or decrease in the separation of the carbons gave a larger change in the working resistance and also in the light. A similar extent of movement of electrodes toward or from each other in the long arc would have scarcely any effect. The floating arc condition was then quite essential to practical operation of these short arcs. The lamp mechanism was indeed a "regulator" in this case.

The various forms of series arc lamps involving differential lamp magnet mechanism of one or another type secure in greater or lesser degree the floating arc condition as distinguished from the "fixed separation", a feature of some types. As exemplifying the importance of this regulating feature upon current stability,

the writer may perhaps be permitted to recall an early experience in arc dynamo design. The first series arc machine designed and built under his supervision, was also the first dynamo having the three-coil (star three-phase) winding with three-segment commutator. This machine, at a speed of 750 rev. per min., ran a series of eight lamps at 10 amperes, with 45 to 50-volt arcs, and survived room temperatures of a bakery in the midsummer of 1879 in Philadelphia, the thermometer on the wall nearby showing sometimes 120° up to 140° fahr. in the early evening with the ovens in full operation. The machines had been carefully proportioned in the light of all available information which could be discovered at the time. There was very little to guide one in this task. Nevertheless, the current of the machine was stable at the current for which it was designed, although no experimental machine had preceded it. Further, there was no change needed in the windings and it remained in service for years thereafter.

Several similar machines were made after the trial of this first one. Following these there was constructed in 1880 a small compact machine for a circuit of 16 arcs in series, but the proportions were less conservatively chosen, and the armature strength was less in proportion to the field, so that regarded from a later standpoint the armature reaction was too small to give a proper droop in the characteristic at the current value of 10 amperes, for which it was designed. The iron sections were also liberal. It was found capable, when driven at 800 rev. per min., of maintaining its full quota of lamps, 16 in series, but although the lamps had the series-magnet regulating feature, the current was decidedly unstable, and only became stable at higher values, about 15 amperes, too high for the safety of the windings. It was observed, however, that if in the circuit there was introduced a single supersensitive lamp, or one in which comparatively large extensions or shortenings of arc length followed quickly upon minor current changes, an artificial stability was produced, which, in spite of the evidently unsuitable characteristic, as now regarded, allowed the lamps to be operated at their intended current of 10 amperes, or even less. The behavior of the dynamo as to heating and efficiency, etc., was then satisfactory.

On account of the relatively high duty of the machine, less weight of wire and cost, it became actually a debatable point whether the supersensitive regulating lamp might not be shut

up in an iron box and maintained as a permanent part of the circuit in practice. Later work, however, developed the fact that by changing the manner of commutation, the same machine became quite stable at the current intended, but at a sacrifice of output which could only be met by running at a somewhat higher speed. Naturally the stability obtained was accompanied by a somewhat lowered efficiency, but when attained was sufficient to render the circuit independent of any adjustment of arc length by the magnets of the lamps, except that of feeding as the arcs were lengthened by consumption of carbons. The actual change in the commutation made as above referred to consisted in rearranging the brushes of each pair, positive and negative, around the commutator from what was called forward regulation position to that which became standard in the Thomson-Houston arc machines afterward built. In the forward regulation the slots in the three-segment commutator were straight and parallel to the axis, in which case a pair of connected brushes on each side separated by a definite angle was used. Sometimes a single brush at each end of the diameter of commutation was applied, and the slot between the commutator segments offset so as to cause the segments to overlap or cover considerably more than 120° each, giving like results in commutation. In the later method, and that which became standard, there were two pairs of brushes. The commutator was made with straight slots parallel to the axis, and a positive pair of brushes placed on one side, and the negative pair on the other side. The brushes of each pair were spaced apart 60°, so that virtually a segment of the commutator was in contact with the positive or negative brushes through 180° in the full-load position. In regulating for a decreased number of lamps in series, the brushes in forward regulation were moved progressively forward, while in the later method the forward brush of each pair was adjusted forward only so far as to cover the spark developed, while the rearward brush of each pair was moved backward to the position where the same value of current was given to the circuit. It was found that if in either form of regulation the current was stable at full load it never became less so on diminished load, but rather more stable. This was to be expected. It is interesting to note in passing that if the arrangement of the brushes was intermediate between that of forward regulation and that of the later standard placing referred to, the circuit was character-

ized by great instability, so great as to make it impracticable to use the machine at all for an arc-lamp load.

These matters were, of course, the result of experimental trials of available apparatus. The explanation is that the needed droop in the characteristic curve of the two forms or arrangements was the result of opposite or conflicting causes incompatible with each other, so that an arrangement intermediate between them was not practicable. There is no space here for a more detailed analysis, which would itself be an interesting and instructive subject. It may be noted that one of the reasons for discarding the setting of the brushes as in "forward regulation" was that the wear of the brushes tended to extend the arc of contact and virtually to increase the overlap of the segments or the arc of revolution during which a segment was in contact with a positive or negative brush. This followed as a result of the tangential position of the brushes, which were made of slit strips of hard-rolled copper, as is well known. Lack of care in trimming and adjusting the brushes would, in fact, so extend the arc of contact that instead of stability, great instability would result. This showed clearly that stability depended on a restricted arc of contact or overlap of contact between adjacent segments under the brushes. If the field were relatively strong this overlap needed to be diminished. With a weaker field and greater armature reaction the arc could be extended. In consequence of this, and for the reasons above stated, the second or later arrangement was the final one adopted.

Another and curious stumbling block so far as stability of current was concerned was met later. Several sizes of Thomson-Houston arc dynamos were designed and built at New Britain, Conn., in 1882–1883, giving perfectly stable current without dependence on the regulating action of lamps or floating arc condition. The manufacture was removed to Lynn, Mass., in the fall of 1883, with the result of importing instability of current. The same patterns and proportions were used, all elements of construction apparently being retained. A change had been made in the lamp mechanism whereby there was now no longer any dependence of arc-length on the current fluctuations, the control of arc-length and feed being obtained by a derived circuit or shunt-potential magnet only. At first it was thought that this change in the lamp mechanism was the cause of instability, and as assisting to that conclusion, experiments showed that the Lynn-built machines could operate a circuit

of lamps having differential magnets (differential lamps) smoothly, and without instability. These were the old so-called D lamps; the new lamps being the old well-known M & K type.

As a remedy for the trouble, and one resorted to while the true cause was being investigated, there was established a short-circuit band of copper wire around the field cores parallel to the winding. This device at once gave stability to the current with both types of lamp. Meanwhile it had been determined that machines built in New Britain could operate with a stable current with either type of lamp in circuit, showing that they at least were independent of the floating-arc condition, or did not need the regulating effect of the lamps to insure stability. It was further found that machines built in Lynn from castings made in New Britain gave stability with either type of lamp. This led to the suspicion that in the cast iron itself lay the true cause. Analysis showed manganese in the New Britain iron and very little in the Lynn foundry iron. Special melts made in Lynn with the same mixtures of pig iron as were used in New Britain gave at once stable current and ended the difficulty without recourse to the closed copper band on the field, which demanded the sacrifice of considerable copper.

If anything will serve to show the pitfalls to which the arc-dynamo designer was subject, the above incident should suffice. The difference in the iron was evidently a difference in magnetic stability, rigidity, or hysteresis. In a dynamo machine the iron of which is magnetically stable or rigid, as when the hysteresis loop is wide, there is an absence of response to minor current fluctuations, which response is one of the causes of instability in series-wound machines. Put differently, it may be that there is an *ascending* characteristic curve, derivable from current and potential determinations while these are increasing, and a *descending* characteristic derivable from similar determinations, while these are diminished or diminishing. The greater the difference between these two the greater the stability of current, and the further down toward the origin one may work without instability. In machines with large iron projections on the armature core, such as the Brush arc machine, it is likely that there would be but little difference ascending or descending, owing to the magnetic vibration accompanying the operation of the machine; but it is conceivable that with steadily-driven, smooth-core armatures the hysteresis of the field iron would be a controlling factor. Doubtless, in the case above described,

a study of the magnetic qualities of the two irons would have disclosed the difference, so important in its practical effect. But it was then too early and there was too much else to be done at that time.

In apparatus in which potential differences of thousands of volts had to be maintained with only a single slot in the commutator between the brushes at such great potential differences, it can readily be understood that flashing or arcing over a slot might short-circuit or reduce the line current so much as to introduce a different kind of instability. Such a flashing or arcing in the slot was of the nature of an alternating arc. A segment of the commutator left the brush with a small trailing spark, showing a diminishing current reaching zero at a small distance in advance of the brush. Immediately thereafter the potential of the segment was reversed and attained a high opposite value, which, as an alternating arc, would jump the zero current position, especially if any heated gas remained in the slot. The writer has been frequently credited with having adopted as a bold expedient the well-known air-blast mechanism to blow out the sparks at the commutator. Nothing could be further from the fact or more mistaken. The real function of the air blast was to ensure the non-existence of heated gas at the zero current, to replace it with cold air and prevent the possibility of the next alternation re-establishing the arc and short-circuiting the commutator over the single slot between positive and negative brushes. The action to be secured was carefully reasoned out in advance of its trial and application. Without it the instability of circuit conditions would have rendered operation entirely impracticable, owing to incessant alternating arcs between commutator segments. Although these considerations apply to apparatus which is rapidly being replaced by other means of distribution, the writer feels justified in here presenting them as matters of history not generally understood.

The arc-light dynamo devised by Charles F. Brush was the pioneer machine in series-arc work, and it is proper to inquire as to the features which made it successful so far as stability of current was concerned. It was early shown to possess a drooping external characteristic, but it is safe to say that in the earliest types there was still dependence upon the regulating feature of the lamp or floating-arc condition. The lamps were of the differential type. It is also a fact that the peculiar

property conducing to stability both in the earlier and more recent designs was largely due to an almost critical proportioning of the proper extent and relative sections of the pole horns adjacent to the armature on each side. In a conversation with Mr. Brush in 1878, the writer remembers that he laid emphasis on the fact that even in the single-arc machines then before us, a comparatively slight variation in spacing or the arc covered by the pole pieces would destroy the stability of current, so that, as he said, he could do nothing with the machine. This was before the day of characteristic curves. The later Brush machines of large capacity for multicircuit work up to 125 arc lights or more, many of which have been built at Lynn, Mass., depended more than ever on the close proportioning of armature and field, and more particularly upon the form section, material, and arc covered by the pole horns. By careful adjustment an external characteristic curve having an almost vertical droop at a current value a little beyond the working current strength was obtained, rendering the current stable regardless of the regulating effect of lamps, and leaving but little to be done in regulating for variations of the number of lamps in circuit.

There is given in Fig. 2 an approximate showing of the relation of external characteristics of three Brush machines of different periods. The curves are not accurate, but merely indicate general types. Curve A shows the general form of characteristic of one of the earlier bipolar types. Curve B is that of a later type of larger capacity (125 arcs in series) built in Cleveland. Curve C gives the approximate curve of a still later machine of the same capacity built in Lynn, the two latter machines being of the four-pole type. There was a progressive saving in copper obtained. In the machine of curve A, the copper was approximately 50 lb. per kilowatt output, in the machine of curve B, about 40 lb. and in the machine of curve C, about 28 lb.

In Fig. 3 are given three characteristics taken from one of the most recent Brush machines constructed for working magnetite arc lamps at 4 amperes at a total circuit voltage of 15,000, multicircuit plan. In regulation the machine is shunted more or less and brushes adjusted automatically for variations of load. Curve A with a little over 15 per cent of the field current shunted and spark at brushes of fixed length, indicates that the normal current point is well over on the droop which at 5 amperes corresponds

to short-circuit or no voltage. With curve B, or unshunted field, the normal current point lies on the horizontal part of the curve which then droops rapidly at about 4.75 amperes and would indicate a probable short-circuit current at about 5.5. Curve C shows the normal current value of 4 amperes at no voltage and indicates the condition when all lamps are shunted and the field shunted, in this case to the extent of 34 per cent

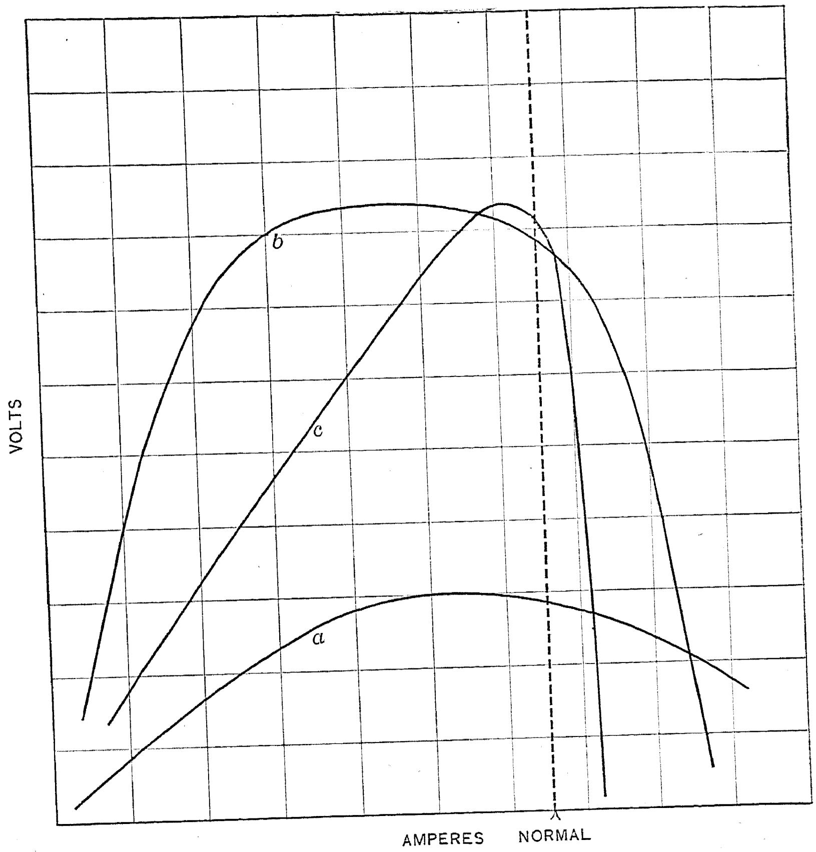
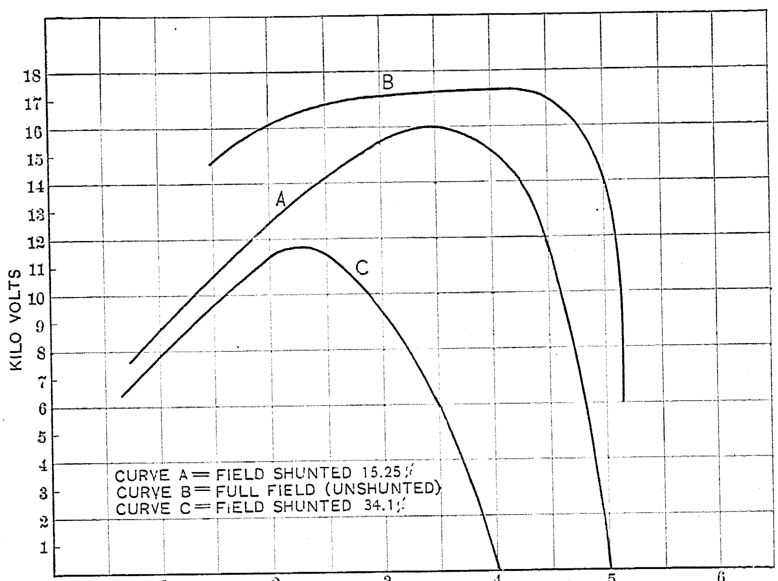


Fig.2

of its current. Curve A shows ample stability. Curve B indicates an approach to instability, probably capable of being prevented by the regulating feature of the lamps or floating arc condition. Curve C is, of course, an indication of most perfect stability and it will be readily understood that as the load is cut off in passing from A to C the margin of stability will increase steadily.

It is interesting to note that in most of the early Brush machines there was applied around the field cores a closed copper band to increase the stability or prevent surging. The temporary use of this expedient in the Thomson-Houston construction has been alluded to. The closed band served to render the field magnetism less sensitive to quick changes occurring in the current of the machine, by permitting the induction of opposed currents in the band. The magnetism was thus rendered more stable and the changes in it more sluggish.

The complete theory of the various types of arc dynamos



CHARACTERISTICS 13 D FORM H-BRUSH ARC DYNAMO 500 R.P.M.

Fig.3

has not been worked out, and much of the designing has been of necessity of the cut-and-try order. The later work on the Brush dynamo has been ably carried out by Charles M. Green at Lynn, and the machine, on account of lending itself readily to multicircuit working, is still constructed, though the output is now relatively small. On account of the great dependence of the working and current stability upon the relations and proportions of the pole horns, it may almost be characterized as a " pole-piece " machine in contradistinction to one in which there is either an absence of pole horns or in which, if they exist, their form and disposition exert no vital influence on

the working, particularly as concerns the stability so essential where the load consists of arcs.

It seems probable that on account of the simplicity of the wiring, the series constant-current circuit will be employed for a long period in the future for arc lighting in street service and for large spaces. The advances recently made in high-efficiency luminous flame arcs will doubtless be a potent factor in future development.

When the enclosed carbon arcs were substituted for open arcs on direct-current circuits, it was recognized that a sacrifice of efficiency of light production with a given energy was involved. This was offset, however, by the other well-known advantages accruing. No noticeable change in the stability of the circuits was discovered when the substitution was made. When, however, the alternating-current open arc, itself less efficient than the direct-current arc, came to be enclosed it was found that the sacrifice in efficiency was such that aside from the white color of the light of the arc there was no advantage over incandescent lamps using the same energy. It may be conceded that no artificial light equals the carbon arc in its approach to an ideal white light, or that of the sun; and doubtless we see to better advantage with such a quality of light even at a diminished intensity of illumination.

The introduction of alternating-current arcs in lighting occurred very early, as in the lighthouse lamps before mentioned. The introduction of the so-called Jablochhoff candle in 1878 was, however, the first attempt to apply it in general illumination. The candles had some of the features of the luminous arc, inasmuch as they smoked and consumed a solid substance such as kaolin besides the carbon. The candles were operated several in a series. Little is known of the stability conditions in such circuits, but in Paris in 1878 the alternating dynamo current was not carried directly to the lamps, but the lamp circuits were operated through huge condensers in such manner that several series of lamps were fed from the single current of larger value split up through the condensers. It is possible that this use of condensers in the distribution served to limit the current values and secure stability. Later attempts to apply open arcs of the alternating type on series circuits from constant-current transformers, met with little encouragement. Not until the success of the enclosed arc on direct current had been fully demonstrated was there any con-

siderable opportunity to make use of series alternating-current arcs. The enclosed alternating arc operated from constant-current regulating transformers gave the opportunity. The system permitted the employment in the station of generators of large capacity with constant-potential mains, to which the primaries of the transformers could be connected at conveniently located sub-stations, while the secondary arc circuits became independent, so that leaks or grounds on one did not affect the other. This was a manifest advantage.

Fortunately no difficulty arose in securing stability of current values in constant-current transformer distribution. The reactance, or more properly the general impedance of the system including the alternating lamp magnets, which themselves assist materially, resulted in a fall of potential with increased current, and the reverse, at a rate sufficient to insure stability. The characteristic curve plotted with current as abscissas and volts as ordinates would in the constant-current transformer droop as in the arc machine, but it would droop steadily from zero current outward. In other words, the potential is at a maximum with zero current, while in the series field arc machine, except for residual magnetism, zero current corresponds to zero potential. Were, however, the primary and secondary coils of the transformer allowed to come too near together the condition would approach too closely that of a constant-potential transformer. The leakage magnetism between primary and secondary coils, when as close together as permissible, will give rise to considerable reactance values, which, as the load on the secondary is cut off, will gradually increase as the secondary is repelled to a greater and greater distance. It is necessary that the movement of the coil be effectually damped, by dash-pots or the like, or an instability due to overrunning akin to hunting will take place during changes of load. The direct-current arc dynamo with its heavy series-field coils is likely to have so much self-induction as to tend to steady the current values so far as very rapid fluctuations are concerned. When, however, the field, as in the Brush machine, is shunted more or less for regulation, this influence of the field inductance is neutralized, at least to a large extent. With the alternating-current circuit there is, of course, an entire absence of such influence upon stability; but in this case the resistance and self-induction in the whole circuit traversed by the current is sufficient for securing stable values of current. These conditions

giving rise to stability naturally involve a lessened power-factor in the system, and, as is well known, the distortion of the current wave in an alternating arc, while not in the nature of a true lag, reduces the power-factor. It is probable also that the tendency to instability in an alternating-current arc is not so great as in the direct-current arc, for the arc undergoes extinguishment at the zero and does not relight until after the potential difference between the electrodes has reached a considerable fraction of the maximum.

In the more recent work with magnetite arcs and with lamps having electrodes which, for satisfactory operation, demand the employment of direct current, there was at one time a tendency to return to the use of motor-driven arc dynamos giving small current values and high potentials. The development of the mercury-arc rectifier with its apparent special adaptability to the rectification of small currents at high voltage has naturally led to its employment in conjunction with the constant-current transformer for securing direct currents suitable for series-arc circuits. The practical results so far obtained with oil-immersed mercury rectifiers have been so generally satisfactory as to indicate an extended future application. The new factor which has thus been introduced into constant-current working may of course be the cause of disturbances and instability under certain conditions, such as too high or too low a temperature in the rectifier, change of vacuum, or momentary flashing from anode to anode, indicating failure to rectify. Any cause which disturbs the circuit voltage is productive of results which may be classed as instability.

Oil-bath immersion has given control of the working temperature of the rectifiers. If too low, the condition imports instability. If too high, there is danger of breakdown. Fortunately it appears that normal working demands such moderate temperatures as are easily maintained in the oil bath. Alteration of vacuum by leakage, or by gradual giving out of gas may result in the accumulation of a body of insulating gas around the anode, interfering with the normal ionized mercury-vapor stream which conducts the current. This may cause a fading out of the rectified current, in turn causing the lamp to lower or drop the electrodes together, and thus again allow increase of current, which again separates them, and so on; a condition of surging or oscillation somewhat akin to the instability of an arc circuit when working too close to the critical point on the

characteristic. Lowered vacuum also endangers the rectifier by causing excessive stresses near the anodes, which may puncture the glass. Suitably spaced shunting gaps may be provided around the rectifier as a safeguard against excessive stresses. The temperature of the rectifier tube can also be maintained by circulation of air by a fan where oil is not used. The plant includes, besides the rectifier and constant-current transformer, additional alternating-current reactance, and a direct-current reactance in the rectified current circuit, the proportions of which are chosen to insure as far as possible steadiness in the working.

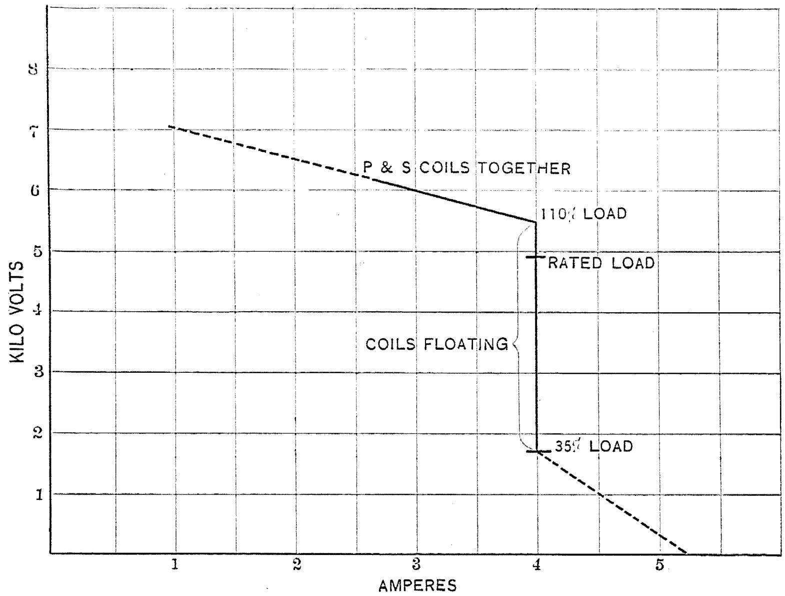


FIG. 4

If a characteristic be plotted it takes the form given in Fig. 4, which is properly the regulation curve. The vertical line at 4 amperes shows the range of constant current between 110 per cent load and 35 per cent of full load in kilovolts, the latter point corresponding to the greatest separation of primary and secondary coils as provided for in the construction of the transformer tested. Even at short-circuit the current will then not exceed 5.25 amperes. As a rule the circuit is adapted to the operation of lamps in which the arc length is either " floating " or fixed, without involving instability of current. Hence

the lamps may be constructed to lift the upper electrodes and separate them to a definite distance and to drop them together at feeding, which in the case of long-burning electrodes would occur very infrequently. This manner of operating was known in the early days as " drop-and-lift " feeding, and was probably first used on any scale in the obsolete Wallace plate lamp. As before shown, such operation requires a current which is inherently stable, as no adjustment occurs in the lamp itself. As indicating the endurance of the mercury-arc rectifier itself, it may be incidentally mentioned that some of these in service have attained upwards of 5000 hours' life and are still operating. The system has the advantage of permitting arc lamps to be operated from 25-cycle circuits, and while the direct-current is then more pulsating than in the case of higher frequencies, it is still satisfactory. The power-factor at full load ranges about 70 per cent and the efficiency is about 90 per cent. Increase of the power-factor would probably result in instability of current.

Before closing, the conditions governing the stability of an arc or arcs in branches from constant-potential mains may be briefly referred to.

As before stated, instability is then at a maximum and current values may rise to the proportions of a destructive short-circuit. If the regulating mechanism of the lamp extends the arc or the reverse in response to current variations, a violent surging or oscillation with extinction results. Hence the necessity of the choking-coil, whether there be one or more arcs in series across the mains. With continuous currents a dead resistance which will cause a drop of from 20 to 30 per cent of the total potential is generally inserted. The condition for stability is that the resistance shall drop the potential at the arc with increase of current at a greater rate than that caused by increase of current in the arc itself, and *vice versa*, as is well known. This involves a serious loss of energy and many efforts have been made to reduce or minimize it. It is customary to allow the floating arc to exist and in such a case the dead resistance may be cut down, if in the arc branch a large self-induction be inserted, a reactive coil which introduces a time-lag in the current variations, allowing the regulating action of the lamp magnets to come into play. Better yet, if the lifting magnet be made a shunt of many turns around the more coarsely wound reactance or reactive coil (as invented by Higham) a supersensitiveness in the lamp mechanism is

obtained which causes it to anticipate current changes or at least compensate therefor in increasing and diminishing the arc length and so preventing the fluctuations attaining any considerable value. Still more effective is the arrangement in which, as in Fig. 5, the arc-adjusting coil C in shunt to the reactive coil R is made to move in the field of the latter used as a field magnet M. These arrangements permit a direct-current arc to be operated between constant-potential mains with a minimum of dead resistance in series with it. Unfortunately, the saving with the carbon arc is of little practical value where a single

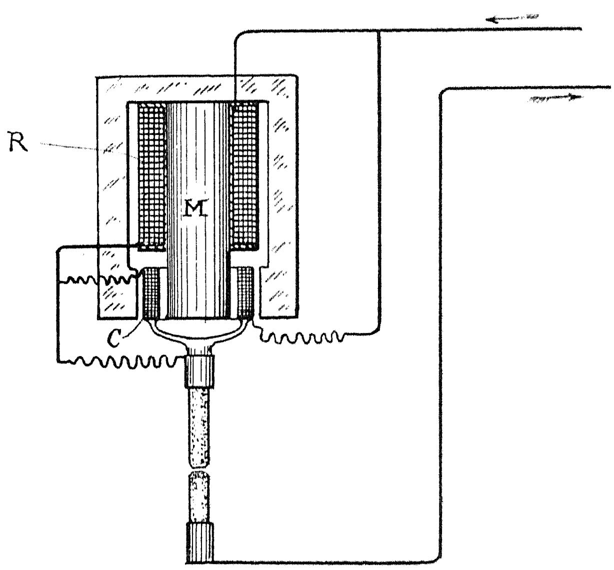


Fig. 5

enclosed arc is put in a branch across 110-volt mains. While the voltage at the arc may be thus increased from 80 volts to 100 or thereabouts, the gain in light does not correspond to the increased arc energy, on account of the comparatively low luminosity of the arc flame itself. It is possible that where the arc flame is the source of light, as in luminous arcs, the expedient may give practically valuable results, though that remains to be seen. In the case of the alternating arc across lines at constant potential, stability is readily obtained by the introduction of an impedance, or reactive coil, which need not involve any considerable loss of energy, but of course its effect

is to diminish the power-factor of the system, to which also the distortion of wave in the alternating arc itself may add. Both the Nernst glower and the mercury-arc-lamp share with the ordinary arcs the disability of invoking instability of current unless special means are provided to prevent it. No special consideration needs therefore to be given to them in this general review of the subject.

Discussion on "Conditions Affecting Stability in Electric Lighting Circuits." New York, January 8, 1909

Elihu Thomson: The paper I have to present is in large measure incomplete. It is merely a sketch or outline, and much of it is historical, going back to the early days of the inception of the art which this Institute represents. It was thought it would be well to bring together in one paper and place upon the records of the Institute some of the historical facts connected with the subject treated of in the paper.

President Ferguson: The Institute is especially fortunate in having been able to induce so distinguished a member as Professor Elihu Thomson to present this paper to you this evening. There is probably no man in this country who is able to talk more authoritatively on this subject, or who has done more in the development of electric street lighting, than the author of this paper. The work which he did was in the early days, when electrical engineering was comparatively unknown, at a time when it required not only skill but courage and patience to accomplish results. I have the distinguished honor, gentlemen, not to introduce to you, but to call upon Professor Elihu Thomson, who will present a paper on "Conditions Affecting Stability in Electric Lighting Circuits."

A. E. Kennelly (by letter): Professor Thomson's interesting paper discusses a subject which has not received from engineers the attention it deserves; namely, the automatic maintenance of current stability in circuits which contain collapsible resistances; that is, resistances which tend to diminish with current flow.

Two principal automatic methods are discussed: namely, ballasting the circuit with extra impedances, and providing the generator with a drooping external characteristic.

The first method is especially applicable to constant-potential circuits. Its weak point is its wastefulness of power.

The second method is especially applicable to constant-current circuits.

The arithmetical relations that determine the restoration of stability to such a circuit are simple enough, if we regard the matter merely from the standpoint of static equilibrium; but in practice the problem is complicated by kinetic actions, such as the time-interval of arc-mechanism response, the time-interval of engine-governor response, the time-interval of field-magnet response, and other time-intervals alluded to in the paper. Consequently, the complete practical problem, as presented in a series-arc circuit, is arithmetically very complex, even with the accumulated experience of 30 years. It must have appeared yet more intricate in the early years of arc lighting.

It is very interesting to learn that a supersensitive arc-lamp mechanism in the series-arc circuit was found sufficiently effective as a stability regulator to make its practical adoption, in the early days, worthy of consideration.

The remarkable degree of current-stability in series-arc circuits supplied through constant-current or tub-transformers, which is familiar to all who have operated such circuits, is clearly explained in the paper. The external electromotive force characteristic shown in Fig. 4 would fall much too precipitately for effective use in a generator, as may be seen by comparing it with curve B in Fig. 3. The current stability in the case of the tub-transformer is only rendered possible, with the vertical drop of voltage characteristic in Fig. 4, by the automatic balancing of gravitational force against electromagnetic repulsion in the manner well known.

It is a curious fact that the property of diminution in resistance with current which may bring about current instability in an arc circuit, is the property which is utilized in wireless telephony to throw a vertical conductor into sustained electric oscillation, by means of voltaic arcs shunted by condensers.

Alex Dow (by letter): It is well known that any cause which disturbs the circuit voltage is productive of results which may be classed as instability and, conversely, that circuit conditions of apparently minor importance may be effective in maintaining stability. For instance, a constant-current Brush machine on a Mississippi River steamer was used to operate two 20-ampere, 50-volt arcs in series during the loading and unloading of the steamer, and, when the steamer was under way, to operate a 40-ampere, 50-volt searchlight. The field windings of the machine operated in straight series on the former service, and in multiple-series on the latter service, so that the field ampere-turns were constant and the armature ampere-turns were double when the searchlight was operated. The latter condition should have tended to produce stability. Nevertheless the two arcs in series were beautifully steady, while the single searchlight behaved sinfully. The completely successful remedy applied was the insertion of inductance in the searchlight circuit, in the form of some large iron wire wrapped over asbestos paper on a gas-pipe core, in which gas-pipe core sundry pieces of old iron found on the steamer were inserted to make the final adjustment.

Another instance. Two new circuits of M and K lamps in which the arc-length was controlled by a shunt round the arc, were unsteady when operated from their proper Thomson-Houston spherical armature machines. They were entirely steady when operated from Brush machines, and the Brush differential lamps were entirely steady when operated from the Thomson-Houston machines. Consequently, this exchange was adopted as the regular rule of operation. Subsequently it was found that if one-quarter to one-third of the lamps on the circuit were of the Brush differential type, the circuit would be steady on either machine.

In my experience the worst case of circuit instability was that of several circuits each intended to carry 150 arcs. They

behaved very badly, and it was necessary to adopt a quick and a cheap remedy. The generators had not the drooping characteristic. Their short-circuit current was about twice the normal current. But they had an exceedingly effective regulator which would control closely the current supplied to a circuit naturally stable. The constructing engineer had combined those machines with shunt-wound arc lamps, not the lamps usually operated in conjunction with the generators. Each string of 150 lamps was served by rubber-insulated, lead-covered underground cables of high insulation resistance, high static capacity, and inadequate resistance to puncture. The capstone, so to speak, of the combination was that the lead cover of each length of cable was intentionally kept out of contact or connection with the lead covering of the succeeding lengths and was, because of frozen soil, virtually insulated from earth.

When these 150-lamp circuits were operating, all the telephones on the west side of that large city went out of business. One hundred modern wireless installations could not have been more effective in disturbing the ether. But the circuits would not stay in operation. The machines spilled, the cables punctured, and the arc-lamp posts burned out. The remedies applied were, first, the earthing of each section of lead sheath; second, a rearrangement of the circuits so that some old differential lamps were included in each string of shunt-wound lamps; third, the steadying of the machines by various devices known in the art; and finally the substituting of different machines with marked drooping characteristics for the original efficient but very sensitive generators. The last remedy was, of course, final, but the effect of the connection of a comparatively few differential lamps was surprising.

E. W. Rice, Jr.: This subject is of great interest to me, because I had the rare privilege of association with Professor Thomson in the early days of which he speaks. I still have vivid recollections of the difficulties surrounding the design and manufacture of electric generators that would satisfactorily operate a load of the inherent instability possessed by the electric arc. It is easy for us with our present-day knowledge to discover the cause of those early difficulties. I doubt if the younger generation of electrical engineers can form a proper mental picture of the conditions surrounding the work of the pioneers in electric arc lighting. In those days practically no electrical measuring instruments were to be had; it was years before the ammeter, voltmeter, and wattmeter were invented. Besides this, the confidence of financiers in electrical inventions was not particularly great, and cash was notably scarce.

Without seeming to depreciate to the slightest extent the achievements of other workers of that period, I believe I am merely stating a historical fact in saying that the author of this paper had, of all his contemporaries, the clearest view and the strongest mental grasp of the electrical problems then press-

ing for solution. For some reason, probably because of his characteristic modesty, Professor Thomson has failed to call attention to the fact that the first scientific investigation of the properties of the electric arc was made by him with, I believe, the assistance of Professor Houston some time in 1878 or 1879, and that a paper setting forth the result of the investigation was published in the *Journal of the Franklin Institute* at the time. This paper clearly sets forth, among other interesting facts, the laws governing the variation of the resistance of the electric arc as an inverse function of the current, from which the inherent instability naturally follows.

I am also able to say that as far back as 1880, when I first became professionally associated with Professor Thomson, he frequently explained to me, in much the same language as that employed in the paper he has just read, the characteristics necessary for a dynamo successfully to operate a circuit of arc lamps. However, for the reasons he has explained, it was not always possible to realize these views in practice. I remember we often wished that an arc lamp would behave like a respectable dead resistance.

It is interesting to note that the early machines operating but a single arc lamp undoubtedly owed a large portion of their success to the high internal resistance, which improved the stability factor, and that the problem, as usual, became more serious when an attempt was made to improve the efficiency. Unfortunately, it seems to be a general law of nature that the highest efficiency is likely to be associated with the greatest instability. However, it is worth while to try for efficiency, as in the long run the most efficient system, if practicable, will survive, as instanced in the relatively short vogue of the inefficient so-called "short-arc" system described in the paper.

Referring again to the earlier one-lamp machines which were operated at widely different currents, I think this variation was probably partly due to the fact that stability was not always found at the designed current, and machines, being small and of relatively large radiating surface, could be operated at that current which gave stability without serious overheating. Professor Thomson's description of the early machine which operated satisfactorily only when artificial stability was produced by means of a single supersensitive arc lamp, and his suggestion that it might be desirable in practice to place such a lamp shut up in an iron box as a permanent part of the circuit, recalls to my mind that at a considerably later date I actually made use of this early experiment. My application of this device was made at a time when I was placed in charge of an exhibition of arc lamps which was made by the Thomson-Houston company in Boston some time in the year 1882. For some reason the particular machine was unstable. After trying various expedients, I concluded that our exhibition was doomed to failure unless some immediate remedy were applied. As it

was impossible to make any general changes in the machine or arc lamps, I arranged one of the lamps as a supersensitive stability regulator, and kept it operating in a part of the space where it would be unobserved by the public. The remaining lamps were, as I remember, entirely satisfactory. After the exhibition the machine was properly tinkered until it worked in a satisfactory manner.

Professor Thomson mentions one method of improving the stability, or even creating stability, in a machine of unstable characteristics, which consisted in establishing a "short-circuit band of copper wire around the field cores parallel to the winding." I remember this device as a sort of powerful medicine that could be used successfully only in small doses. It, of course, had the objection mentioned by Professor Thomson, of increasing the amount of required copper, but I remember it also had the added objection that if the short-circuit band were made of too large cross-section, so as to become a fairly good secondary circuit, the machine had the characteristics of a separately-excited machine. Under such conditions any flashing of the commutator was enormously increased in intensity and prolonged in time. So great was this fault that for this reason alone the machine would become uncommercial. On one occasion one of these machines gave so much trouble that I was sent to investigate its cause. I found the machine, of 10-lamp capacity, entirely stable and satisfactory so long as flashing was prevented; but if for any cause a flash occurred at the commutator, the flash would continue for so long a time as actually to extinguish the arcs, seriously interrupt the service, and exhaust the patience of the mill owner. The machine had to be replaced. After the short-circuit band was reduced to proper proportions, the same machine was found to operate in an entirely satisfactory manner.

It is pleasing to note the valuable and timely suggestions made in the closing paragraphs of the paper, which point out the possibility of increasing the efficiency of the already extremely efficient flaming arc lamp, by employing methods to maintain stability with minimum loss.

Dugald C. Jackson: When I was at Cornell University the laboratories possessed one of the old Wallace arc lamps with plate electrodes made of gas-house carbon. The arc formed between adjacent edges of these plates was prodigious, and absorbed a corresponding volume of current. It wandered from right to left and from left to right at will, along electrode edges perhaps a foot long. It was sometimes here and sometimes there, at its own volition, and the illumination afforded was too erratic to be useful.

Arc lighting with cylindrical carbons $\frac{5}{8}$-in. or less in diameter had even then come to stay; but there was a contest on between the short or hissing arc of approximately 20 amperes and perhaps 35 volts, and the long or silent arc of approximately 10

amperes and perhaps 50 volts. Each had its ardent supporters. But the hissing, flickering short arc returned so much less light per unit of energy supplied to it that its commercial life was doomed, notwithstanding the good workmanship, for that day, in the Weston shunt-wound dynamos, and the gems of mechanicians' skill comprised in the Weston lamps, which among some others, supported the short-arc or low-potential arc lighting system.

It required a bold and original man to stake out a feasible path into the domain of the unstable long or high-potential arc. But the talents of Mr. Brush and Professor Thomson, each in his own way, brought the desirable end. Mr. Brush's dynamo, judged by the dogmatic theory of that day, was viewed as a curiosity; Professor Thomson's dynamo was looked upon as a monstrosity and contrary to the proper theories of dynamo electric machines. But both dynamos abundantly proved their right to exist, and both proved the lack of imagination of their critics. The fault of the critics lay in trying to apply an incomplete theory of dynamo electric machines to one element (the dynamo) of a complete system comprising several elements, all of which react on each other in a complex manner.

In viewing the magnetic flux entering the armature of a direct-current constant-potential machine, we think of the aggregate flux as a determining feature of construction and operation. In the case of alternating-current machines, we generalize further and recognize that the position of the flux with respect to polar faces and arrangement of the armature conductors affects the form of the pressure wave and the output and functioning of the machine. A like condition seems to be even more marked in the case of the Brush arc-light machine, and *the positioning of the flux* which enters the armature core may be of an importance comparable to the importance of the aggregate strength of field. For example, in some Brush machines, I have understood that changing the polar tips from soft steel to cast iron has actually increased the capacity of the machines. These are machines in which, of course, armature coils are changed from series to parallel relations, and the reverse, by the action of the commutator in each revolution of the armature. A rational theory includes such effects; and its guiding finger points to such improvements as the reduction of copper required per kilowatt of capacity referred to in the paper, By "positioning" of the flux, I mean the distribution as it enters the armature core, say at full load, and the variation of the distribution with changing current.

My first intimate acquaintance with the Thomson-Houston arc-light machine was made with a 30-lamp machine in the later 80's in the middle of the Mississippi valley at a place where repair parts were unknown. A sleepy attendant allowed the lubrication of the blower to become defective, the runner stuck

in the case, and the blower case was torn from its bolts and entangled with a mess of wreckage of the links and levers of the regulator mechanism. The accident was no fault of the machine itself. To show the essential simplicity of the type, I will say that new bolts and some links and levers roughly shaped from iron rods by the aid of a country blacksmith's forge put the machine in satisfactory and prompt commission, and it was continuously used while new parts of orthodox construction were being obtained from the factory in the East.

My observation of that machine, and many others of the same or of larger size, has led me to view the function of the blower a little differently from the manner stated in the paper. I differ with Professor Thomson, but the difference may, after all, be only a matter of definition. A close observation of the working of a Thomson-Houston arc machine shows an obvious blowing out of the arc. The blowing out is not like that of a candle, where the flame is blown from the wick by a blast; but the air blast crosses the path of an arc existing between commutator segments, stretches it to the point of rupture, and replaces the heated gases by air which is cool and unpolarized (Professor Thomson prefers the word un-ionized) and therefore will not support a renewal of the arc until it is re-established by the action of the brushes. A similar effect may be produced by blowing a brisk blast across the arc of an arc lamp.

The constant-current transformer, associated with alternating-current lamps, referred to in the paper, brought arc lighting within the scope of economical central station practice, since it made the operation of arc lighting from current generated by great generating units an assured fact. The addition of the mercury-arc rectifier has done much to extend this improvement, but it is apparent that the end is not yet. Indeed, improvement must be vigorously prosecuted if electric arc lighting is to maintain a satisfactory margin of supremacy over the new gas-burners for outdoor lighting.

C. M. Green: The early observation of armature reaction is, in my opinion, the basis of all of our arc machines of to-day. Under full-load conditions the point of commutation is under the following pole piece, and as the load is decreased, or lamps are cut off, the point of commutation goes still farther under the following pole piece. The regulation of various machines is accomplished in various ways. In the Thomson-Houston and Fort Wayne machines the final adjustment or regulation is accomplished by means of varying the armature reactions—by overlapping the double brushes in the Thomson-Houston machine; by short-circuiting the armature during part of the revolution and separating the double brushes in the Fort Wayne machine. In other words, in the Fort Wayne machine with constant current through the field there are less active conductors in the armature under full load than under no load. The

regulation in the Brush, Schuyler, and Excelsior machines was accomplished by varying the field strength—in the Brush and Schuyler machines by shunting more or less current through a rheostat and in the Excelsior machine by varying the number of turns on the field.

The short-circuit bands around the fields common to both the Thomson-Houston and early Brush machines undoubtedly had a very beneficial effect. We have found other ways of obtaining stable operation as may be seen from the following:

Brush arc generators			
No. of machine or class	7.5	12	13
Volts rating	2500	8500	15000
Amperes	9.6	6.6	4
No. of commutator	3	4	.4
Volts per commtuator section	833	2125	3750
Temperature rise of pole shoe above air for	270° cent	11° cent	30° cent
Material of pole shoe	Cast iron	Laminated	Cast iron
Closed copper band on fields	Yes	No	No
Pounds of copper per kw. output	53.7	29.5	26.2
Efficiency full load	70.5	88.0	85.4

For stability of operation of machines on an arc-lamp load, one of the most essential features is the saturation of the iron. An examination of Fig. 3, curve *C*, shows that there is very little saturation until the curve drops very abruptly. At this point the armature bobbins are commutating under the following pole tip, and the pole shoe becomes very highly saturated just beyond the magnet core. Into some 0.5 in. holes bored in some of the shoes I inserted a thin steel rule; the rule was attracted very strongly to the iron, showing a heavy magnetic field across the hole. The magnetism held the rule so tightly that it could be twisted about 30° in 3-in. lengths. I doubt if there are any machines manufactured to-day in which the saturation of the iron is higher than in the Brush machine.

I desire to add another characteristic curve to the two which are given in Professor Thomson's paper, Fig. 2 and Fig. 3; one showing the effect or characteristic of varying the length of spark on the commutator, entitled "External Characteristic No. 11 Brush Arc Generator." When the generator is running on short-circuit, if the spark length be increased, the current rises; this however, continues to a less and less degree, as the full arc-lamp load of the machine is approached. At the point of about maximum output a variation in the length of spark has very little if any effect upon the current output of the machine. If the machine is still further loaded, the brushes move

backward, and if the spark is lengthened out the current falls off rapidly; in other words, a short distance beyond the point at which changing the length of spark does not affect the output of the generator, is the maximum load of arc lamps which the machine will carry. In addition to this, it is the maximum output which can be obtained from the machine. Either short-circuits or open-circuits will decrease the load and the power required to drive the machine. Generally speaking, arc machines do not operate well at reduced current, and the kilowatt capacity is practically in proportion to the square of the current, and the operation is poor, due to the fact that the iron is not saturated at lower current.

The first 120-lamp 6.6-ampere Brush arc generator was developed in Lynn, worked satisfactorily, and was sold to the Boston Electric Light Co. Shortly after that they installed 18 more machines. When about one-half of them were delivered I was told that they were unsatisfactory. The difficulty in the machines was corrected after extensive tests by changing the material in the pole shoes from steel casting to cast iron, taking 360 lb. of copper from the fields, rewinding the rheostats, and increasing the rating from 8000 to 8500 volts.

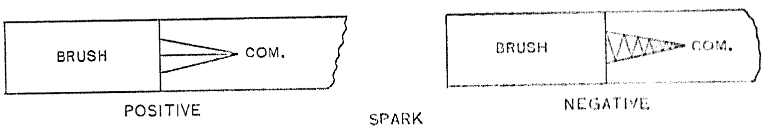


Fig. 1

Later additional changes were made, reducing the amount of copper in the original machine by 636 lb. or more than 25 per cent.

Experiments were conducted at Cleveland and approximately 500 lb. of copper were taken from the fields and the magnet cores were increased from 5.25 in. to 6 in. in diameter. The generator operated satisfactorily on a resistance load, but when the machines were operated on an arc-lamp load they were not satisfactory. They would run satisfactorily to about 5000 volts, but not to 6250 volts, their rated load. Characteristic curves were taken up to about 5000 volts and were satisfactory for the operation of arc lamps, but when the field strength was increased to obtain a higher voltage the peak of the characteristic curve went out to 12 or 13 amperes. Arc lamps could be operated at this current, but it was not what the customer wanted. About two years later further experiments were carried on, and we succeeded in reducing the copper on the fields 636 lb. and the machines worked satisfactorily.

I thought we could reduce the magnetic leakage in the machine and increase the output by removing the sharp

corner from the back ends of the pole shoes by making the radius of the curve about 0.25 in., but the output was reduced 5 per cent or 300 volts. I also conducted further experiments, displacing the pole shoes somewhat, about 2 in., from the center of the magnet cores. I found on these machines that a change in the direction of rotation would give an output of four to one, depending upon whether the long end of the pole shoe was leading or trailing. With the long end in leading position, the machine gave the larger or maximum output. The commutation, however, was not satisfactory over the necessary range. Another peculiar thing in arc generators is that the brushes move backward with increasing load, which is apparently the

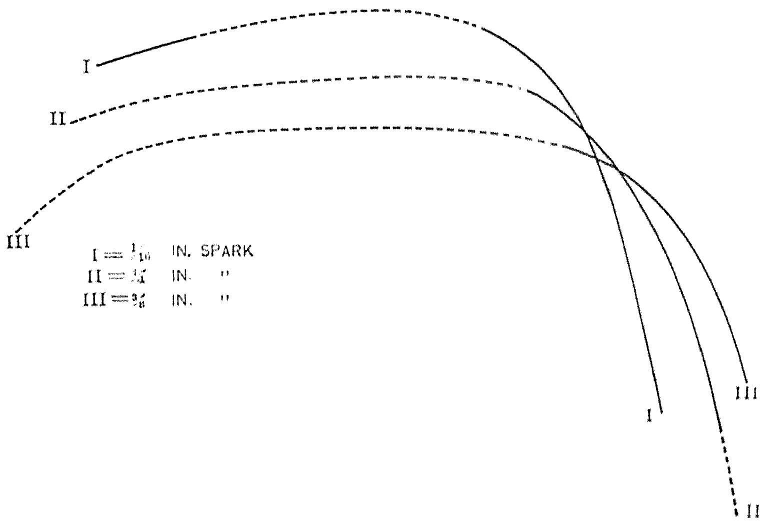


FIG. 2

reverse of constant-potential machines; but in both machines the brushes move forward when the resistance in the load circuit is reduced.

The Brush and also the Thomson-Houston arc generators possess the peculiar property of allowing us to change the direction of rotation of the armature, keeping the direction of the current through the field the same and maintaining the same relative position for the positive and negative brushes on the commutator. This seems wrong, but theoretically it is right. The instantaneous fluctuations in current of the Brush arc generators amount to about half a per cent. I do not know exactly what it is with the Thomson-Houston machine, but I know it is many times greater. The steadying effect and alternating voltage across the fields of the Brush machine are negligible. In the Thomson-Houston machine the steadying effect I believe

to be very appreciable and essential, and if my recollection is right, the alternating voltage across the field amounts to about 75 per cent of the rated direct voltage. This varies somewhat under different conditions of load.

I think the Brush multicircuit arc machine has interfered more with telephone service and done more to improve it than any other machine, by forcing the telephone companies to put in metallic circuits. We have also experienced trouble in operating these machines on underground cables. I should like to bring to the notice of the Institute the use of a transformer on a fluctuating direct Brush circuit for the purpose of preventing surges between the generator and the circuits, which

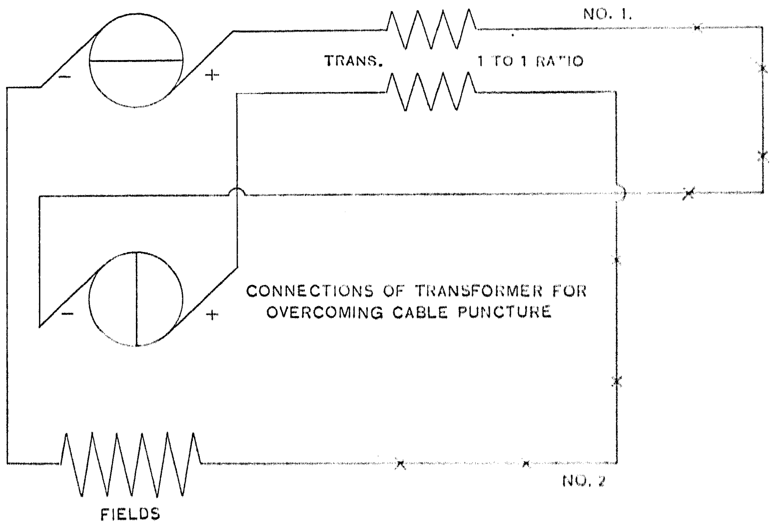


FIG. 3

tend to puncture the insulation on the cables but as yet have had no effect, so far as I have been able to find out, on the generator. This seems strange when a rated 8500-volt machine usually breaks down on about 10,000 volts, alternating current, jumping from the armature bobbins to the pole pieces.

The transformer is so connected that the currents through the windings neutralize each other. If we put capacity in the circuits, with the varying voltage which is generated between different commutator sections, we will have an extra current flowing through the transformer due to the capacity, and this will transfer part of the high electromotive force from one loop or section of the generator circuit over to the other so that

there will be very little alternating voltage between the negative brush in the figure and that end of the transformer winding connected to load in series. On the 12 Brush arc generator rated at 8500 volts, 6.6 amperes, two-circuit machine, the alternating voltage across the transformer is about 800 at full load. The larger the amount of capacity or length of underground cables in the circuit, the greater the alternating voltage across the transformer, so that we will have changed from an unstable to a stable condition by the use of the transformer, and the electrical surges in the circuit which punctured the cables are cut out, but they had no effect so far as we could observe upon the machines or upon the lamps.

The disturbances on the telephone lines, either grounded or common metallic return, were due to the electrostatic effect, or to the wave of voltage which continually traveled along the lighting circuit. They were not due to the current or magnetic inductive effect, which was proved by the fact that telephone disturbances were experienced on multicircuit machines when one of the circuits was short-circuited so that there was no current flowing in it, only the variation in potential existing. The difficulty was overcome by the telephone companies putting in metallic circuits, which should have been done long before, as the service at that time was unsatisfactory, owing to stray currents from electric railway and other circuits.

John B. Taylor: Mr. Green has just referred to fluctuation or pulsation in value of the current, or more properly (since we may have pulsations of widely different frequencies due to various causes) the extent to which the direct-current arc circuit becomes an alternating-current circuit with frequency determined by a number of points of commutation. At first sight, an armature with a three-part commutator might be expected to introduce a considerable alternating-current component, but a rough analysis will show that this alternating component is not very great, and there is also the reactance of the circuit and apparatus tending to smooth out the alternations. A neighboring telephone circuit, however, as already pointed out, gives evidence of the alternating feature, and in addition I was surprised to find a perceptible alternation in illumination.

This I noticed about eight years ago while making some stroboscopic tests on alternating-current motors by the aid of a striped cardboard disc rotating, and illuminated by an alternating-current arc. About the same time some old Thomson-Houston direct-current open arcs were being replaced by 60-cycle enclosed arcs supplied from constant-current transformers; during the process of changing, both systems were in operation at the same time. I rotated the disc on one street corner near a 60-cycle arc for the edification of a friend, and on the next corner advised that no such stroboscopic effect would be apparent on account of the direct-current arc. I was surprised, however,

to find that there was a decided alternation in intensity of illumination as evidenced by the disc appearing stationary at the proper speed, although, of course, the contrast between light and dark stripes was not so definitely marked as with the 60-cycle lamp.

I wish to ask if tests have been made which can be quoted to show how much alternating effect there is in the Thomson-Houston circuit with the machine under average load, and if these alternations are sufficient appreciably to affect figures for efficiency for such machines where the output has been measured by voltmeter and ammeter of the D'Arsonval or Weston type. Such meters give the mean value of current and voltage rather than the root mean square value, and in the extreme case of a rectified sine wave, ammeter and voltmeter individually indicate but approximately 0.9 of the proper value, while output in watts, as determined by their product, would show a little more than 0.8 of the true value. If this has not been taken into account, some values of efficiency on such generators might properly be increased by a small value, possibly 1 or 2 per cent.

There is one feature of stability rather outside the scope of this paper; that is, the stability of illumination as distinguished from stability of current producing the illumination. Incandescent lamps are probably the most stable form of artificial illumination which we have, while the illumination from an arc, in spite of cored-carbon and such devices, still leaves much to be desired.

As Dr. Kennelly has pointed out in his letter, the inherently unstable feature of the arc itself is now being turned to good use in wireless telegraph and telephone service.

I believe that Professor Thomson, as usual, has anticipated many other experimenters in this direction and I hope he can tell us something of his early experiments with the "singing" or high-frequency arc. It seems possible that we may have in some of the underground cable distribution systems for arc lighting a condition not unlike that which is purposely arranged for the singing or oscillating arc; that is, a capacity (made up of cable) around the arc in series with more or less reactance (principally in the lamp mechanism). If both reactance and capacity should chance to have the proper values, and the resistance and other losses were not too great, this combination might build up sufficient voltage to account for some unexplained cases of breakdown on such systems.

H. G. Stott: I would call attention to a very important historical note in the paper, where reference is made to one of the first series arc dynamos. It is as follows:

The first series arc machine designed and built under his supervision was also the first dynamo having the three-coil (star three phase) winding with three-segment commutator.

I believe Professor Thomson can enlighten us very consider-

ably on the point, which was recognized by him and his associates, that this was a three-phase machine, and that power could be transmitted from it and supplied to another machine by putting on what were then called slip-rings. I think from an historical point of view, it would be interesting for Professor Thomson to bring this out when he closes the discussion.

Elihu Thomson: Reference has been made to the use of a partial load of differential lamps. I remember that this was resorted to as a steadying influence in times when unstable circuits were found. It was also noticed that with an increase in line-length on some of the early Thomson-Houston arc machines, the machine would carry more load. A 50-lamp machine with a long line would often carry 55 arc lamps. Undoubtedly in that machine the inductance of the field windings had a good deal to do with the operation of the machine, and this inductance was added to by a long line. It was a machine which when first sent abroad evoked the opinion that it ought not to work—but did work. One must take into account the fact that although at the commutator during regulation there was a definite short-circuit repeated six times during a revolution, the inductance of the windings kept the current from too great fluctuation.

Mr. Rice has adduced several interesting matters. I remember the instance he has told of in which he had to resort to the use of a supersensitive arc to keep the current stable on the circuit. I also remember very well the condition he referred to in which we had too much of a copper band on the field—the field magnetism did not vary enough. It is well known that in the case of machines of the open-coil type, if the field is maintained and any flashing occurs, such flashing becomes permanent; they can not be used as separately-excited or even as shunt-excited machines.

Mr. Stott has asked about the original three-phase machine. When we first applied for a patent on the three-coil machine, it was illustrated in the patent specification with a commutator and also with slip-rings, as alternatives, since the novelty of the structure was well appreciated; but the Patent Office was unwilling to recognize our arguments with relation to the winding with slip-rings, and to save time we finally gave up and took out the patent with claims for the winding with commutator segments, and left out the slip-ring combination. As shown in the specification originally with slip-rings, it was, of course, an alternating three-phase machine. In 1881, in New Britain, we were experimenting with a machine which had slip-rings. We had carried the terminals of the armature to three slip-rings, in addition to the commutator. The object was to control the sparking of the machine by putting condensers between the commutator segments, and to do that we carried slip-ring connections to condensers outside. We found that the trouble was that the condensers would not live, but go off in a flash.

At the same time we noticed high-frequency effects with the condensers, getting sparks between turns of wire leading to the condensers instead of the current going round. The desirability of having another such machine with slip-rings was discussed, as we knew then that we could transmit power without commutation from one machine to the other machine; but as we were not doing power distribution at that time, and no transmissions were then contemplated, we put the matter aside.

In regard to Professor Jackson's statement with relation to the function of the blower, I quite agree with him that the blower would not work unless we had a spark to blow upon. We must have the terminal spark from the brush forward as the segment leaves the brush so as to be able to knock it out of the way with the air blast, and insert the cold air layer. The residual arc was deflected into a new position. The air blast, acting synchronously with the motion of the segment, would drive the tapering end of the spark away down under the segments of the commutator, air insulated as it was, thus allowing plenty of room to put in the cold air. We may perhaps account for the old anomalous so-called Ball unipolar machine by supposing that it was at first made with the four pole-pieces, and that the stability was not then what it should be. By taking off two of the four pole-pieces, thereby lessening the effect of the field, and increasing the armature reaction, stability was probably secured. It then became necessary to speed up the machine to obtain a fair output. The average arc dynamo speed at that time was something like 700 or 800 rev. per min., but this machine ran up to 1600 rev. per min.

Mr. Green referred to the remarkable effect of rounding the corners of the pole-pieces in the Brush arc machine, thus adding evidence to show that it is properly characterized as a pole-piece machine. The question in regard to the fluctuations of current in the Thomson-Houston machine I have virtually answered by saying that the field-coil circuit itself acts as a high inductance and tends to smooth out fluctuations. There would otherwise be a very ragged current curve. In fact an interrupted current would exist if it were not for the large inductance of the field coils. I may mention that when the closed copper band was put on the field, the current was considerably more fluctuating than when it was not used, as would naturally be expected. The current fluctuation was greater with partial load than with full load, from the fact that during regulation there was a longer interval of short-circuiting with lighter loads.

Mr. Taylor mentioned stability of illumination. That is a little outside of the paper, and it would take some time to discuss it. I do not think we have time to consider the high frequency arc matter now, and its application to wireless telegraphy.

E. A. Sperry (by letter): I was so fortunate as to have been born almost under the eaves of Cornell and was a youth when our late lamented founder, Professor William A. Anthony, was in the

midst of his early activities at that institution. It will be remembered that he was the first one to build a Gramme electrical generator in this country, the machine exhibited at the Centennial Exhibition in Philadelphia in 1876. I watched some of the construction of this machine with eager interest, as well as many of the experiments that were tried with it, and some of the overhauling in 1878. I feel that we never have accorded sufficient credit to Professor Anthony. He was a source of inspiration to a great many engineers of the present generation, combining as he did in a most extraordinary degree keen perception in theory as well as wonderful mechanical ability and unusual skill as a pure mechanician.

In 1879, I had built my first dynamo and arc lamp in Cortland, New York, near Ithaca. Then in Syracuse, I completed a 15-lamp machine which in 1880 was taken to Chicago, where we built a plant and started manufacturing a system of arc lighting, which included a regulator by means of which the lamps could be turned on and off. It was shown that a 20-lamp circuit could be cut down to five, and later we found means for reducing a 30-lamp machine to one lamp.

The early machines were all of the short-arc type. We started to build the long silent arc of about 9 amperes in 1884. As I remember, one of the first 6-ampere machines was sent to Cornell for testing purposes. After we were established in Chicago, there came upon the horizon a new lamp, about which I had already quite thoroughly informed myself. The current was supplied by a Thomson-Houston arc-light machine. The first night the lamp started I was present as an observer. I thought at that time the lamp ought to work; there was a case, it seemed to me, of a pulsating current, in which the current supplying the lamp was forced along the line in impulses which were positively divorced from each other, and, through the vigorous shaking and vibrating which the regulator of the arc was constantly undergoing, no wonder the lamp could operate well and feed continuously. With our finely divided commutator and smooth flowing current, however, it was not an easy matter to cause a continuous and uniform feed of the carbons. Before many years the Thomson-Houston system became formidable, and we found great difficulty in maintaining our standard in competition with it. This system was developed by Professor Thomson.

The tests at Cornell in 1885 showed us that the armature reaction and the drooping characteristic of the machine were the secrets of stability and regulation in our machines. But in those days it was a matter very largely of cut and try methods.

About that time I carefully constructed an ammeter and sent it to Charles F. Brush at Cleveland, asking him kindly to mark 9.6 amperes on the scale. Correct measuring instruments were difficult to obtain and even more difficult to keep in calibration.

Chas. P. Steinmetz: The interesting phenomena of instability of electric circuits, discussed by Professor Thomson, have an importance far beyond the arc circuit. Owing to the work of Professor Thomson and other investigators, the arc circuit is the first and therefore the best known type of unstable circuit; but instability occurs frequently also in other electrical and mechanical systems, and the study of the conditions leading to instability thus is one of the most important subjects of electrical engineering. Thus we find that synchronous and induction motors drop out of step, or we find such motors starting from rest, but failing to run up to their proper speed; we meet with surging of synchronous apparatus, and find electrical apparatus misbehaving, apparently without reason, at the end of very long transmission lines.

In all these phenomena, the same characteristic is the cause of instability, which was investigated so ably by Professor Thomson in the early days of the arc circuit. This characteristic is: if the effect of a cause is in opposition to the cause, the system is stable; if however the effect assists the cause, it becomes accumulative, and instability results. When dealing with energy, as a corollary to the law of conservation of energy it follows that the effect always opposes the cause, and thereby limits itself. When not dealing with the question of energy, however, the effect may assist the cause and thus give instability. The typical case is that of an arc in a constant-potential circuit. The voltage consumed by the arc decreases with increase of current. The current depends on the resultant e.m.f., that is, the difference between impressed e.m.f. and the voltage consumed by the arc. An increase of current causes a decrease of the arc voltage and thereby an increase of the resultant e.m.f., hence a further increase of current etc.; that is, instability: either a short-circuit, or extinguishing of the arc.

The cause of this phenomenon is the volt-ampere characteristic of the arc, shown in Fig. 1, as *I*, which decreases with increase of current. Assuming now a metallic resistance r inserted in series with the arc, the voltage consumed by r is shown by the straight line *II* in Fig. 1, and the total voltage consumed by arc and series resistance is curve III. This curve III has a minimum point for a certain critical value of current i_0: above this value, the conditions are stable; that is, an increase of current requires an increase of voltage, and thereby limits itself. Two values of current, i_0 and i_2 thus exist at the same supply voltage, of which the upper one, i_2, gives stability, while at the lower one, i_0, the arc is unstable, and either goes out, or the current increases, until the upper value of current i_2, and thereby stability, is reached.

Similar relations between the arc volts and amperes are frequently met in electrical engineering, for instance in the relation between speed and torque of the induction motor. The speed-torque curve of a polyphase induction motor is shown

as D in Fig. 2, with the speed as abscissas, the torque as ordinates. Assuming such an induction motor operating on a load requiring a constant torque, as when driving a reciprocating pump

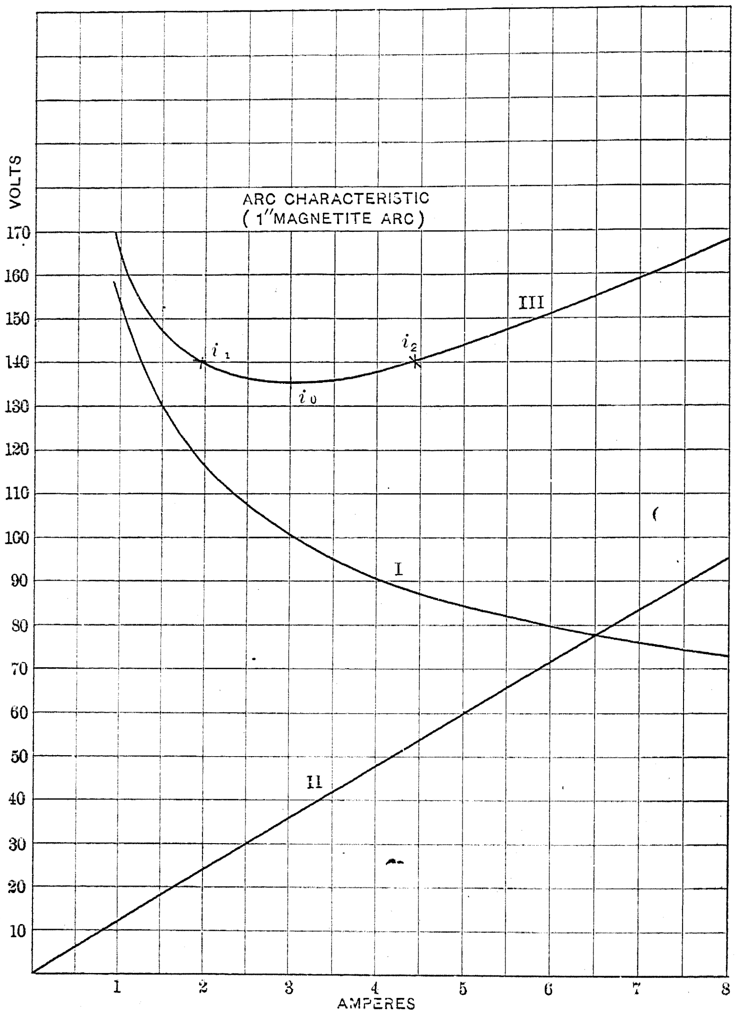


FIG. 1

working against a constant head of water, then in the range between synchronism and the maximum torque point m, the speed of the motor is stable. Thus at the load L, the speed c is 94 per cent of synchronism, and any momentary increase of

speed decreases the torque, a momentary decrease of speed increases the torque, and thereby limits itself. The same torque is reached again at a lower speed *d*, of 55% of synchronism. This speed, and the entire range from standstill *t* to the maximum torque point *m*, is unstable; that is, at the point *d*, the motor either drops down to standstill, or runs up to the stable speed *c*, but can not continue to revolve at *d*, on a load requiring constant torque *L*.

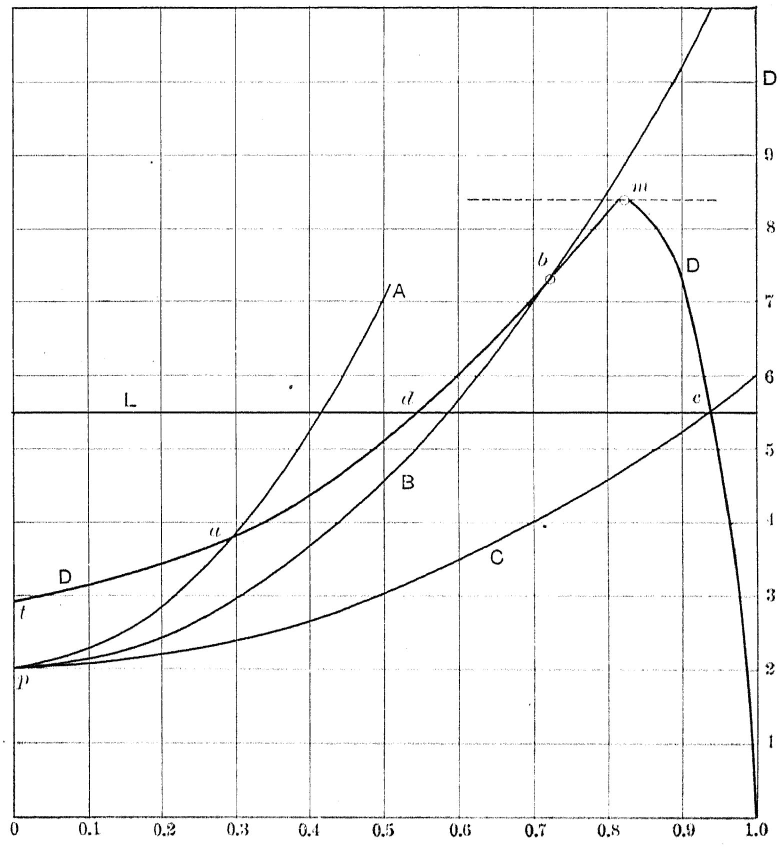


Fig 2

The conditions of stability or instability of induction-motor speed, as discussed here, apply only to the case of a load requiring a constant torque, just as the stability consideration of the electric arc applies only to the case of constant voltage supply. The question of stability of electric and mechanical systems therefore involves two conditions: the circuit or system, and the external conditions under which it is stable or unstable;

that is, in the present instance, the volt-ampere characteristic of the electric arc, and the condition of constant voltage supply; or the speed-torque characteristic of the induction motor, and the condition of constant torque of the load.

Assume for instance a load requiring a torque proportional to the square of the speed, as when driving a centrifugal pump, or a ship propeller. In this case, if the required starting torque p (Fig. 2) is less than the starting torque t of the motor, the motor always starts and runs up to the speed corresponding to the load: a high speed c for moderate loads C, a lower speed b for

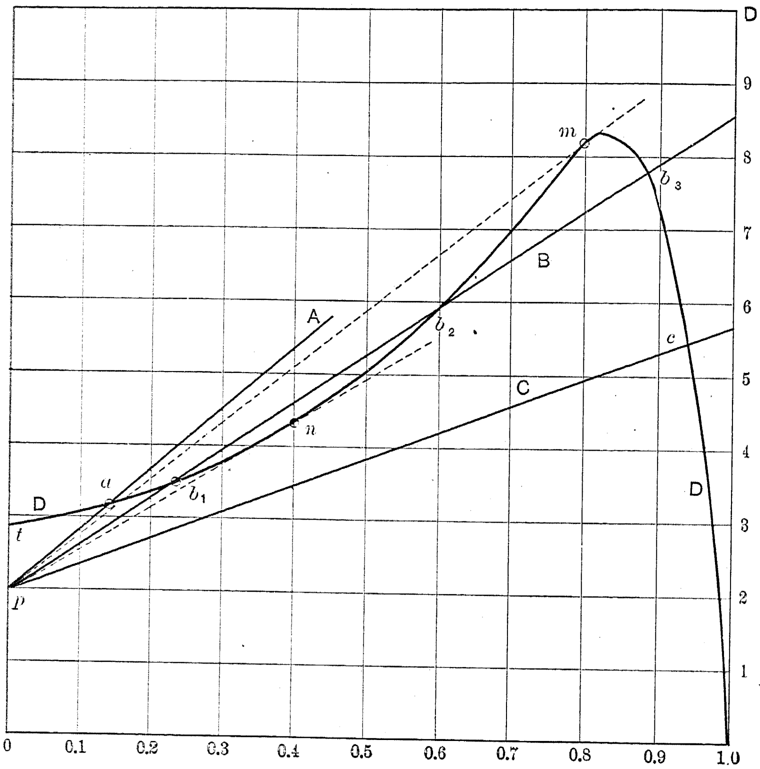


FIG. 3

overloads B, and a low speed a for excessive loads A; but the speed-torque relation is always stable, that is, with this character of load the induction-motor-torque curve has no unstable branch.

Still more interesting are the speed-torque relations of the induction motor on a load requiring a torque proportional to the speed, as when driving an electric generator at constant field excitation and loaded by a constant resistance, as shown in Fig. 3. The motor always starts from rest. At moderate load,

the speed runs up to near synchronism, on a straight line C. At excessive overloads, the motor still starts, but runs up to a low speed a only, and keeps revolving in stable condition at this low speed; that is, in a range of the motor curve, which on a load requiring constant torque is unstable. Especially interesting is the case of an intermediary load, represented by line B in Fig. 3. B intersects the motor curve D at three points, b_1, b_2, b_3; that is, three different speeds exist, at which the motor gives the torque required by this load, at 24 per cent, 62 per cent and 88 per cent of synchronism respectively. Of these, the intermediate speed b_2 is unstable, and so is the entire range of the motor curve between the points of contact n and m of the two tangents from p on curve D, while the low and the high speed are both stable. That is, if started from rest, on this load, the motor runs up to speed b_1 and stays there. If the load is taken off and the motor is allowed to accelerate near synchronism, and the load then put on again, the motor slows down to speed b_3 and stays there. At speed b_2 however, the motor can not continue to run, but either slows down to speed b_1, or speeds up to speed b_3.

We have here a case where a motor can carry the same load at two different speeds, a high speed, with moderate current—the proper running speed—and a low speed, with excessive current. In this case, the motor operates satisfactorily at speed, it always starts, but it does not run up to its proper speed, and shows a "dead point", or rather a "dead range" at intermediary speed. Phenomena of this character are frequently met with in single-phase induction motors, polyphase synchronous motors etc., though more or less complicated by secondary actions, as the effect of the motor-starting device, the magnetic locking of stator and rotor teeth, the induction machine action of the synchronous machine, etc.

Still a different form of instability occasionally occurs in induction motors. On constant voltage supply and at a moderate drop of speed, frequently far below the maximum torque point, the motor suddenly drops out of step without previous warning. This phenomenon depends upon the relative momentum of the induction motor and its load and of the generating system, and on the rapidity of voltage regulation of the generating system, and may become marked with a sluggish system of voltage regulation.

Of special interest also are different forms of instability which lead to surging of synchronous machines, as the surging due to electromechanical resonance, which is frequently eliminated by lowering the motor excitation, that is, running with lagging current, while the surging due to the limited power supply of the generating system is intensified by lowering the motor field excitation, that is, running with lagging currents, and may disappear with over-excitation of the motor and operation with leading current. The study of these phenomena is of the highest importance, as apparently the same phenomena may

be produced by entirely different conditions, and the remedy in one case thus may intensify the troubles in another case which resulted from different conditions. However, the subject is too vast to be dealt with in a short discussion, but may form a separate paper.

An interesting feature of the Brush arc machine is the great constancy of current independent of speed fluctuation: even speed variation of 50 per cent or over with such machines does not appreciably vary the current, but merely reduces the maximum voltage of the machine and thereby its output in proportion to the speed reduction. It is startling to observe in such a machine, at light load, when shutting it down, how perfectly constant the current remains even down to speeds of a very small percentage of normal and until the very moment when the machine stops.

Another type of arc machine, a constant alternating-current generator, deriving its constancy of current from armature self-induction, was once introduced into the industry, the Stanley constant-current alternator. This was afterwards modified by the addition of a rectifying commutator into a constant direct-current arc machine, but disappeared after some years, apparently owing to the unsatisfactory character of the arc lamps operated from it. It would therefore be of interest to hear something of this development of by-gone days.

Similar to the interesting case of the induction motor, above discussed, conductors which can carry the same load at three different speeds are occasionally met with. Thus for instance there are conductors among the so-called " pyro-electrolytes ", which at the same constant-supply voltage can pass three different currents. The volt-ampere characteristic of one such conductor, a magnetite chromite alloy, is shown in Fig. 4. As seen in Fig. 4, at the same voltage, as 20 volts, the current can have three different values $i_1 = 0.45$; $i_2 = 5.0$; $i_3 = 36$ amperes.; and of these three values, the low one, i_1, and the high one, i_3, are stable, the middle one, i_2, with the entire range of the curve, between maximum m and minimum n, is unstable.

However, even the volt-ampere characteristic of the arc is not always the curve shown in Fig. 1, but may be considerably modified.

This curve in Fig. 1 is fairly well represented by the equation:

$$(e - a)^2\, i = \text{const.}$$

but applies only to the arc at constant vapor pressure, as the arc in air. A different volt-ampere characteristic results, if the vapor pressure in the space in which the arc is produced varies with the current, as for instance is the case with a vacuum arc, as the mercury arc. Thus the volt-ampere characteristic of the mercury arc in stationary conditions is of the character shown in Fig. 5, that is, has a range of minimum voltage at intermediate currents, while for smaller currents as well as for larger

currents the voltage increases. Especially interesting is the latter range for larger currents, where an increase of current requires an increase of voltage. From our previous considerations, such a characteristic should give stability on constant-potential supply; that is, for currents above the minimum voltage range, the mercury arc should be capable of operating steadily on constant-potential supply, without any series resistance, by any variation of current limiting itself. This however is not the case, but the mercury arc in a vacuum is unstable on constant potential over the entire range, just as any other arc is, and for the same reason that an increase of current causes a momentary decrease of voltage and thereby becomes

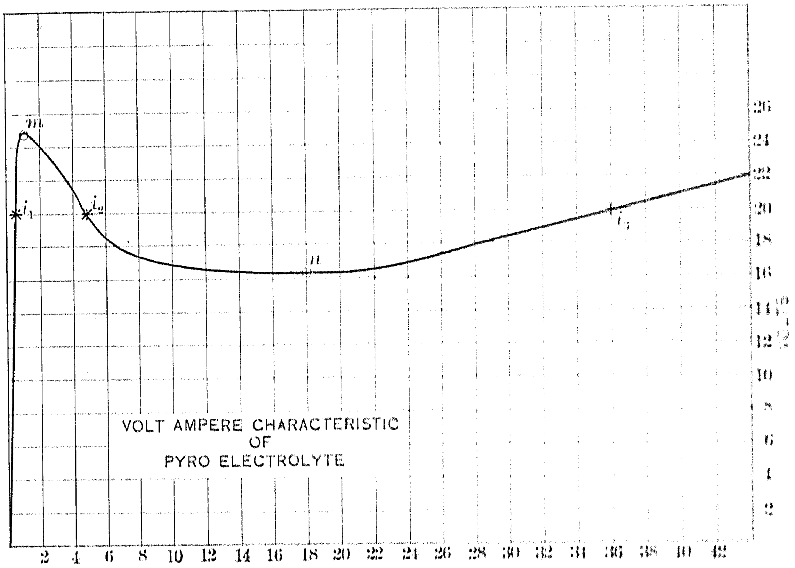


FIG. 4

accumulative. Thus, in the mercury arc of Fig. 5, an increase of current from 5 amperes to 7 amperes raises the voltage consumed by the arc from 33.2 to 36.6. In the first moment, however, after raising the current from 5 to 7 amperes, the voltage drops, as indicated in Fig. 5 in dotted lines, to about 30 volts for a short time, a few seconds, but still long enough to be observed by the voltmeter, and then gradually, during some minutes, increases to its stationary value of volts. This leads to the distinction between the volt-ampere characteristic of the arc, shown in Fig. 5, and the momentary volt-ampere characteristic, and it is the latter which determines stability or instability. The latter is always dropping, and thus the arc is always unstable on constant potential.

In reality, the period of voltage drop with increase of current is preceded by an extremely short period of voltage rise; that is, with an increase of current in the mercury arc, the voltage first rises, then drops, and then gradually rises to the final value, about as shown in Fig. 6. The first period of high voltage lasts for a thousandth of a second or less, but may be observed under favorable conditions by the oscillograph. It is the conduction through the vapor stream of previously existing lower current. With a velocity, possibly of some thousand feet-seconds, the conducting vapor stream of the arc adjusts itself to the change of current and hence drops in resistance. The arc voltage thus decreases approximately by the above equation

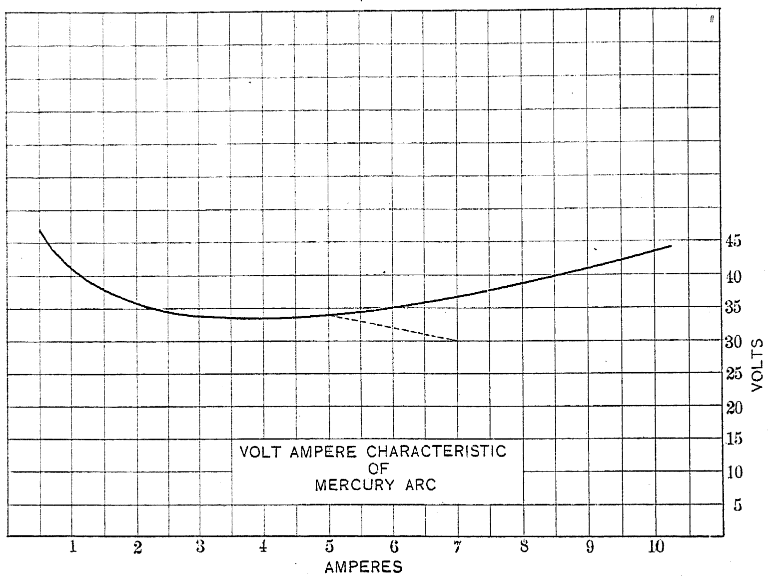


FIG. 5

$(e - a)^2 i$ = constant. The vapor pressure in the space rises gradually and the resistance of the arc stream thereby increases and thus also the voltage. With an arc in enclosed space, the second period of low voltage thus lasts a few seconds only, while with arcs at constant pressure, as in air, it is the permanent condition, and the third period is never reached.

The initial period of very short duration, where the arc conductor has not yet reached the stationary condition corresponding to the current flow, can be observed indirectly by superimposing on a direct-current arc a small high-frequency current and gradually increasing the frequency of the latter. Ultimately then a frequency is reached where the power-factor of the superimposed alternating current is unity; that is, the arc stream acts

as ohmic resistance, or in other words, the variations of current are faster than they can be followed by the vapor stream of the arc. This transient period of the arc is of very great industrial importance. Extending back the arc characteristic Fig. 1, it must intersect the zero line of current at the disruptive voltage;

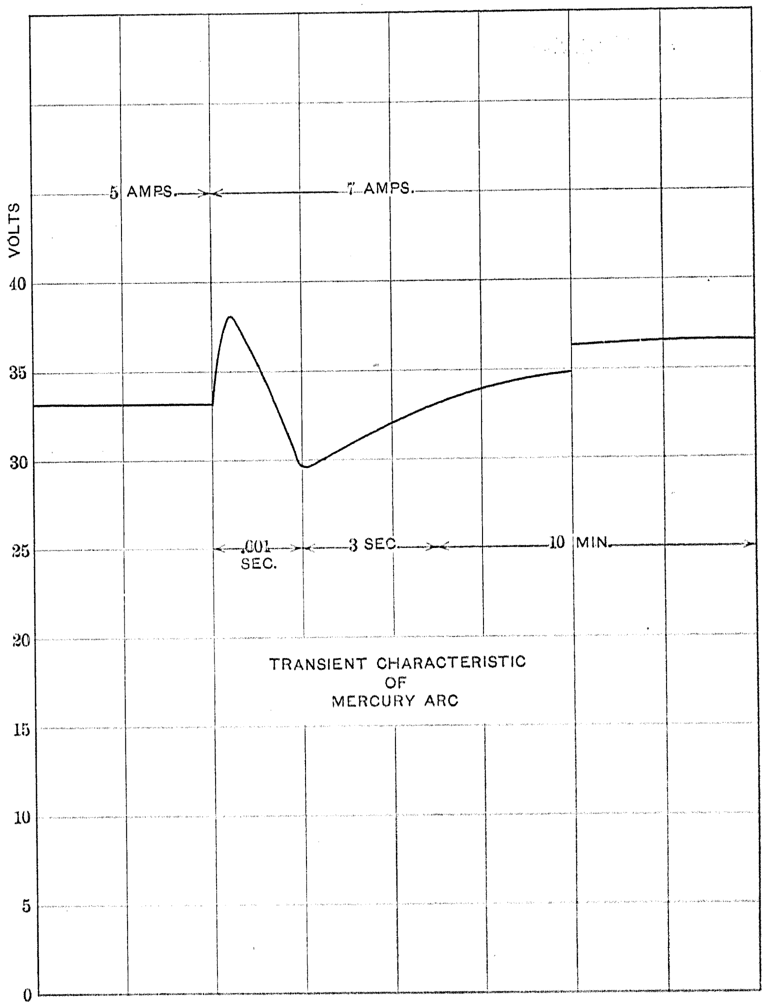


Fig. 6

that is, the voltage required to jump a spark across the terminals. If then the arc is started by a spark, the volt-ampere characteristic traverses a curve starting with the disruptive voltage at the moment of the passage of the spark, and rapidly decreasing in voltage with increase of current. This transient

volt-ampere characteristic however is not the curve in Fig. 1—which is reproduced as *I* in Fig. 7—but is higher; the more so the more rapidly the current increases, as shown by II and III in Fig. 7. In consequence thereof, if the high voltage lasts only a very short time, that is, decreases very rapidly, the voltage may fall below that required by the transient arc characteristic, and

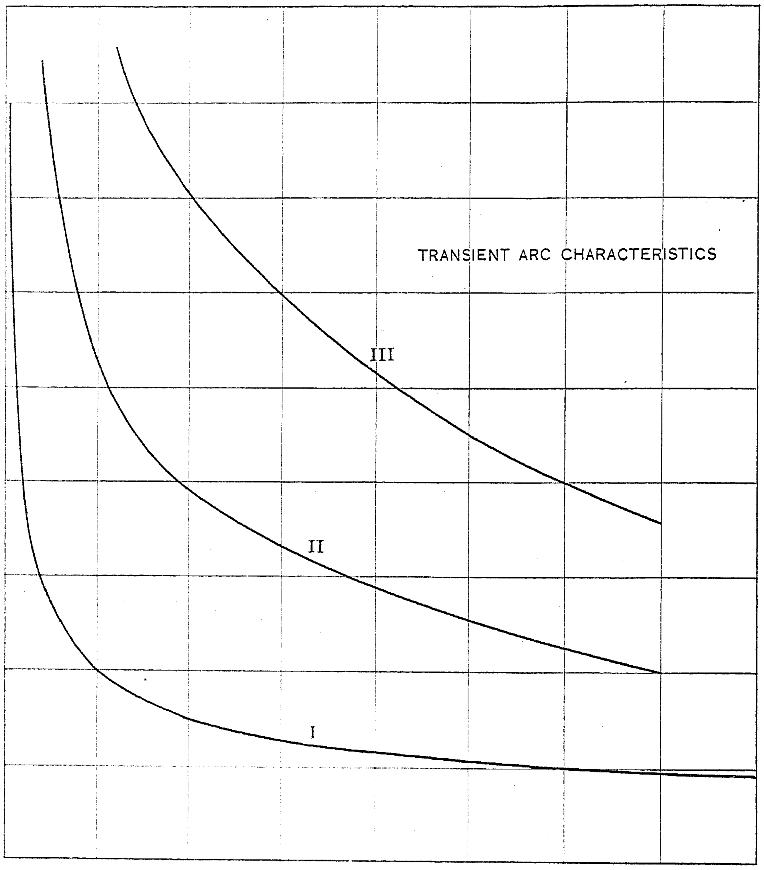


Fig. 7

the arc goes out, or in other words, will not be established by the electrostatic spark, even if the voltage between the terminals is sufficiently high to maintain an arc when once established. This is the explanation of the phenomenon that a static spark between terminals is not always followed by an arc, even if the voltage at the terminals is sufficiently high to maintain an arc when once established. Thus, while it requires about 100 volts

to maintain an arc between terminals one inch apart, with 2000 volts potential difference between these terminals, static sparks may be jumped across these terminals, without any arc following if the duration of the voltage which sends the spark across the terminals, is less than the time required for the transient arc characteristic in Fig. 7 to drop to 2000 volts. To start an arc disruptively, it therefore is not sufficient to have an electrostatic spark, but the spark voltage must be maintained sufficiently long to establish the arc; that is, supply the power required to produce the arc conductor at least so far that the maintained voltage between the terminals can hold the arc. How much power this is, depends on the permanent voltage between the terminals: the lower the permanent voltage, the greater must be the energy of a spark for starting an arc, and while a very powerful spark discharge may start an arc between terminals of moderate voltage, sparks of very high frequency and therefore very small power are incapable of starting an arc even at high voltage between the arc terminals. Thus, with sparks of such low power, insulation does not break down instantly, but numerous successive sparks are required before an arc follows. This is the reason why the effects of high voltages are so much more severe, if considerable power is back of the voltage, than are the powerless discharges of static machines.

This phenomenon of the time element required to establish the arc, is and has been for many years industrially used in the multigap lightning-arrester. This lightning-arrester consists of a series of spark-gaps between line and ground, over which high voltage discharges, but the arc can not be established during the short duration of the static discharge. It also explains the occasional failure of the earlier types of such lightning arresters, the number of spark-gaps being such that the machine voltage could maintain an arc across them, if the arc be once established, and the operation is based on not allowing the arc to be established. If however an exceptionally heavy discharge passes, the energy of the static discharge may be sufficient to establish the arc, and then the lightning-arrester burns up; for this reason this multigap type of lightning-arrester has recently been modified by adding various shunting resistances of different discharge capacity, so as to divert and destroy the stability of the arc.

In testing apparatus with high voltage, this phenomenon of low power static sparks must be carefully guarded against. Because of the distributed capacity of the high-potential circuit, oscillations of very high frequency are produced, which, superimposed on the permanent alternating testing voltage, cause sparks to jump across the testing spark-gap long before the permanent voltage has reached the required values, and thus long before the specified insulation strain has been reached. In using a spark-gap in high-voltage testing, it is therefore necessary to insert resistances in series or in shunt to eliminate such

oscillations. A discharge of the spark-gap indicates the existence of the required test voltage, only if the spark is followed by an arc, but not otherwise. Thus I have seen sparks jump a one-inch gap, with the static voltmeter indicating only 10,000 to 12,000 volts, and by raising the voltage of the step-up transformer the voltage indicated by the static voltmeter remained the same, but merely the frequency of the sparks increased, without however an arc following. As soon as an arc follows, the static voltmeter instantly drops to zero: short-circuit.

Elihu Thomson. It is gratifying that the paper served to get Dr. Steinmetz on his feet. We always learn a great deal from his discussions. May I suggest that it would be a capital idea for him to present a paper on other forms of instability. He has given half a promise that he would do this.

I agree with Dr. Steinmetz that it is a large subject and one can approach it from various sides. I have dealt with the most decided example of instability, the arc circuit, where all the conditions tend to instability, and effective provision has to be made to avoid instability. The other conditions Dr. Steinmetz has alluded to in the discussion are of course in similar lines, very suggestive indeed, and I think should be followed out. We should have a good exposition, such as I know he can give us, of the different forms of instability depending not only on the current, but instability of torque and speed, etc. There is, in fact, material here for several good papers.

Following Dr. Steinmetz we see the reason why it has been very difficult to work arcs of very small current value. They must exist if at all on the very steep part of the characteristic curve, and require high voltage and potent influences to secure stablilty.

Dr. Steinmetz has pointed out the fact that there is an instantaneous effect before settling down, when a change is made in the current strength in an arc. He shows that this effect is in fact in an opposite direction to that which is the final result, and without doubt has indicated the true reason of the effect, in the time it takes for the shift in the volume of the arc stream or hot gases to take place. On an increase of current a slight excess energy must be absorbed while the arc is expanding, and this can only occur with a momentary increase of opposition measured as a resistance.

A paper presented at the Cleveland Section of the American Institute of Electrical Engineers, February 8, 1909.

THE PURCHASE OF FUEL ON A BRITISHTHERMAL UNIT BASIS[1]

BY LAWRENCE P. CRECELIUS

The boiler plant of the traction company which operates the street-car system of Cleveland, Ohio, is representative of modern practice in both the nature of the equipment installed and the means employed in handling and firing coal. Of the total cost of manufacturing a kilowatt-hour in this plant, exclusive of investment charges, taxes and insurance, the fuel item amounts substantially to 70 per cent. The calorific value of the fuel used is obviously of the utmost importance, and to insure a supply of coal having a high thermal value is therefore greatly to be desired. This may be accomplished by making use of one of three means which are at our command:

First, to locate the power plant at the mine where good coal lies; second, to bind by contract the supply from certain districts or mines producing coal of known heat value; and third, to purchase coal on a calorific value basis.

The heat efficiency of the power plant referred to in this paper is approximately eight per cent; this means that out of every 100 tons of coal burned, the energy of but eight tons is delivered to the switchboard in the form of electricity. Probably the greatest factor entering into the cost of coal in the United States is the cost of transportation, and as this could be entirely eliminated by locating the power plant at the mine this plan seems attractive. Most unfortunately, however, this is impossible as yet, not so much on account of the geographical

1. The author acknowledges with thanks the work of Messrs R. W. Emerson and E. H. Haynes in calculating and experimenting to ascertain the values contained in this paper.

limitations as of physical conditions. The second plan, which probably is in general use, and no doubt will continue for some time to be employed, was the method by which coal was purchased for this power plant. This plan, however, proved unsuccessful, mainly owing to the fact that both goo dand poor coals exist in the same district or mine, and also because stock coal could be sent in on this contract.

The method of purchasing coal on the basis of calorific value has finally been adopted and with very satisfactory results. Payment is made only according to the B.t.u. contained in the coal, and suitable restrictions are imposed on the maximum amount of ash and sulphur which the coal may contain.

Ohio bituminous slack, of a heat value of 12,500 B.t.u. and with 15 per cent ash and 3.5 per cent sulphur per pound of dry coal, has been specified in the contract because of its relatively low cost, although, at the time when this contract was drawn up it was necessary to take local conditions into consideration to such an extent that the table was made flexible enough to make it possible to supply coal from other districts on the same table. It may be interesting to note that while the purchasing of fuel on a calorific value basis has been attended with good results, it was prompted by a different cause than that which brought about this practice in New York City. There, the predominating influence which effected its introduction, as the writer understands, was to restrict the volatile matter contained in the coal, on account of the stringent smoke regulations.

Here, upon the threshold of two great coal-producing districts, with a difference in freight rate from them to this city and with a considerable difference in the price of the coal from the two districts, it occurred to the writer that some severe restrictions must be placed upon the quality of coal sent in. The city of Cleveland has in effect a heat-value contract under which is purchased the slack required by the Water Department. The heat value of the coal is placed rather high and the coal is required to be up to or above this standard when the contract price is paid. But for coal running less in heat value than that called for by the contract, a deduction in price is made in proportion to the amount less than the standard. Aside from this, however, no information was at hand or available, as far as the writer knows, covering the purchase of slack on a heat-value basis. The case of the Interborough Rapid Transit

Company of New York and several others within the writer's knowledge covered a very high grade of fuel, and, therefore, a good many allowances had to be made in the nature of restrictions which should be imposed upon the amount of fixed carbon, volatile matter and moisture contained in the coal. In lieu of this the company abandoned the idea of placing a premium upon a certain minimum of ash in the coal.

Early experience with the grade of fuel to be supplied under the heat-value contract had shown that coal which runs high in heat value is usually low in ash, sulphur and moisture, so it became only necessary to limit the coal to the maximum permissible amount of ash and sulphur; subsequent experience has confirmed this hypothesis.

A point worthy of being mentioned here is that the standard in the contract has been made rather low. This was done in order to get a low basic contract price, for when it is understood that premiums and penalties are of an accelerating rate on either side of the standard, and based on the contract price, it may be better understood. As a matter of fact it was expected to pay slightly more than the prevailing contract price of slack as a guaranty of good coal, and in the light of the contrast with the quality of coal received under the old contract in force prior to the inauguration of the purchase of coal on the heat-value basis, this policy has been justified.

Another point of interest, and one which has a very marked influence on the success or failure of this kind of contract, is the personality of the coal dealer. Only the most responsible parties should be dealt with.

There are available in the Cleveland market in extensive quantities two principal grades of bituminous slack, one coming from the Western section of Pennsylvania, the best of which is known as Youghiogheny gas slack, and the other from the Eastern part of Ohio.

For a period of about two months before the contract was put into effect, the company made a series of evaporative tests to determine the relative steam-making qualities of Ohio bituminous slack and Youghiogheny gas slack. These tests clearly indicated that in the furnaces as constructed, and with the draft available, both Youghiogheny slack and Ohio slack can be burned with equally good results. Average chemical properties of the two kinds of slack, per pound of dry coal, are as follows:

	Youghiogheny gas slack.	Ohio bituminous slack.
B.t.u.	13,185	12,614
Ash	11.6%	13.8%
Sulphur	2.03%	3.53%
Volatile combustible matter	31.95%	36.62%
Fixed carbon	53.52%	45.55%

The relative evaporation per pound of coal as fired was 7 per cent in favor of the Youghiogheny gas slack. The inherent moisture contained in the coal in the first case is 1.52 per cent and in the second case 2.7 per cent.

Basis upon which payments are to be made. In order to illustrate the work of sampling shipments of coal received, and to describe how adjustments of any disputes which may arise between the contracting parties are made, the section of the contract relating to these matters is herewith reproduced. This general method was devised and put into effect by the Interborough Rapid Transit Company of New York, a few years ago, and the essential features, modified to suit local conditions, have been adopted by the traction company.

Each day's consignment of coal furnished to each power plant by the contractor during the continuation of the contract, is sampled by the superintendent or his authorized agent and analyzed to determine its heat value, and the price paid by the company, per ton, for each car of coal is based on a table of heat values in excess or deficiency of the standard therein contained, but subject to further deductions for ash and sulphur.

A small quantity of coal is taken from at least five different places in each car received, by driving into the coal a five-foot ram (Fig. 1), before the car is unloaded. The quantities thus received from each car of coal of the day's consignment are thrown into a receptacle provided for this purpose, and thoroughly mixed, and a properly selected sample of this mixture is taken for chemical analysis. One-half of the sample of the average mixture is labeled and held at the company's laboratory for a period of two weeks after unloading the cars. The other half of the sample taken from the average mixture is analyzed as soon as possible after being taken. No other sample is recognized.

Tests of the sample taken from the average mixture are made by the company's chemist under the supervision of the superintendent. Should the contractor question the results of the company's test, a copy of which is mailed to him, the company will, if requested within three days after the copy of test has

been mailed, forward the other half of the sample, held for this purpose, to any laboratory in the city of Cleveland which may be agreed upon by the superintendent and the contractor, and have the said sample analyzed by it, and the results obtained from this second test shall be considered as conclusive and final. In case the disputed values, as obtained in the company's test, shall be found by the second test to be two per cent or less in error, then the cost of the second test shall be borne by the contractor; but if the disputed values shall be found to be more than two per cent in error then the cost of the second test is to be borne by the company.

Should there be no question raised by the contractor within the three days specified as to the accuracy of the company's

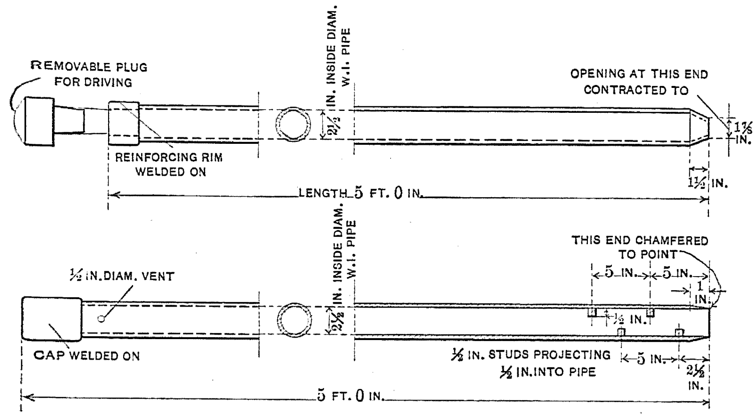


FIG. 1—Slack sampling rams at the Viaduct and Cedar avenue power stations

analysis, the second sample of coal is destroyed at the end of two weeks from the date of unloading the cars from which it was taken. Should a second test be made of the sample of the average mixture, as provided, any penalties to be made will be based on the results as obtained from the second test.

Penalties for excess ash and sulphur. Coal which is shown by analysis to contain less than 15 per cent of ash and 3.5 per cent of sulphur is accepted without a deduction on that account. Where the analysis gives amounts of any neutral elements in excess of these quantities, deductions are made from the basic contract price in accordance with a schedule provided, independent of the corrections made for departures from the standard B.t.u. value.

It will be noticed that each day's consignment of coal delivered

to each power plant is analyzed, instead of each individual carload of coal, although a sample is taken from each car. The object of this plan is to obtain an average of the whole day's supply in order to allow the dealer the benefit of cars of coal running low in ash and sulphur offsetting those running high.

PREMIUMS AND PENALTIES.

For coal in any car which is found by analysis to contain per pound of dry coal

13,975 and above	35 cents per ton above standard
13,850 to 13,974, inclusive—	35 " " " " "
13,725 " 13,849 "	30 " " " " "
13,600 " 13,724 "	25 " " " " "
13,475 " 13,599 "	21 " " " " "
13,350 " 13,474 "	17 " " " " "
13,225 " 13,349 "	13 " " " " "
13,100 " 13,224 "	10 " " " " "
12,975 " 13,099 "	7 " " " " "
12,850 " 12,974 "	5 " " " " "
12,725 " 12,849 "	3 " " " " "
12,600 " 12,724 "	1.5 " " " " "
12,475 " 12,599 "	Standard
12,350 " 12,474 "	1.5 " " " below "
12,225 " 12,349 "	3 " " " " "
12,100 " 12,224 "	5 " " " " "
11,975 " 12,099 "	7 " " " " "
11,850 " 11,974 "	10 " " " " "
11,725 " 11,849 "	13 " " " " "
11,600 " 11,724 "	17 " " " " "
11,475 " 11,599 "	21 " " " " "
11,350 " 11,474 "	25 " " " " "
11,225 " 11,349 "	30 " " " " "
11,100 " 11,224 "	35 " " " " "
10,975 " 11,099 "	40 " " " " "
10,874 " 10,874 "	45 " " " " "
10,725 " 10,849 "	50 " " " " "
10,600 " 10,724 "	55 " " " " "
10,475 " 10,599 "	60 " " " " "
10,350 " 10,474 "	65 " " " " "
10,225 " 10,349 "	70 " " " " "
10,100 " 10,224 "	75 " " " " "
9,975 " 10,099 "	80 " " " " "

9,975 and below, an additional 10 cents per ton per car will be deducted for each one per cent variation below standard.

The plan also offers the advantage to the company that, on account of having a fewer number of samples to analyze, the chemist is not hurried in his work and can therefore devote more attention to the preparation of samples and to the checking of his determinations.

The table of allowances referred to in this section of the contract is so proportioned as to incite the dealer to supply Ohio bituminous slack of a value ranging from 12,500 B.t.u., the standard in the contract, to 13,125 B.t.u., 5 per cent above the standard; this range gives him the best profit obtainable. In order to guard against the supply of fuel having a high B.t.u. value, but which, by reason of its chemical composition, is not

Deductions on Ash.	
00.0 to 15%, inclusive—	Standard
15.1 " 16 "	3 cents below standard, per ton
16.1 " 17 "	6 " " " " "
17.1 " 18 "	9 " " " " "
18.1 " 19 "	12 " " " " "
19.1 " 20 "	15.5 " " " " "
20.1 " 21 "	19 " " " " "
21.1 " 22 "	22 " " " " "
22.1 " 23 "	25 " " " " "
23.1 " 24 "	28 " " " " "
24.1 " 25 "	31 " " " " "
25.1 " 26 "	34 " " " " "
26.1 " 27 "	37 " " " " "
27.1 " 28 "	40 " " " " "
28.1 " 29 "	43 " " " " "
29.1 " 30 "	46.5 " " " " "

Deductions on Sulphur.	
0.0 to 3.9%, inclusive—	Standard
4.0 " 4.4 "	2 cents per ton below standard
4.5 " 4.9 "	5 " " " " "
5.0 " 5.4 "	7 " " " " "
5.5 " 5.9 "	9 " " " " "
6.0 " 6.4 "	12 " " " " "
6.5 " 6.9 "	14 " " " " "
7.0 " 7.4 "	16 " " " " "
7.5 " 7.9 "	19 " " " " "
8.0 " 8.4 "	21 " " " " "
8.5 " 8.9 "	23 " " " " "
9.0 " 9.4 "	26 " " " " "
9.5 " 9.9 "	28 " " " " "
10.0 " 10.4 "	30 " " " " "

well adapted to the stoker equipment, a limit has been set above which no further premiums are paid.

To guard against a supply of low-grade cheap coal or slack which has been kept in stock, the B.t.u. value table is so arranged that the value can fall but 10 per cent below standard, where by reason of the addition of penalties due to ash and sulphur on this sort of fuel, the price is cut to such an extent that only the freight charges are realized.

Almost immediately after the contract went into effect, a marked improvement in the cost efficiency of the entire plant took place, accompanied by an improvement of some eight per cent in the consumption of coal per kilowatt hour.

In conclusion it may be said that the installation of the scheme of purchasing fuel on a heat-value basis has been justified.

Taking samples. In the preceding paragraphs reference has been made to a ram by means of which samples are taken from the car. The efficiency of the ram when used for this purpose is indicated by the table below.

	Sample taken by hand at regular intervals while car was being unloaded.	Sample taken from top of car.	Sample taken by means of ram.
Wt. of sample	600 lbs.	20 lbs.	20 lbs.
B.t.u.	12,843	12,918	12,777
Ash	10.96%	10.4%	11.3%
Sulphur	2.51%	2.53%	2.56%

A sample obtained in instalments during unloading, taken with great care and diligence, may be said to be truly representative of the entire contents of the car. It was, however, soon found to be unsatisfactory in practice as the men, when unloading cars, unless closely watched, would take enough coal at once, in order to get through with the job as soon as possible.

The sample obtained by ram is shown to be very satisfactory, and owing to the fact that the human element involved is cut to a minimum, fairly accurate samples can be easily and quickly secured right after the cars are placed by the railroad company.

Description of analytic work in laboratory. After the coal in the cars has been sampled and ground up into coarse particles, it is brought to the laboratory for analysis. About 2 pounds of the sample from each car are thoroughly mixed with a large spatula, in order to obtain a fair average of the whole day's shipment. Slack coal and run-of-mine are not mixed, however, but treated as separate samples. The mixed coal is now quartered down until about 0.5 pound of the original amount is left. This part is again mixed, spread out in a thin layer and about 20 to 25 grams of it removed from various points with a small spatula, to be used as the sample for analysis. The remainder is put away in an air-tight sample can, properly labeled for future reference. The smaller part

is ground repeatedly with a muller and bucking board until the whole will pass through a 100-mesh sieve. The pulverizing and sifting thoroughly mixes the sample, which is put into the heater for one hour at a temperature of 100 degrees cent. The heater consists of a copper oven surrounded by a water jacket, in which the water is kept at a boiling temperature by a bunsen burner. The coal, now dried out, is ready for analysis. Sulphur, ash and British thermal units are the constituents usually determined—although moisture and volatile combustible matter are sometimes required for boiler and stoker tests.

Sulphur determination. For the sulphur, 0.5 gram of the dry sample is weighed. Two determinations are carried on simultaneously on each sample, in order to have a check on the results. The 0.5 gram of coal is put into a 20-gram platinum crucible half-filled with Eschka mixture, then stirred together, covered with a little more of the mixture and placed over a bunsen flame turned very low for fifteen minutes. Eschka mixture consists of two parts (by weight) of calcined magnesium oxide, (MgO) and one part (by weight) of dried sodium carbonate, (Na_2CO_3). At the end of 15 minutes the flame is gradually turned up to full power, and the mixture heated for 45 minutes, meanwhile being stirred several times. After the whole has been fused and the coal completely burned out, the contents are turned into a clean beaker, that part of the fusion adhering to the crucible being washed out by means of distilled water and a rubber-tipped stirring rod. 50 c.c. of distilled water are added to the beaker, the whole allowed to boil for a minute on the hot plate, then filtered through a single filter (Munktell 9 c.m.), the precipitate washed with distilled water, 15 c.c. of 1 to 1 hydrochloric acid (HCl) of specific gram, 1.19 added to the filtrate, the solution boiled for a minute and stirred, then 10 c.c. of saturated barium chloride solution ($Ba\ Cl_2$) added, and the resulting precipitate of barium sulphate ($Ba\ SO_4$) allowed to settle for two hours or longer, while the beaker rests on the hot plate. When settled, the precipitate is filtered off, (using double filter papers), and washed three times with hot distilled water. The filters are then folded around the precipitate, both placed in a clean platinum crucible, and heated over the bunsen flame until burned out. This takes about half an hour. From the weight of precipitate found is subtracted the weight of $Ba\ SO_4$ obtained as the average of two or three blanks previously run on the Eschka mixture. The corrected weight of sulphate is doubled, (as but 0.5 gram of sample was taken), multiplied by 0.1373—the sulphur factor—

and then by 100 to obtain the percentage of sulphur in the coal sample. A sulphur table for half-gram weights is then furnished for reference, thus saving time in figuring results. With six burners and six crucibles at hand, six determinations on three samples of coal are carried on at one time.

Ash determination. When the fused mass in the foregoing sulphur determination has been removed from the crucible, the latter is thoroughly washed out, and in it is placed a gram of the dried coal, accurately weighed for the ash determination. This consists in merely allowing the coal to completely burn out over a full bunsen flame. Sometimes the sample is heated to constant weight, but if burned for two or three hours with frequent and careful stirring, the extra weighing is unnecessary. From the weight of residue found, the percentage of ash in the sample is calculated in the usual way. A half-gram of sample may be used instead of the whole gram, though any error is doubled in this way. Two determinations are made at a time on one sample, and they should check within 0.2 per cent.

While the samples are being heated in the crucibles for sulphur and ash, the B. t. u. determinations are made.

British thermal unit determination. This is accomplished with a Parr calorimeter, which measures the rise in temperature of 2 liters of distilled water, due to the heat absorbed from the combustion of a fixed amount of coal sample with an oxydizing agent. This combustion takes place in a metal bomb, which is immersed in the water, and caused to rotate at about 100 rev. per min. by a small motor. The coal and oxydizing agent are made to combine by passing a current from a 40-ampere-hour storage battery, giving 6 volts, through an electric fuse in the bomb mixture. The fuse wire is of german silver, No. 36, which melts on passage of the current and starts combustion of the coal. The water used in the calorimeter for the heat absorption is protected from outside heat influences by two fibre tubs with air spaces between.

To determine the heat units in a sample of coal, the bomb is first taken apart, all the parts washed and thoroughly dried, and the fuse-wire fastened to the inside terminals in the form of a loop. 0.5 gram of potassium chlorate ($K\,Cl\,O_3$) and 0.5 gram of coal sample are weighed and placed in the bomb. To this is added a measure (10 grams) of dry sodium peroxide ($Na_2\,O_2$). The bomb is now put together, screwed tight with a wrench, well shaken, the water veins adjusted, then immersed in the 2 liters of water, and made to rotate slowly to secure

uniform temperature by the circulation of the water. After about three minutes, the temperature of the water is read from a Fahrenheit scale thermometer inserted through the cover of the calorimeter. The thermometer is calibrated in twentieths of degrees, but the readings are estimated, with the aid of a magnifying glass, to hundredths. After the mercury column has become stationary, the charge in the bomb is exploded, and the thermometer carefully watched, as the mercury rises, until the maximum temperature is reached. The difference between the first reading of the thermometer and the last reading is the rise due to the heat absorbed by the water.

A rise of 0.014 degrees fahr. is due to the heat of the fuse wire used, and a further rise of 0.171 degrees fahr. is caused by the accelerator ($K\,Cl\,O_3$). The sum of these two (0.185) is subtracted from the total rise in degrees, and the remainder is multipled by 3117 to obtain the number of heat units in the sample.

The number of heat units is the number of grams or pounds of water that 1 gram or 1 lb., respectively, of coal sample during combustion, will cause to rise 1 degree fahr. in temperature.

The above factor, 3117, is obtained in this way: the 2000 c.c. of distilled water used in the calorimeter weighs 2000 grams; the metal bomb and metal containing-cylinder and pivot together are the equivalent of 135 grams of water on the basis of heat absorption. 2000 grams plus 135 grams gives 2135 grams. This number multiplied by the rise in degrees would give the number of gram calories in fahr. degrees given off when 0.5 gram of coal was burned. Therefore, twice this amount represents the calories of heat from 1 gram of coal. It has been experimentally determined that when coal, $K\,Cl\,O_3$, and $Na_2\,O_2$ are exploded, only 73 per cent of the heat evolved is due to the combustion of the coal. $2135 \times 2 \times 0.73 = 3117$, which represents the number of gram calories due to a rise of 1 degree.

After each explosion, the bomb is washed out, dried, and wired for the next charge. The 2 liters of water used should be about 4 degrees lower than the temperature of the room, and fresh distilled water is used for each explosion. Two determinations for heat units are always made on each sample, and should agree closely. If the rise in temperature in one case varies more than 0.02 of a degree with the rise on another explosion of the same sample, (corresponding with 62 heat units) then successive determinations are made on this coal until a check or a good average is obtained. A calorific table does away with the calculation and results are obtained directly.

Moisture determination. To find the moisture content the coal in question is first air-dried, a gram of it is weighed, placed in the heater at 104 degrees cent. for one hour and weighed again. The loss in weight divided by the weight of air-dried coal gives the percentage of moisture.

Volatile combustible matter determination. For finding the percentage of volatile combustible matter in coal, a.1 gram sample is placed in a clean weighed platinum crucible, the cover is put on tightly, and the coal heated over the strongest heat of a bunsen burner for three and one-half minutes. At the end of this time the bunsen flame is removed and a blast lamp put under the crucible, care being taken to have a flame below the crucible while making the change, so the contents will not cool. The coal is heated by the blast lamp for three and one-half minutes more, the crucible is allowed to cool in a desiccator and weighed. From the loss in weight from heating, the weight of moisture found is subtracted, and the difference divided by the weight of the sample taken to find the percentage of volatile combustible matter. This difference also contains one-half the sulphur, and this value, found in another determination, is subtracted, leaving the per cent of volatile combustible matter.

Fixed carbon determination. Remove the cover and burn off the remaining carbon over a bunsen burner until nothing remains but the ash—the loss is fixed carbon with the remainder of the sulphur.

Coke and ashes are analyzed in much the same way that coal is, except that tests for sulphur and moisture are not required. In determining the heat units, however, it is difficult to get an explosion with the ordinary chemical, so the use of benzoic acid is resorted to. 0.5 gram of $K\,Cl\,O_3$, 0.25 gram of coke or ash sample, 0.25 gram of benzoic acid ($C_6H_5\,COOH$), and 10 grams of Na_2O_2 are used for each charge. A correction must be made not only for the wire and accelerator, but also for the benzoic acid. This correction is found by running two or three blanks of 0.25 gram $K\,Cl\,O_3$, 0.25 gram acid and 10 grams Na_2O_2 for each lot of chemicals and taking the average. From the total rise in degrees after the explosion of coke or ash sample, the blank correction is first subtracted, and the remainder multipled by 6234 (2×3117, as but 0.25 gram of sample instead of 0.50 gram was used) to obtain the number of heat units. For the determination of ash in coke or ashes, but 0.50 gram is taken, as the combustion is slower with these substances.

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PRIME MOVERS

BY CHARLES P. STEINMETZ

I.

Electric energy is not a primary energy; that is, it is not found in nature nor directly producible to any appreciable extent from the stores of energy available in nature—water power and the energy of fuel. To become available for conversion into electric power, the energy found in nature must first be converted into mechanical rotation by some form of prime mover. The engineering characteristics of these converting apparatus may be classed under two main groups, those referring to economy and reliability, respectively. In both, the electric machine, whether generator or motor, ranges very high: its efficiency is virtually unity; its size, first cost, and maintenance small; its reliability great. In the cost of electric power the electric machine plays only a subordinate part; the essential element in determining the cost and the reliability of electric power is the prime mover; that is, the intermediary step between nature's stores of energy and the dynamo shaft.

The cost of electric power consists of three parts.

A. The fixed cost or permanent cost; that is, the cost depending on the size of the station, but not on the amount of power supplied by it.

B. The proportionate cost; that is, cost proportional to the amount of power delivered.

C. The reliability insurance; that is, the additional cost required to assure the desired reliability or continuity of service.

A. Fixed Cost

1. *Interest on the investment in the plant*. This factor varies very greatly with the form of the available energy. It is frequently

very large with water-power, owing to the hydraulic development required—dams, pipe-lines, land to be flooded for water storage, power stations and prime movers, transmission lines, etc.

It also depends on the prevailing rate of interest and on local conditions: whether a market is waiting for the power and capital for the development is easily obtained; whether a market is within reach and can be developed; or whether a market has first to be created.

2. *Depreciation of the plant.* This factor is different for different parts of the plant. For instance it is very low for buildings and for line copper; it is high for apparatus such as railway motors, which by the nature of their service operate on a narrow margin.

In considering depreciation, the " useful life " of the apparatus must be recognized. It may often be much shorter than the total life. The ratio of the useful life to the total life varies with different apparatus. Electrical machines and prime movers and other parts of the plant, for instance, may still be in condition to give many years' service, but nevertheless their further use is uneconomical, and their useful life is thus ended. This condition may be due to the advance of the art, which makes other types of apparatus so much more economical that the cost of the difference in economy properly capitalized exceeds the cost of reconstruction of the plant. It may be due to the fact that with the increase of the market the size of units initially chosen may have become too small, or the space-economy too low, or the voltage too low for economy. Furthermore, the cost of attendance or of maintenance may be so much lower with the more modern apparatus as to make it uneconomical to retain the existing plant.

Just as it was realized many years ago that with the incandescent lamp there was a difference between the total life and the useful life, that after a definite period it did not pay to maintain incandescent lamps in service, even if still operative, so with all engineering apparatus a similar " smashing point " exists, a point where economy requires renewal of the plant, even if still operative. At this point the useful life of the old plant is ended; it should be thrown out with whatever salvage is feasible—either by use as reserve plant, or to take care of peak loads, or to be operated elsewhere when space economy limited the useful life. In considering depreciation, the useful life of the apparatus should thus be considered.

3. That *part of the operating expense* which does not depend on the use of the plant, as superintendence, some supervision, etc., such repairs as maintenance of dams and hydraulic development, etc.

B. Proportionate Cost

1. This comprises the *cost of energy* and *accessories* to its conversion—fuel, condensing water and lubricating oil in thermodynamic engines, water (where a charge is made for the water power) and oil in hydraulic plants, etc.

2. *Labor and attendance*, including that part of supervision which varies with the power utilized.

3. *Maintenance, repair and depreciation* of the plant, as far as they depend on the use of the apparatus; for instance, brush renewals, commutator repairs, railway motor depreciation, etc.

C. Reliability Insurance

1. This comprises the *overload capacity* of apparatus in power, voltage etc., provided to take care of emergencies.

2. *Duplication of parts* of the plant, as reserve units, duplication of exciter plant, of lines, and feeders, etc.

3. *Additional plant*, as steam reserve with water-power plants, storage-battery reserve, tie lines with other systems, etc.

Some of the items of the cost may belong partly in one, partly in the other class; depending on local conditions, it may properly be chargeable to either one or another subdivision. For instance, the cost of developing water storage would belong under *A*, if the power capacity of the plant depends on it, while it would be chargeable to reliability insurance *C*, if provided to guard against failure of the power in unusually dry seasons.

The relative weights of *A* and *B* depend largely on the *load-factor*; that is, the ratio of the average to the maximum power. With a poor load-factor, item *A* is far more important. It recedes in importance with the improvement in the load-factor.

Attention is drawn to the recognition of Class *C*, the reliability insurance, as a distinct and essential part of the cost of power, separate from *A* and *B*. Item *C* again consists in part of the character of fixed cost, and in part of the character of proportionate cost. Nowadays, when the lay public is interested in the cost of electric power, and is comparing plants which may be very different in regard to reliability of service (as supply from a water-power over a single transmission line

without steam reserve compared with a city steam station with storage-battery reserve), it appears desirable to recognize the insurance of continuity of service as a separate part of the cost of electric power.

The great difficulty met with in discussing reliability of service is the absence of an established standard; for what one engineer may consider as perfectly satisfactory service, may be considered as entirely unsatisfactory by another engineer.

A classification of electric power supply regarding its reliability may be made by the number of shutdowns per year. Four classes of shutdowns may be distinguished, by the time of their duration:

a. Less than one second; that is, less than the time which would throw synchronous apparatus out of step. A few seconds' failure of power in the supply of a synchronous motor or converter means re-starting and synchronizing, and thus for the power users depending on the synchronous machine represents a very much longer shutdown.

b. Less than 20 minutes; that is, the time in which synchronous apparatus in a well organized plant can be put back into service. In lighting circuits, while the failure of the lights is annoying, it is not usually serious, and the loss of time by failure of the power supply is moderate.

c. Less than three hours; that is, the time sufficient to start anew a steam plant, replace or repair damaged apparatus as transformers, repair lines, etc., and a time which, if the shutdown occurs only rarely, in general does not warrant the installation of a separate system of light and power supply, as gas or a private steam plant.

d. More than three hours. This represents a complete breakdown of the system. If this is likely to occur, it requires the provision of private lighting or power-plant service wherever power is used from the electric supply system.

In estimating the reliability of service the shutdown of a part of the system would be considered as a part of a shutdown, in proportion to the connected load. Thus a shutdown of a section comprising $1/n$ of the total connected load of the system would count as $1/n$ of a shutdown. This method of counting gives the average number of shutdowns per customer.

Voltage variation beyond the limits permissible in good service—about 2% in lighting, and 10% in power supply—would be considered as a partial shutdown, about in the following manner:

A voltage variation of 2 per cent to 10 per cent as one-tenth of a shutdown in a lighting system, but does not count in a power-supply system.

A voltage variation of 10 per cent to 50 per cent as one-half of a shutdown.

A voltage variation of over 50 per cent counts as a complete shutdown.

It appears reasonable to omit from consideration any shutdown made for purposes of change, repair, etc., provided the users have been notified beforehand.

By the yearly number of shutdowns, an electric power supply would be classified as:

1. First-class service:

$a < 4$ per year; $b < 1$ per year; c and d absent.

2. Good service:

$a < 12$ per year; $b < 4$ per year; $c < 1$ per year; d absent.

3. Second-class service:

$a < 52$ per year; $b < 12$ per year; $c < 4$ per year; $d < 1$ per year.

4. Third-class service:

$b < 52$ per year; $c < 12$ per year; $d < 4$ per year.

5. Unsatisfactory service, suitable only as auxiliary power, etc.

Some such schedule of classes of service appears urgently needed, to give a meaning to statements and discussions on character and reliability of service.

The above proposition is probably the best that can be expected at present, without putting too large a majority of the electric systems into the second and third classes. It must be realized, however, that the standard set by it is far lower than that maintained in most gas or water-supply systems, that what is here called good electric service would be entirely unsatisfactory in a gas service. However, this lower standard is permissible, owing to the far greater safety of electric power, in which a temporary interruption of the supply is not liable to such disastrous results as would be the case with gas service.

II.

In discussing the features of prime movers it is necessary to distinguish between those which are inherent in the type of apparatus—as the dependence on meteorological conditions in the hydraulic plant, the high temperature in the gas engine—and those which are incidental. The incidental features are due to the particular form of design, and therefore defects from this

cause are usually temporary. While a defect in the prime mover may be just as serious to the operation of the plant, even if not inherent in the type of apparatus, in a comparative discussion such features cannot be given the same weight, as they are not permanent. They are eliminated with the advance of the art, or avoided when recognized; as, for instance, the difficulties found with the convection of superheated steam, due to the considerable expansion and contraction of piping caused by the high temperature differences.

Most of the features of prime movers pertain either to economy or to reliability.

A. Economy

1. *Power economy or efficiency.* With a thermodynamic engine the total efficiency is the product of mechanical efficiency, thermodynamic efficiency, and producer efficiency. The thermodynamic efficiency is the ratio of the available energy in the engine to the total heat energy supplied to it. The mechanical efficiency is the ratio of the mechanical output at the engine shaft to the available energy in the engine. The producer efficiency is the ratio of the total energy of combustion of the fuel to the heat energy supplied to the engine; it thus comprises furnace efficiency, boiler efficiency, superheater efficiency in a steam engine, or producer efficiency in a gas-producer plant, etc.

The thermodynamic efficiency depends essentially on the temperature range utilized by the engine, and to a lesser extent on the thermodynamic cycle used by it. It increases with increase of the temperature range.

The mechanical efficiency depends on the cylinder volume per kilowatt output, on the temperature range, the pressure and the pressure differences, the momentum of the moving parts, their velocity and the nature of the velocity, whether reciprocating or rotating, etc. In general it decreases with increasing size and mass per kilowatt, with increasing speed, (especially when reciprocating) and with increasing temperature and pressure differences. It thereby depends largely on the available energy per unit volume and per unit weight of the working fluid.

Consideration of the producer efficiency opens such a wide field as to be beyond the scope of this paper.

As the cost of the energy is only a part of the proportionate cost of power, the importance of the efficiency varies with the proportion which this part of the cost bears to the total cost.

Efficiency thus depends upon numerous considerations—the size of plant, load-factor, etc. In general, high engine efficiency increases in importance with increasing size of plant, and with increasing load-factor, and in such plants becomes the most important factor. In small plants with a poor load-factor it may drop into secondary rank, compared with the fixed cost A, and the cost of maintenance and repair, especially if in the design of the plant the mistake is made of giving too much attention to high efficiency, and thereby too little to reliability and low maintenance.

2. *Space economy.* This factor depends essentially on the engineering skill and judgment in the design of the plant, and on the type of the prime mover. It affects the fixed cost of power, and varies greatly in importance; for instance, space economy in the huge plants of our large cities is of very great importance, while for smaller country plants, or plants in locations where unlimited space is available, it becomes of very little importance.

3. *Investment economy.* This depends largely on the available market, and the desired quality of the power. It affects the choice of the prime mover. With an assured market, requiring a high grade of service, the large investment of a first-class modern plant with the best type of prime movers and ample reserve is most economical, while in many of the earlier railway plants the conditions were such that single-cylinder, non-condensing engines of the cheapest type gave the most economical and occasionally the only feasible arrangement. But even in the large first-class plants, in those parts of the plant that are only occasionally called upon for service, that is, which have a very low load-factor, good judgment may suggest a cheaper and correspondingly less efficient type of apparatus than would be economical in the engines which are continuously in service. Such a reserve and emergency plant thus offers a means of utilizing machinery which in the main plant has finished its useful life. To some extent this applies to the thermodynamic reserve of a water-power plant, especially where it is only rarely called into service, and economy in first cost and in space and simplicity may far outweigh high efficiency.

4. *Labor economy, as determined by the amount and character of attendance required.* With a high price and poor quality of skilled labor, prime movers which require a higher grade of attendance become less economical than less efficient prime

movers which can be operated by unskilled labor, or require only very little skilled labor, and especially in our country, the reduction of the amount of high-grade skilled labor is economically necessary. It is this feature which has forced the designers of prime movers and electrical apparatus to devote their attention chiefly to make the apparatus as "fool-proof" as possible, even if efficiency and other characteristics have to be sacrificed to some extent. This fact has to a considerable extent determined the trend of development of the industry, so that types of apparatus which have proved entirely satisfactory in other countries were a failure here, until they had been re-designed to meet the requirements of being operated by the available class of labor. A characteristic instance of the effect of price and character of labor is the general introduction of the enclosed arc lamp in this country, while abroad the open arc is universally used and the enclosed arc has never been accepted, due to its lower efficiency. As a result, the experience gained abroad with different types of prime movers and other apparatus is not directly applicable in our country, and conversely.

5. *Maintenance, repair, and depreciation.* This depends largely upon the type of prime mover, and in thermodynamic engines on the cycle used in the engine. Higher temperature differences, greater cylinder volume and piston pressure, heavier moving (especially reciprocating) masses, higher speeds (especially of reciprocating masses) – in short, greater mechanical and temperature stresses in general, other things being equal, tend to greater maintenance and repair cost and more rapid depreciation. Thus this item is to some extent proportional to the mechanical efficiency, and inversely proportional to the thermodynamic efficiency of the engine cycle.

The economic item of depreciation depends, however, not only on the wear of the apparatus, but also on the limitation of its useful life. In this respect it must be kept in view that with apparatus which is in a state of rapid development, as the steam turbine or the gas engine, the useful life should be expected to be shorter than with apparatus in a field in which no great development occurs. With a rapid advance in the development the time soon arrives where older apparatus is so far left behind as to make it more economical to replace it. This has been well illustrated by the history of the electrical industry. It would be an argument against the use of newer

types of apparatus, and thus antagonistic to progress, if it were not for the fact that in the types of apparatus with which the new development competes, the useful life is shortened in the same or probably still greater degree, as either an extra rapid advance in their design must occur to meet the competition, or they would be replaced by newer types. Thus the useful life of the steam turbine, due to its rapid advance, must necessarily be shorter than the life of the steam engine would be if there were no steam turbine or gas engine; but owing to the existence of steam turbines and gas engines, the useful life of the steam engine, as limited by the advance of the art, is correspondingly shortened, as it faces displacement by steam turbine or gas engine or also has to advance more rapidly.

It will be seen, therefore, that in the economic characteristics of thermodynamic prime movers in general, efficiency and the other features are to some extent mutually conflicting. Engineering judgment must therefore decide on the relative weights of the different economic factors which enter into the choice of the prime mover in the individual case under consideration.

B. Reliability

Features, which are of importance for the reliability, that is, continuity of service, are:

1. The absence or presence of external influences beyond the control of the operating force, as meteorological effects, etc.

2. The design of the plant. This is an engineering problem outside of the scope of this paper. It is obvious, however, that an error in the design can rarely be remedied afterwards; therefore, before the plant is constructed and before the buildings are designed, the arrangement of prime movers and electrical apparatus should be designed to afford the greatest safety of operation. Frequently this has not been done. For instance, the architect of the buildings has overlooked the fact that the switchboards and controlling devices of our modern large, high-voltage electric plants require considerable space, and such apparatus—on which the safety of the system depends—has been crowded together in an altogether insufficient space, to the great detriment of the reliability of operation of the system. A marked improvement has taken place in this direction in the last few years.

3. The probable frequency of shutdown of the prime mover, and the liability of involving other machines by it, and the number of other units and reserve plant available. To some

extent this depends on the mechanical and temperature stresses in the prime mover; it is related to the economic item of maintenance, repair, and depreciation.

4. Rapidity of starting of apparatus, and getting them into service, to cope with such emergencies as a shutdown of the system or a part of it, as a breakdown of a transmission line or feeder, an accident to one or several units, etc., or even an unexpectedly rapid increase of load. Rapidity of reaction to changes of load; that is, speed regulation and voltage regulation, etc. These are important features in their effect on the reliability of service. They depend to a considerable extent on the type of prime movers and other apparatus, but to a still greater extent on the preparedness for such emergencies, and the organization of the operating force for it.

III.

The suitability of the different types of prime mover for the various economic and other requirements of electric service, as discussed above, are best seen by considering their characteristic features.

The available types of prime mover are:

A. The hydraulic turbine. The impulse type, the reaction type, and their various combinations.

B. The steam engine. Condensing and non-condensing, with and without superheat.

The steam turbine. Condensing and non-condensing, with and without superheat.

The gas engine. Using the available fuel directly, as natural gas, blast furnace gas or liquid fuel, and

The gas engine. With producer plant converting solid fuel into gas.

The gas turbine does not yet exist, but as it undoubtedly will arrive sometime, some consideration is given to it in the following.

A. Hydraulic Prime Movers.

The most prominent characteristic of hydraulic power is its dependence on meteorological conditions, dry seasons, freshets, ice in the water supply; and mechanical and electrical interference with the transmission lines connecting the power with its market. As a result the reliability of hydraulic power is inherently far below that of steam power. To give the same class of electric service, therefore, the reliability charge, in the form of duplica-

tion of plant, of steam reserve, etc., is far higher than with a steam plant close to its market. In this respect climatic conditions naturally exert a great effect.

The fixed cost is usually very high, due to the expensive hydraulic development required. It is especially high if water storage has to be provided; but this also depends largely on the local conditions of the water-power.

The proportionate cost, is, however, very low, owing to the absence of cost of fuel. This is the redeeming feature of the hydraulic plant, which gives it an advantage over steam power. Nevertheless, even with an extremely low proportionate cost, the high fixed cost and the high cost of reliability insurance greatly reduce the difference in the cost of steam power and of water-power of the same quality. Under very unfavorable conditions, it may even make the water-power more expensive than the steam power.

Owing to the high fixed cost and low proportionate cost, water-power depends on a good load-factor still more than does steam power, to such an extent that it may occasionally be economical to oversell the water-power, thereby keep constant load on it, and take care of peak loads, etc., by auxiliary steam plant.

B. Thermodynamic Prime Movers.

The thermodynamic efficiency is essentially a function of the temperature difference, on which the engine operates. The mechanical efficiency, maintenance cost, and reliability depend largely on the mechanical and temperature stresses in the prime movers, the available energy per unit weight and unit volume of the working fluid—steam or air—and on the speed of the engine.

To compare the different types of prime movers in this respect to determine how far certain features, as high mechanical stresses, low space economy, high thermodynamic efficiency, etc., are inherent in the type of prime mover, requires a study of their thermodynamic cycles.

In Fig. 1 is plotted, with the temperature in degrees centigrade as abscissas, the available energy in kilojoules per kilogram of the working medium: steam or air, for the three typical air cycles, and for steam used non-condensing and condensing, in the latter case with a condenser pressure $p = 0.1$ kg. per cm^2, or about 27 in. vacuum; that is, about the average between that of the steam turbine and that of the reciprocating engine. The

corresponding pressures are shown in dotted lines, and the thermodynamic efficiencies in the upper part of Fig. 1. Figs. 2 to 6 give some typical cycles, and Tables I and II the constants of these cycles.*

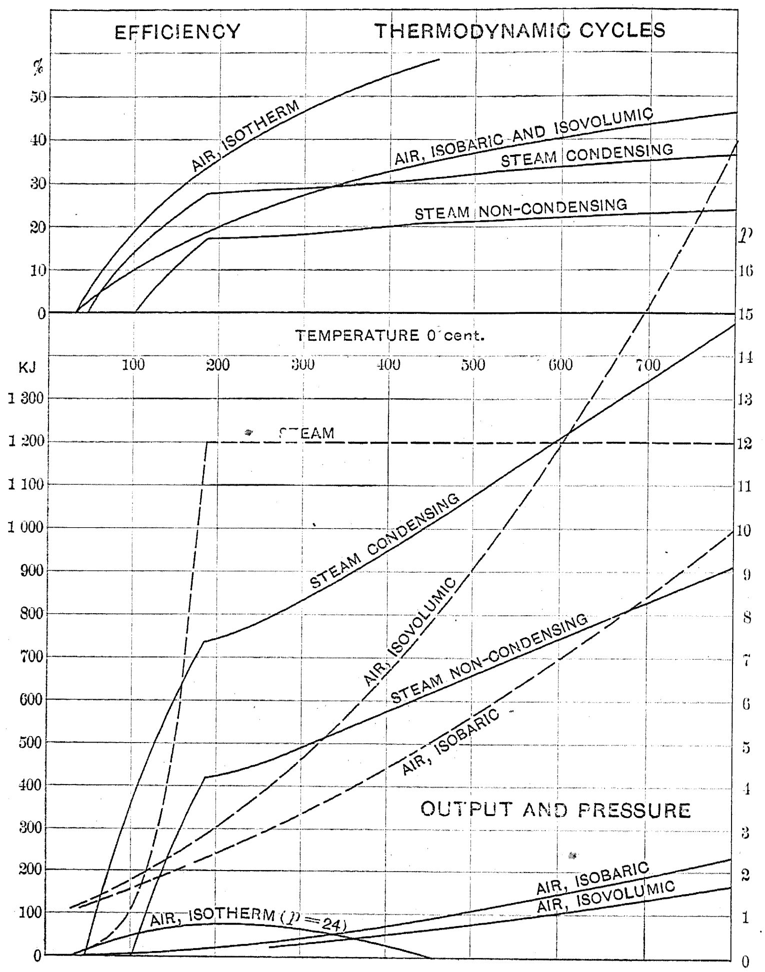

FIG. 1

A pressure limit of p = 12 kg. per cm² (about 170 lb. per

* Obviously, these curves and tables are approximate only, as thermodynamic calculations do not allow of the same exactness as electric engineering calculations, since some of the essential constants, as the specific heat, the adiabatic constant, etc., are not yet known with the exactness familiar in electrical determinations.

sq. in.) is assumed for steam, while for the air cycles a maximum of twice this pressure has been allowed.

As will be seen in Fig. 1, the isothermic or Carnot cycle—that is, constant temperature admission, adiabatic expansion, constant temperature exhaust, adiabatic compression—does not give any appreciable output, nor can it use considerable tem-

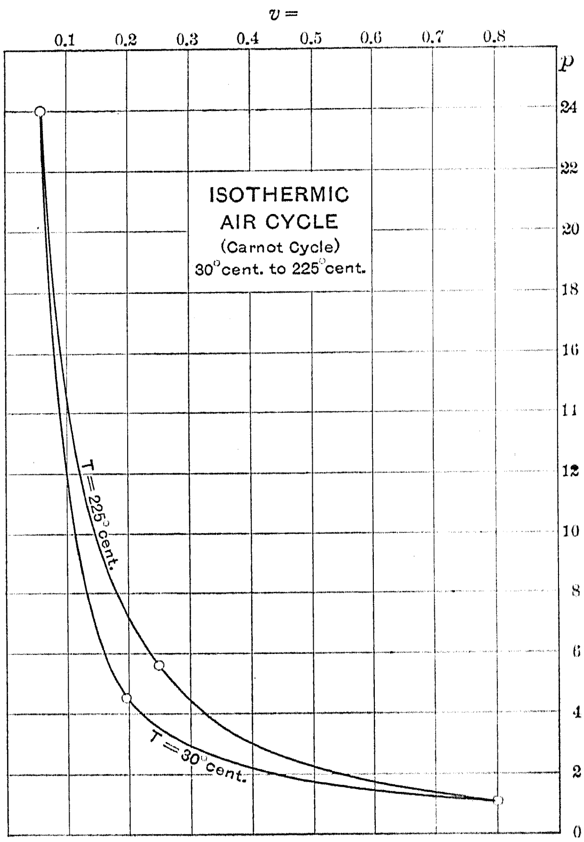

Fig. 2

perature differences without going to impracticable pressures. Thus its existence is limited to the text books of thermodynamics. With a maximum pressure $p = 24$, the maximum energy available for the Carnot cycle is 78 kilojoules per kilogram of air, or about one-tenth of the available energy of saturated steam at half this pressure, and occurs at the relatively low upper temperature of 225° cent. This cycle is shown in Fig. 2, and is the "fattest"

Carnot cyclè which can exist within the pressure limit of 24 kg. per cm^2. At higher temperatures the efficiency of the Carnot cycle increases, but the output decreases; the output vanishes, by the cycle collapsing into a line, at about 455° cent., or at a temperature far below that permissible in gas engines.

The curve for the isovolumic air cycle—that is, constant volume admission (constant volume combustion) and constant volume exhaust is the cycle of the theoretical or ideal gas engine. The curve for the isobaric cycle—that is, constant pressure admission and constant pressure exhaust would be the cycle of the gas turbine. These curves are calculated for the conditions of maximum available energy between the given temperature limits. Under this condition the thermodynamic efficiency is much lower than the maximum possible thermodynamic efficiency as represented by the Carnot cycle. It is: $\eta = 1 - \sqrt{\frac{T_2}{T_1}}$ for the isovolumic as well as the isobaric cycle of maximum energy, while the efficiency of the isothermic cycle is $\eta = 1 - \frac{T_2}{T_1}$,

where T_1 is the maximum, T_2 the minimum absolute temperature reached during the cycle. The thermodynamic efficiency of these cycles can be increased by going to higher pressures, but the output and thus the mechanical efficiency are decreased thereby.

It is interesting to note that the available energy of the air cycle per kilogram of air is insignificant until higher temperatures are reached; but even at a temperature of 800° cent. it is still much lower than that of the non-condensing steam cycle. It must be considered, however, that the volume of steam per unit weight is 58 per cent greater than that of air at the same temperature and pressure. The thermodynamic efficiency of these cycles is less than that of the steam cycle for lower temperature, but rises far above it at higher temperatures.

The relatively low pressure is interesting. This pressure gives the maximum output in an isobaric air cycle, and shows that the development of the gas turbine will take place at relatively moderate pressures.

In Figs. 3 and 4 are shown the isovolumic and isobaric cycles for 800° cent. maximum temperature and the same conditions of maximum available energy as in Fig. 1. Obviously, the

high-pressure peak at the beginning of the expansion, in Fig. 3, is cut off by delayed combustion, etc. Even at the highest temperature the maximum output pressure of the isovolumic

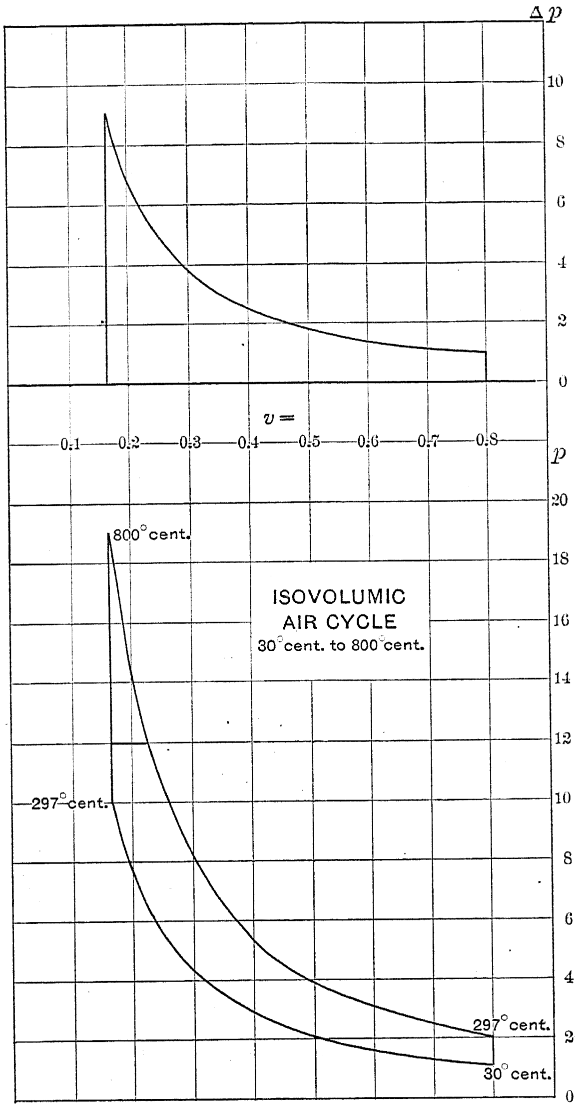

Fig. 3

cycle does not reach that occurring in the Carnot cycle at an insignificant output.

The steam curves in Fig. 1 are interesting from the very rapid

rise and very high values of available energy. Thus in the temperature interval of 141° cent., between boiler and condenser temperature, the steam cycle gives per kilogram more than four times the available energy of the isovolumic air cycle in the temperature interval of nearly 800° cent. Obviously, the cause is the partial condensation of the steam, which makes the enormous latent heat of steam available.

The efficiency of the saturated steam cycle is virtually the

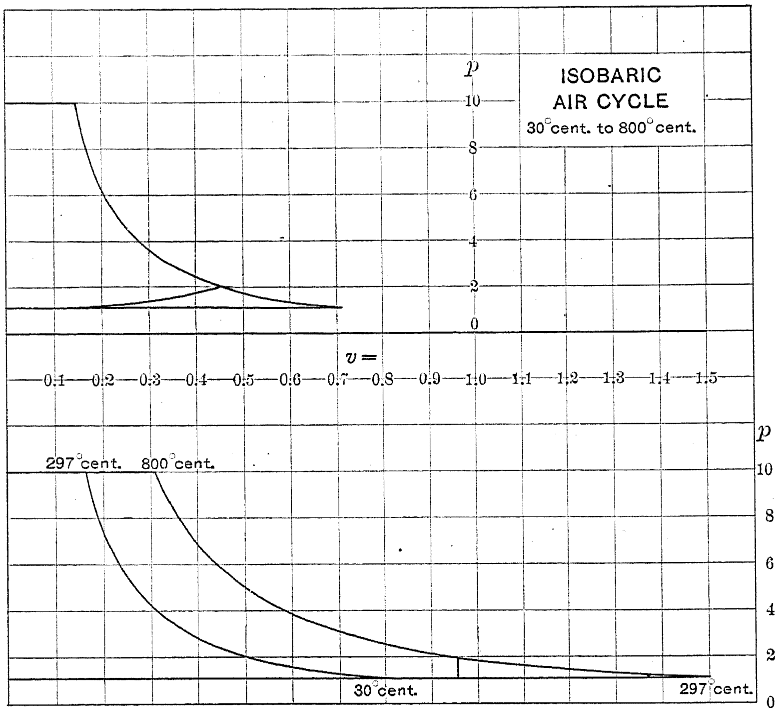

FIG. 4

same as that of the Carnot cycle; that is, the theoretical maximum possible for the utilized temperature interval. However, the pressure rises very rapidly, and the temperature interval which can be used in the saturated steam cycle is limited thereby. Hence from the temperature of 187° cent., corresponding to the steam pressure of $p = 12$, the further increase of temperature occurs not at saturation and with rising steam pressure, but at constant steam pressure, by superheat.

These curves of superheated steam* show the small increase of available energy of superheat, until very high superheats, gas-engine temperatures, are reached. They show especially the very small increase of thermodynamic efficiency resulting from superheat. Thus doubling the temperature range (141° cent.), of condensing saturated steam, by 141° cent. superheat increases the available energy only by 17.5 per cent, and the efficiency by 1 per cent. Increase of the temperature range of steam by superheat, therefore, does not increase the thermodynamic efficiency of the cycle correspondingly, and the steam cycle, while more efficient than the air cycle at low temperature, drops below it in thermodynamic efficiency at higher temperature. While the efficiency of the temperature range beyond saturation temperature is higher in accordance with the thermodynamic laws, the amount of work done in this range is such a small part of the total work that it does not appreciably increase the total efficiency.

The increase of efficiency of the steam cycle by superheat is not due to an increase of thermodynamic efficiency of the cycle, but is due to the decrease of losses, mainly those resulting from the condensation of steam; that is, it is essentially an increase of what has previously been considered as mechanical efficiency.

The available energy of superheat becomes considerable only at very high temperatures, and the thermodynamic efficiency, therefore, increases. However, for the total range from boiler temperature of 187° cent. to gas-engine temperature of 800° cent., the additional available energy of superheat about equals that of the steam in the 141° range below saturation, and the thermodynamic efficiency rises by only 9 per cent., thus dropping far below gas-engine values. This is important as indicating the losses of thermodynamic efficiency, resulting by limiting the temperature of combustion in gas cycles by injecting water.

Beyond the point marked by a circle on the steam curves, the steam remains superheated even at exhaust pressure; that is, the cycle occurs entirely within the gas range. Obviously, this point is shifted to lower temperatures in the turbine engine by the re-evaporation of moisture by mechanical losses.

It is interesting to note the large amount of available energy of steam below atmospheric pressure, 350 kilojoules per kilogram of steam, which is used in the low-pressure steam turbine.

* Calculated with the specific heat $c_p = 0.525$ at $p = 12$. and the adiabatic constant $a = 1.286$.

Fig. 5 gives the cycle of non-condensing saturated steam, and Fig. 6 the turbine cycle of condensing saturated steam.

Comparing Figs. 4 and 5 the greater width of the area of the steam cycle is illustrated. In Figs. 3, 4, and 5 the cycle is shown in the upper part of the figure, modified by subtracting the compression curve from the expansion curve, as this gives the area, and thereby the available energy, in a form more convenient for comparison.

Characteristic of all thermodynamic cycles, Figs. 4, 5, 6, is

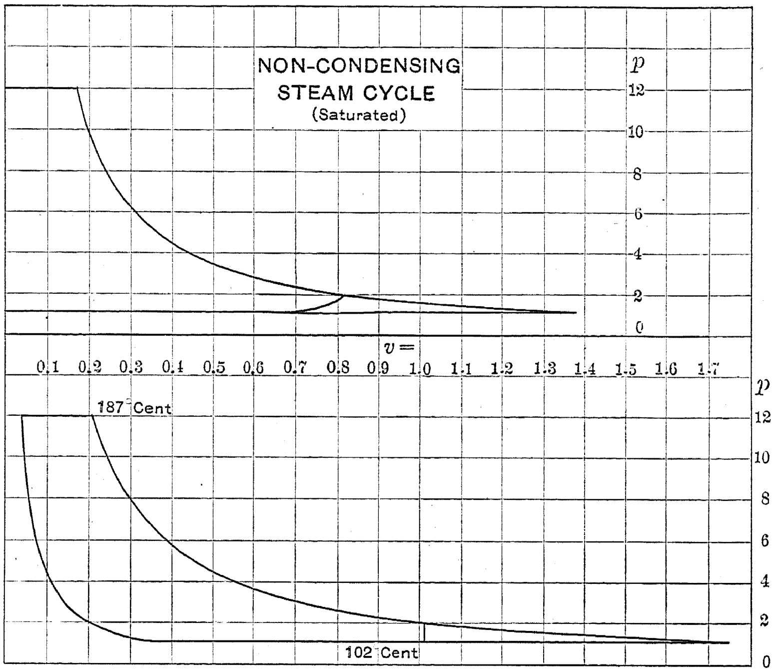

FIG. 5

the very large volume at the end of the expansion; that is, the area runs out in a long and low triangular shape. To carry the adiabatic expansion down to exhaust pressure, therefore, would require abnormally large cylinder volumes in a reciprocating engine, and correspondingly larger mechanical losses; therefore the expansion has to be broken off long before exhaust pressure is reached, about as indicated in the curves, or even at still higher pressures. As a result the reciprocating engine cannot get the full benefit of very low condenser pres-

sures, while the turbine engine is not limited in this respect. For instance, between $p = 0.07$, or 28 in. vacuum, and $p = 0.2$, as the pressure at the end of the expansion of a reciprocating engine, the available energy per kilogram of steam is 153 kilojoules, or about 37 per cent of the total energy of the non-condensing steam cycle. Of this only 100 kilojoules, or about two-thirds is used is the steam engine, while the total is used in the turbine. Hence the steam turbine gains far more by high con-

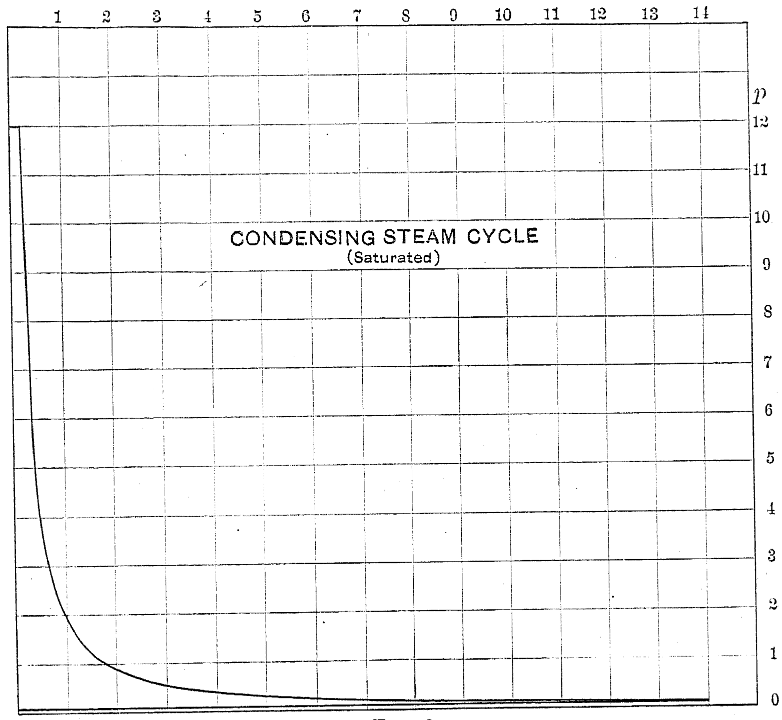

FIG. 6

denser vacuum than does the reciprocating engine. With a steam turbine of condenser pressure $p = 0.07$ (28 in. vacuum) the available energy of the steam would be higher by 42 kilojoules, than the steam curve in Fig. 1.

To show the relative magnitude of the quantities, the results of six typical cycles are calculated in Tables I and II.

As seen from Table II, the available energy E per kilogram of working medium is highest for steam with condensation, lowest for the Carnot cycle of air. The weight w of steam or

air required per 1000 kilojoules of available energy, is given in column 6, and varies between 0.68 kilogram for steam superheated to 800° cent., and 12.8 kg. for the Carnot cycle. Column 7 shows the maximum volume v_0, per 1000 kilojoules, assumed by the working medium at the end of the expansion. Here 2*a*, 3*a*, and 4 are such as to show the impracticability of these cycles for an engine

TABLE I.

THERMODYNAMIC CYCLES

1. *Saturated steam, non-condensing.* Fig. 5.
   - *a.* Expanding from: $p_1 = 12$; $v_1 = 0.166$; $T_1 = 187°$ cent.
     to: $p_2 = 1.1$; $v_2 = 1.38$; $T_2 = 102°$ cent.
   - *b.* Expansion terminated at: $p_1' = 2$; $v_2' = 0.81$
   - *c.* Compound engine, low-pressure cylinder: $p_1' = 3.5$; $p_2 = 1.1$
2. *Saturated steam, condensing.* Fig. 6.
   - *a.* Expanding from: $p_1 = 12$; $v_1 = 0.166$; $T_1 = 187°$ cent.
     to: $p_2 = 0.1$; $v_2 = 11.5$; $T_2 = 46°$ cent.
   - *b.* Expansion terminated at: $p_2' = 0.2$; $v_2' = 6.2$
   - *c.* Triple-expansion engine. Expansion terminated at: $p_2' = 0.2$; $v_2' = 6.2$. Low-pressure cylinder: $p_1' = 0.8$.
3. *Superheated steam of gas-engine temperature*, 800° cent.
   - *a.* Expanding from: $p_1 = 12$; $v_1 = 0.38$; $T_1 = 800°$ cent.
     to: $p_2 = 0.1$; $v_2 = 17.5$; $T_2 = 98°$ cent.
   - *b.* Expansion terminated at: $p_2' = 0.2$; $v_2' = 9.4$
   - *c.* Triple-expansion engine. Expansion terminated at: $p_2' = 0.2$; $v_2' = 9.4$. Low-pressure cylinder: $p_1' = 0.8$.
4. *Air, isothermic, (Carnot) cycle.* Fig. 2.
   Expanding from: $p_1 = 24$; $v_1 = 0.06$; $T_1 = 225°$ cent.
   to: $p_2 = 1.1$; $v_2 = 0.8$; $T_2 = 30°$ cent.
5. *Air, isobaric cycle.* Fig. 4.
   Expanding from: $p_1 = 10$; $v_1' = 0.31$; $T_1 = 800°$ cent.
   to: $p_2 = 1.1$; $v_2 = 1.5$; $T_2' = 297°$ cent.
   Compressing from $p_2 = 1.1$; $v_2' = 0.8$; $T_2 = 30°$ cent.
   to: $p_1 = 10$; $v_1 = .166$; $T_1' = 297°$ cent.
6. *Air, isovolumic cycle.* Fig. 3.
   Expanding from: $p_1 = 19$; $v_1 = 0.166$; $T_1 = 800°$ cent.
   to: $p_2' = 2.1$; $v_2 = 0.8$; $T_2' = 297°$ cent.
   Compressing from: $p_2 = 1.1$; $v_2 = 0.8$; $T_2 = 30°$ cent.
   to: $p_1' = 10$; $v_1 = 0.166$; $T_1' = 297°$ cent.

in which the maximum volume is of importance; that is, a cylinder engine. Cycles 2*a* and 3*a* thus are turbine cycles.

At the same piston speed, that is, the same number of revolutions and same piston stroke, the product of maximum pressure (column 4) and maximum volume (column 7) is proportional to the piston pressure, and these relative piston pressures: $P = p\,v_0$, are given in column 8. This column is an indication

of the mechanical strains which have to be taken care of in the engine design.

Column 8 shows the impracticability of utilizing the full pressure range of a condensing steam engine in a single cylinder—even if it were otherwise permissible—due to the excessive piston pressure resulting in this case; it also shows the enormous reduction of piston pressure by subdivision of the expansion, from 24.6 to 11.6 in the compound non-condensing engine, and from 104 to 7.0 in the triple-expansion condensing engine with saturated steam. Very striking, however, are the much higher piston pressures of the air cycle; that is, the gas engine: 84 in the isobaric and 62.5 in the isovolumic cycles, compared with 7 in the condensing triple-expansion steam engine. While by different assumptions, regarding pressure, temperature, etc., the numerical values may be somewhat varied, the differences between the two types of engine are of such magnitude as to exhibit a characteristic or inherent feature of the cycles.

In the last four columns are given, for 1000 kw. output, the values of cylinder volume in m^3 and cu. ft., of kilograms of steam or air consumed per kilowatt-hour, and the total efficiency, under the assumption of 60 per cent mechanical efficiency of the steam cycle and 50 per cent mechanical efficiency of the air cycle*, for a double-acting engine of 75 rev. per min., under the assumption of 60 per cent boiler and furnace efficiency; that is, ratio of heat units supplied to the engine to heat units representing the chemical energy of the fuel. The efficiencies of the air cycles are given for 100 per cent producer efficiency; that is, the case of gaseous or liquid fuel. In brackets are added the efficiencies for a plant making its gas from coal, by assuming 75 per cent producer efficiency.

Table II shows the characteristic and inherent differences between steam engine, steam turbine, and gas engine. The steam turbine can utilize the expansion as far down in pressure as condenser design permits, and is of smaller size than the steam engine, owing to the higher velocities permissible by the rotary motion.

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* By mechanical efficiency is understood the ratio of mechanical output to the available energy of the cycle. The mechanical efficiency thus includes all of the losses between the available energy and the indicated engine output, as condensation, radiation, delayed combustion, throttling, steam friction, etc., which are not included in the ratio of brake horse power to indicated horse power.

Characteristic of the gas engine, compared with the steam engine, are higher piston pressures; thus heavier reciprocating masses, and thereby heavier construction, larger sizes, and greater mechanical losses, and lower space economy, especially when compared with the steam turbine. From the higher mechanical stresses inherent to the gas engine must be expected a lower reliability of service, or a higher class of the required attendance, or a more difficult and expensive engine design. Also characteristic of the gas-engine cycle is a very much higher thermodynamic efficiency which, even when allowing for the lower mechanical efficiency resulting from the higher mechanical stresses, is still very far above that of the steam engine, especially when gaseous fuel is directly available, and no producer plant is required.

In general, however, in discussing gas engines, the two distinctly different cases must be separated:

a. Where gaseous or liquid fuel is available, as in the blast-furnace gas, natural gas, and the automobile engine.

b. Where gaseous fuel has to be produced from coal by a separate plant.

When balancing the advantages of higher efficiency against the disadvantages resulting from the higher mechanical stresses, the difference between the two cases may often be the determining feature in the choice of the prime mover for maximum total economy.

Discussion on "Prime Movers." New York, February 19, 1909.

President Ferguson: This is a subject in which we all are vitally interested. There probably has been more done in the last five or six years in the development of prime movers than was accomplished in the previous 20 years. This serves to emphasize the importance of considering the question of depreciation in the broad way in which the author has approached it, and I wish to call attention to the happy manner in which he has expressed it. As an illustration of the truth of what he has said, I might cite something that may be of interest to you.

It is now six years since the largest steam-turbine powerhouse was completed in this country. There were at that time four turbines installed. Thirty days ago there was begun the dismantling of the first of these four turbines. That is a very short period of time in which to have had the turbine installation in use, but it bears out exactly what Dr. Steinmetz has said in his paper. The economy of the present turbine, compared with those which were installed six years ago, has so improved that it pays the company to scrap the turbines it put in at that time and install new ones of a more modern type; in other words, the saving to be effected, due to the increased economy of the new turbines, when capitalized, will more than pay for the cost of the new turbines. Not only has the economy of prime movers been improved, but their reliability as well, both contributing to place the producing and marketing of electricity and electrical apparatus on a more firmly established basis. This appeals, naturally, to those of us who are engaged in the operating department of engineering, carrying with it the great responsibilities not only to investors but to the general public, whose servants we really are, and we appreciate most highly the interesting and instructive manner in which the author has brought out the necessity for considering in all statements of cost the value of reliability.

Charles E. Lucke: It seems to me the first part of this paper is mainly a repetition of many elementary facts almost axiomatic, in which the only idea that strikes me as new is the proposition concerning reliability insurance. That seems worth while. As to the rest, I confess I am distinctly disappointed, especially in relation to the thermodynamic efficiency part. I have studied things of this sort a good many years, it is my own specialty, and it may be that I expect too much, but one fact I cannot overlook in this thermodynamic efficiency analysis is that thermodynamic efficiency, as it is taken up in the paper, referring only to the cycle, depends primarily on two things (*a*) the nature of the cycle, that is to say, the sort of phases and order of succession of the phases or processes, to use a better word, and (*b*) the extent to which each phase is carried.

The order in which the processes are carried out will affect

the efficiency, in fact it will determine it, with one exception, and that exception is the equation of condition that fixes the extent of the phases. If it is desired to adiabatically compress or heat at constant volume, the nature of the process is defined, and that is the first step in fixing the possibility of the cycle, the next step being, how much heat is added or how much the gas is compressed. What fixes the thermodynamic efficiency of the cycle, as such, is just these things: the nature of the processes, the order of the succession, and the extent of the heating, the expansion, cooling or compression.

The author has assumed certain cycles, which are very well known, which have been studied repeatedly, and which are in nearly all the elementary books. The study of these cycles is proper subject matter for a third year course, such as we have at Columbia, or, in a not very advanced institution, for a fourth year course. The author has assumed certain limitations to the extent of certain phases by fixing pressures and temperatures, purely arbitrarily, and the results and conclusions are worth no more than the assumptions. Any other assumptions that any one else considers reasonable will bring other results, and we might have endless controversy concerning the conclusions. Every man's conclusions would be as good as the others so far as one set of assumptions is as reasonable as the other.

The author has fallen back in two or three places on that old statement in every text-book, which is generally misunderstood: "The thermodynamic efficiency is essentially a function of the temperature difference, on which the engine operates." This is a variation on the statement of the Carnot cycle law, which is ordinarily set down as follows: the maximum efficiency obtainable for a given temperature range, is fixed by a certain expression, usually $\frac{T_2 - T_1}{T_2}$. As a matter of fact the actual efficiency of any cycle has very little to do with this, has no more to do with it than my income has to do with the wealth of the nation, although the wealth of the nation certainly limits my income. It is quite true that Carnot's cycle yields an efficiency which is the highest possible for the temperature range, and it is likewise true that no gas cycle comes anywhere near the Carnot limit.

We are especially concerned with the gas cycle. The efficiency of any gas cycle is not a function of the temperature range, it is a function of, the nature of, the order of succession of, and the extent of each phase making up the cycle.

As engineers what do we care about the temperature range anyway? Do we approach the gas engine, the question which Dr. Steinmetz has finally interested himself in, by saying let us compress this gas to a certain pressure, and then have a constant volume rise to a temperature of 800 degrees cent. or

700 degrees cent., or 900 degrees cent.? We do nothing of the sort. We put into the gas mixture as much heat as we can, and do not care what the resulting temperature is. We put just as much heat into the gas as we can, to do as much work as possible in the cylinder, and the maximum temperature nobody knows the value of nor cares about. We do know how much work we can get, how much efficiency we can get, and also, that it has nothing to do with the temperature range.

It is possible, by taking proper assumptions for all variables, by setting up mathematical or other criteria by which to judge these results, for each man to arrive at results in his own particular way, and all different. If you will go through the literature on the subject you will find laws enough derived in this way to prove almost anything. Such cyclic analysis is almost as valuable as the Bible to prove things. An expression can be found to prove almost any statement, and another one to disprove it. Change the assumptions, always keeping them within the range of what you can induce some one to say is reasonable, and contradictory results can easily be obtained. I think as a consequence of this situation that this analysis of thermodynamic efficiency is not very valuable.

I have been carefully looking through the paper for a conclusion. I have been trying to find out just what Dr. Steinmetz was trying to show, to throw, if possible, some light on the assumptions he uses, and I have not been able to find any conclusions. If the conclusion is that the gas engine cannot have as high an efficiency as the steam engine then the conclusion is absurd.

Henry E. Longwell: The opening sentences of Dr. Steinmetz's paper constitute a cheering message to those of us whose daily bread depends on the continuance of a demand for prime movers. When one who is not, and who cannot hope to be, versed in the intricacies of electrical science, hears and sees evidence of what he imagines to be mechanical work done by that phenomenon which we call lightning, he perhaps should not be too harshly criticized if he should have conceived the idea that electrical energy does exist in nature. It is not improbable that the school-books of our childhood days are largely responsible for the inculcation of this disquieting idea in the unsophisticated mind. By their persistence many of us have been led to unconsciously accept as classic, the crude experiments and deductions of that archaic philosopher and physicist, Benjamin Franklin.

During a life which is still a little inside the alloted span of three score and ten, I have seen so many impossible things happen, that for my own part, I am pretty well content to keep an open mind on all questions bearing on ultimate possibilities.

One can readily imagine a perfectly flat country, where the inhabitants could have no conception of hills and waterfalls. Such a country might be abundantly watered by lakes and

canals, and yet one could forgive the wise men of this hypothetical nation if they asserted that water-power was not found in nature; that water-power could be available only through the kind offices of a prime mover that would convert the energy of fuel or of air currents into mechanical rotation or reciprocation, thus producing an artificial head or pressure that would enable the water to do work.

Considering the marvelous developments in electrical science during the lifetime of even the youngest member of this Institute, is it safe to set a limit to the possibilities of centuries yet to come? May it not be that we are now coming ourselves only with electricity as a *medium* for the transmission of energy? We can with certainty and precision produce by mechanical agencies, a difference of potential which enables this thing we call electricity to reproduce in a large measure the energy expended upon it. But does this fact make absolutely unthinkable the proposition that there may be natural differences of potential, yet undiscovered? However, with the authoritative assurance that we can never hope to eliminate that troublesome yoke-fellow of the electric generator, the prime mover, who alone prevents the business of operating an electric power plant from being a rapturous dream of profit and pleasure, we can, with all the more satisfaction and determination, apply ourselves with renewed vigor to the task of reforming him.

As regards the determination of costs of electric power, this is not essentially different from cost accounting in any other industrial enterprise. Industrial accounting is the most flexible and accommodating science known to mankind. Set half-a-dozen men to working independently on as many different sets of industrial cost records, and you will get just that number of different costs, and the correctness of each and every one is susceptible of proof by the unimpeachable rules of mathematics.

The total cost of electric power begins with the water on the hilltop, or the fuel in the earth, and ends only with the light emitted from the lamps and the power delivered from the shaft of the motor. A single individual or corporation rarely deals with the entire chain of operations, and the cost of the electric power, and the relative importance of the several items entering into the cost, depend largely on how many links in the chain are under consideration. For example, the author in speaking of the interest on the investment as a factor of the cost, says "It is frequently very large with water power, due to the hydraulic development required," etc. It might be added that it is sometimes very low, in cases where a hydraulic development company delivers water at the plant.

If we are to accept the author's inference that the cost of electric power stops at the terminals of the generator, and that the prime mover includes everything from the generator back to "nature's stores", then we must perforce agree with his statement that "In the cost of electric power the electric machine

plays only a subordinate part; the essential element in determining the cost and the reliability of electric power is the prime mover."

There are some electric power enterprises that include expensive hydraulic development plants, or to speak more accurately, there are some large hydraulic development enterprises that include electric power plants as convenient transmitting mediums. I also know of one concern so happily situated as to be able to dispose of its output to a customer who furnished his own switchboard, transformers and transmission system. On the other hand, in how many instances does the owner or operator of an electric power plant get much nearer to " nature's stores " than the coal car on his railway siding, and in how many instances can he dispose of his output without delivering it on his customer's premises? Surely the instances are not so numerous as to warrant any such sweeping statement as that quoted.

I will not venture to intrude any discussion on the classification of electric power service into five different grades of quality. Your association may have under consideration the wisdom of setting up an official standard scale of badness. It might be useful to a manager of an electric power station as an instrument for convincing a complaining customer that he had no business to complain. To the manager whose plant is giving only mediocre service, it might be a comfort to know that he was still one or two degrees within the boundary of official recognition.

That part of the paper devoted to the thermodynamic consideration of heat motors, seems to be simply a brief resumé of several of the standard propositions that are demonstrated in most of the elementary text-books dealing with the subject. Those who have studied the mechanical theory of heat to the extent required in a modest technical course, will accept the statements put forth without argument. Those who have not a reasonable grounding in the subject must accept the statements as dogmatic.

We are introduced to a new unit of energy, which is referred to as the "kj." I am glad to be confirmed in my suspicion that this is a kilojoule, but it is a little difficult to see why it is used to the exclusion of the more conventional units that are commonly found in text-books on thermodynamics. As the value of a kilojoule differs from that of a British thermal unit by less than 6 per cent, we may accept it as a sort of " esperanto " term which conveys practically the same idea to those who think in metric measures and those who think in British measures. We also meet two old friends of the thermodynamic cycle family with new names. Isobaric and isovolumic would be convenient terms, were it not that the text-books recognize two other air engine cycles, those of the Ericsson and the Stirling engines, in which the isobaric and isovolumic features are re-

spectively combined with isothermal expansion and compression, instead of adiabatic. I wonder if I dare suggest as a solution of the difficulty, isobarabatic and isovolumbatic—words which if not more significant, are more sonorous.

Referring to the efficiency curves in Fig. 1, we naturally expect the curve for the Carnot cycle to stand at the top, as it is our standard of the highest efficiency attainable. We expect the cycles which are designated as isobaric and isovolumic to fall short of the efficiency of the Carnot cycle, because they do not receive all their heat at the highest temperature, or reject all of it at the lowest. We expect the isobaric and isovolumic cycles to be equal in efficiency, for the reason that while they deal with different total quantities of heat, the amounts received and rejected at corresponding temperatures are strictly proportional. We expect the curve for saturated steam to coincide with the curve for the Carnot cycle between the same temperature limits, for the reason that the ideal cycle for saturated steam is a Carnot cycle. The only reason the curves do not coincide in the figure is, that different minimum temperature limits have been selected. We expect a decided change in the steam-cycle curve, at the point where superheat begins, for the reason that we there depart from the Carnot cycle and begin to take in heat at varying temperatures.

At higher temperatures we expect this superheated-steam curve to fall below the isobaric and isovolumic curves, for the reason that a larger proportion of the total heat dealt with was received at lower temperatures.

The curves of relative output per unit weight do not uncover any surprises. It is the heat and not the substance, that does the work. As steam has a greater heat-carrying capacity than air, naturally the work done by the heat that can be put into a unit weight of steam will be proportionately greater than that done by the smaller quantity of heat that can be put into a unit weight of air, temperature limits being the same.

The output of the isobaric cycle we expect to be greater than that of the isovolumic cycle, just in proportion as the specific heat of air at constant pressure is greater than the specific heat at constant volume, which corresponds with the ratio of the figures given in Table II, column 1.

The expression $1 - \sqrt{\frac{T_2}{T_1}}$ for the efficiencies of the isobaric and isovolumic cycles is novel, but applies only to the special case in which the temperatures at the end of adiabatic expansion and compression are equal, which satisfies the condition for maximum output per unit weight of air as the working fluid.

In the Carnot cycle for air, it is quite reasonable that for a given lower limit of pressure and temperature there is for each upper pressure limit a corresponding temperature that gives the maximum output per unit weight of the working substance. If

the temperature were increased to such an extent that it would require the expansion to be adiabatic over the entire pressure range in order to reach the lower temperature limit, then there could be no isothermal expansion, no intake of heat and consequently no work.

It will be also noted that it is not alone with air as a working medium that the use of the Carnot cycle involves high pressures in order to utilize moderate temperatures. By a curious coincidence, the temperature given by Dr. Steinmetz as best for an air cycle with 24 atmospheres maximum pressure, corresponds almost exactly with the temperature of saturated steam of the same pressure. And this is why we use superheated steam. We cease to attempt to approach the Carnot cycle at the higher temperatures, and turn to a less efficient one in order to avoid inconveniently high pressures.

Superheat is not theoretically the most efficient means for utilizing great temperature differences, with steam as the working fluid, but for the present it is the most practical means available, and is much better than nothing at all.

Thermodynamics is a very interesting and almost limitless subject, but thermodynamics and practical commercial engineering overlap only to a moderate extent. The cycle of the heat motor is but a small link in the chain of processes involved in converting the energy of fuel into mechanical energy; and if one concentrates too much on this one link, he is apt to contract his field of vision.

The proper selection of a prime mover depends more than anything else on that valuable human faculty that we call "horse sense",—which, like greatness, one may be born with, or may acquire, but which, unlike greatness, cannot be thrust upon him. It may be taken for granted that no manufacturer who has been prominently in business for ten or twenty years is putting out a distinctly bad heat motor.

The steam turbine may, for obvious reasons, without attaining prohibitive dimensions, be made large enough efficiently to utilize exceedingly low pressures and temperatures, when the low temperatures are available, but it does not follow that it must necessarily be so made. Because of the especial adaptability of the turbine for using large volumes of steam, and the excellent way in which the reciprocating engine handles steam at the small volumes consequent on high pressures, a high-pressure reciprocating steam engine compounded with a low-pressure steam turbine often presents a very attractive combination.

The mechanical difficulties involved in the attempt to utilize the full range of expansion of high-pressure steam in a single cylinder of a condensing engine, have long been recognized as being at least fully as important as the thermodynamic difficulties. We are not confronted alone by high mechanical stresses but by prohibitive clearances, unendurable leakage losses, and

the impracticability of a valve gear for the very short cut-offs that would be necessary.

The pressures in gas engines,—say 25 to 30 atmospheres—have not been found unduly difficult to cope with. The heavy reciprocating masses smooth down the peaks of pressure so that in general the mechanical stresses are well within the limits of conservative engineering practice. Gas producers are now obtainable which may be operated with the same degree of efficiency, reliability and continuity as the best boiler plants.

It can scarcely be claimed that one type of prime mover demands a higher grade of attendance than another, or that there is any serious difference in reliability as between the several types. The skill required to obtain the best results from the turbine or the gas engine, as compared with that required for getting equally good results from reciprocating steam engines is different in kind, but not necessarily in degree. Skilled attendants for steam turbines and gas engines are more uncommon than for reciprocating steam engines, but that is because we have been training attendants for reciprocating engines for 150 years, and the school has been a very large one. We have been training gas-engine and steam-turbine attendants for perhaps less than 30 years, and our educational facilities have until recently, been exceedingly limited in extent. There are dozens of men who can detect symptoms of trouble in reciprocating engines, and who can correct troubles before they become serious, to every one man who can do the same for a gas engine or a steam turbine. Yet the skilled gas-engine or steam-turbine attendant might not be able to display much resource in an installation of reciprocating steam engines. When we see a particularly fine piece of work, do we compliment the workman, or his tools? When we hear of an especially good record of electric power station service, both as regards reliability and overall economy, shall we compliment the prime mover, or the manager? The final overall efficiency and reliability of the plant depends on the human factor as well as the mechanical factor. The variations possible in the latter—considering, of course, only reasonably modern and accepted types of equipment—are so small as to be over-shadowed in the variations of the former.

Always the two-thousand-year-old parable of the talents is recalled to us. The talents differed not in goodness, and yet they produced widely different results, according to the respective efficiencies of the men to whom they were entrusted. And the parable would have been more realistic, and more in keeping with what we often see in our daily contact with the great and growing electric power industry, if he who was entrusted with only a single talent had, by his energy and foresight, equalled or surpassed him that was entrusted with ten.

David B. Rushmore: ' Almost every engineer has to do with a prime mover in one form or another, and questions arise in

that connection, whether the engineer is an operating engineer, or connected with a manufacturing company, or whoever he may be. One of the questions to be considered is, what shall be called the output or the rated capacity of these machines? This question nearly always arises in a comparison between machines manufactured by different concerns, where the basis for the rating has not been definitely fixed.

In the steam engine, for example, which is one of the oldest forms, we have a fairly definite idea when we speak of so many horse power output for the engine. The turbine, which is not as old, has a variable overload capacity, or an overload capacity not always expressed in the normal rating of the machine, and so there is considerable lack of definiteness which is sometimes confusing.

The gas engine is peculiar in a certain way, in that the rated output of the gas engine is one with a very small overload capacity. In comparing estimates on certain installations, which I have recently looked over, the engineers did not bear this fact in mind, and in certain industrial establishments which are now in process of construction, an active discussion is taking place as to which is the more desirable form of prime mover to use, in order to generate electrical energy from the gases of blast furnaces. This is the particular point I desire to speak on, not to reach any conclusion, because there is no conclusion which can be had at present. The factors which are going to form this conclusion are those not to be reached in a way which is positively acceptable.

A steam turbine may have a thermodynamic efficiency under the conditions of operation which will approximate 13 or 14 per cent. A gas engine under the conditions of operation may reach 24 or 25 per cent. The output in the case of the gas engine is approximately 100 per cent more than we will get in the other case, and naturally, as has been the fact, people have wished this output, but the lowest cost of electrical energy is not the real thing that the owners of this establishment are after. It is the lowest cost of their output, and the factors which should be introduced into the problem are depreciation, repairs, and interruptions to service. In making a final comparison we must have more or less of that information at hand. Reliability insurance, which Dr. Steinmetz speaks of, appears not exactly in the way in which he has presented it, but in the choice of machinery to be used. If we have engines which must be out of service frequently, if we have engines with very high charges for repairs, if we have engines on which the depreciation is very high, if we have an installation the cost of which is from 50 to 100 per cent greater in one case than in the other, a thermodynamic efficiency which is twice in one case what it is in the other, will not make up for the influence of the other factors.

Calvert Townley: I understand that our respected President and some of his advisors have inaugurated a movement at

Washington intended to preserve the water-powers of the country for the general public. It is proposed that when water powers are developed there shall be a payment made to the government proportionate to the output of such powers. I take this opportunity, therefore, to say that I believe such a course in many cases will absolutely prevent the development of water-powers, and, instead of benefiting the general public by swelling the government income, will retard the advance of any industries that might be assisted by cheap power.

I presume that the idea of a charge is born of the thought that water-powers are enormously profitable to the owner, and that he should not be allowed to monopolize our country's natural resources for private gain. It has been my experience in New England, gained both from operating some water-powers and in examining the commercial feasibility of others with a view to possible development, that in competition with steam, the majority of power developments are so heavily handicapped by fixed charges and operating difficulties as to make their commercial development of doubtful interest.

There is a decided similarity in many of the conditions affecting water-powers in the several New England states; for example, the rain-fall, the run-off per square mile of drainage area, and the relations between maximum, minimum and average flow. We can be fairly sure, for example, that the maximum flow of any stream will be about two-and-one-half times its average flow, not considering spring freshets, and that the minimum flow will sometimes drop to the vanishing point. Let us examine then the fixed charges of a water-power which it is proposed to develop for the sale of electric energy in competition with that produced by steam. Assume that this power can be developed to 80 per cent of the maximum flow of the stream at a cost of $100 per kilowatt capacity, including hydraulic works, electric machinery and transmission line. This is a low figure for many water-powers, but is purposely so chosen for purposes of illustration. We cannot safely contract to deliver a peak load greater than the average flow of the stream. Therefore, if it costs $100.00 per kw. capacity to develop 80 per cent of the maximum flow, which flow is two-and-one-half times the average flow, the development cost for the average flow will be $200.00 per kw. The load-factor must next be considered. It is a fortunate company that can show a load-factor as high as 50 per cent,—that is to say, where the average 24-hour load is as much as one-half the peak load or station capacity. This means, then, that for the average current sold, which is really the unit that has to carry the full fixed charge, the investment is not $200.00, but $100.00 per kw.

In time of low water, the company must still be prepared to carry its peak load; that is, a load equal to the average flow of the stream. Last year the minimum flow was not to exceed 25 per cent of the average, requiring, therefore, a relay of at

least 75 per cent from some source other than the water-power itself. If this source be a steam plant, and the cost of such plant be taken at $100.00 per kw. rated capacity, there must be added $75.00 per kw. to our investment on which to base fixed charges, making the total development cost for each annual kw. sold, $475.00, or at only five per cent interest, not including depreciation, an annual cost of $23.75 over and above all operating expenses.

By contrast, a steam plant costing to build $100.00 per kw., and having a capacity equal to the peak load and operated with the same load-factor previously assumed (50 per cent), must provide for fixed charges on $200.00 only, or $10.00 per year, taking the same interest as before. This shows at once a handicap of $13.75 per year which the hydraulic plant must overcome by lower operating expenses, before the water-power is on all fours with the steam development. Where the load-factor is lower, or the cost of the hydraulic development is greater per kw. capacity, or where depreciation is to be provided for, or a larger steam relay necessary, any or all of which conditions are frequently met, the hydraulic plant labors under a still greater disadvantage.

It will readily be seen, I think, without pursuing this comparison further, that any arbitrary governmental tax or charge will lessen the attractiveness of water-power investment, and will so seriously handicap the development of a great number of powers as to entirely destroy their commercial availability.

My excuse for speaking of these obvious facts, which are known to many of you, is that there seems to be an entire lack of knowledge on this question by a considerable portion of the general public, resulting in the danger that unfortunate laws may be enacted tending to injure seriously an industry in which many of us are interested. Therefore, if you, as engineers, will, as opportunity offers, take pains to see that the fundamentals are understood, you can have a material effect on forming proper and intelligent public opinion.

Ernst J. Berg:* In concluding the discussion I shall not try to defend Dr. Steinmetz's paper nor the premises used in his arriving at the thermodynamic efficiencies, especially since I feel that it will give him great pleasure to do so himself. I will, however, add that when reading his paper this afternoon I found it most instructive and valuable. No doubt I have seen these thermodynamic cycles described in text-books, but it is so many years ago that I have forgotten them, and I am sure that they were not given in the concrete form that Dr. Steinmetz has presented. I know that there are other members in the same situation, and to them as to me the paper has a charm of novelty and is very interesting.

Dr. Steinmetz says, if I am not mistaken, that the limits of

* In the absence of the author on account of illness, the paper was presented in abstract by Mr. Berg, who replied as follows:

pressure and temperature chosen by him in working out the thermodynamical cycles might be somewhat varied, but he also states that the conclusions will not be much affected by such variations as long as they are reasonable, and I believe him. A temperature limit of 800 degrees cent. seems as high as could well be used, since I believe this to be the temperature at which the cylinders begin to be dull red.

As to the pressure, it seems that 24 atmospheres or about 350 lb. is as high as is good for conservative engineering, therefore, I find nothing wrong about the assumption. The efficiencies are finally based upon cycles which give the maximum work for given cylinder dimensions, which seems also very fair. When finally the tables showed that the efficiencies, mechanical as well as thermoydnamic, were consistent with known facts, I was satisfied there could not be any criticisms on these scores.

Professor Lucke said that there were no conclusions given in the paper. That is true. I do not think Dr. Steinmetz wanted to give any, I do not think he wanted to say that any given cycle or type of prime mover was best. He made a fair comparison between the various prime movers.

Charles P. Steinmetz (by letter): As far as I can understand, Professor Lucke's lengthy but rather unintelligible discussion is a general denial. In short, he says: the assumptions of the paper are wrong, the methods of reasoning are wrong, and the conclusions are wrong; while the second speaker, Mr. Longwell, explains in great detail, that the assumptions of the paper, the methods of reasoning, and the conclusions are old and well-known, and their correctness demonstrated in every elementary text-book, so much so that they do not need to be stated. As the statements of these two speakers thus contradict each other, I do not need to discuss them further, but merely desire to point out to Professor Lucke that the study of typical thermodynamic cycles is not useless, because no heat engine exactly follows any of these cycles. In electrical engineering also, no existing transformer exactly agrees with the typical transformer diagram, nor any existing transmission line with the equations of transmission lines. Even Ohm's law is not rigidly obeyed by any industrial circuit, as secondary phenomena, such as temperature changes, always occur. Nevertheless, we use these laws and methods in design and calculation, and the history of electrical engineering has amply justified us. Phenomena of nature are always complex, and thus can never agree completely with any typical representation; but it is the province of the engineer to find the closest theoretical representation of the actual facts, and then bridge over the gap by engineering judgment.

While Mr. Longwell desires the thermodynamic cycles omitted from my paper as obvious, for Professor Lucke I should have added an extensive discussion on the relations of the theoretical thermodynamic cycles to the actual cycles in heat engines. I

have thus chosen the middle ground, illustrated the reasoning on the typical thermodynamic cycles, and left it to the intelligence of the reader to realize that the actual engine cycles are intermediate between, and combinations of, the typical cycles, and hence are covered by the results of the typical cycles.

Professor Lucke's statement, that the temperature range has no effect on the efficiency of the thermodynamic cycle, obviously is wrong. If his idea of gas-engine design is to put as much heat into the cylinder as possible, and see what power he gets out of it, and he does not care to know what the temperature is; this obviously does not prove that the temperature is immaterial.

In general, an efficient designing engineer endeavors to know not only what is put into, and what comes out of an apparatus, but still more, what takes place in the apparatus. The rapid advance of electrical engineering is to a large extent due to the fact that the designing electrical engineer was not satisfied merely to put as much mechanical power into the machine as possible, and measure whatever electrical output he got, but has carefully studied the internal reactions of the apparatus, and thereby was enabled intelligently to choose and predetermine the actions, with the result that the waste of power in electrical apparatus has been reduced to a negligible quantity.

However, Professor Lucke's method of gas-engine design, by putting as much heat as possible into the cylinder, is merely a repetition, in different words, of the assumption which I made in my paper for the air cycles. These cycles I consider for the conditions of maximum output, where the greatest amount of heat is put into the cylinder, which still increases the output.

The claim made by Professor Lucke and Mr. Longwell, that the assumptions made in my paper are arbitrary, and by a different set of equally justified assumptions any other conclusion can be reached, is wrong; the assumptions made in my paper are not arbitrary, but, as proper in such investigations, I have chosen the conditions most favorable for each case. That is, where the efficiency or output increases with the temperature, I have chosen the highest temperature; where the efficiency or output increases with the pressure, I have chosen the highest pressure permissible by reasonably conservative design. The only feasible variations of the assumptions thus are minor changes: some engineers may consider 24 atmospheres too high, and prefer 20 atmospheres as a more conservative limit, while another engineer may be willing to allow pressures up to 30 atmospheres. As I have stated in my paper, however, the results and the conclusions, are not appreciably changed by any reasonable change of my assumptions.

Incidentally, Mr. Longwell is mistaken in his statement that equality of the two intermediate temperatures of the air cycle fulfils the conditions of maximum output per unit weight of air. This happens to be the case for the two cycles specifically

discussed in my paper, but not in general. For instance, for the air cycle of constant pressure admission and constant volume exhaust, in the conditions of maximum output—which is the assumption of my paper—the two intermediate temperatures are not equal. His statement that the efficiency of the isobaric cycle of saturated steam is that of the Carnot air cycle between the same temperature limits, is not quite correct either, but the efficiency of the steam cycle is slightly lower, though the difference is usually small.

My paper discusses the economy of the production of electric power from nature's stores of energy to the switchboard. If one corporation develops the water-power, another one uses it by buying the water of the former. This obviously does not change the cost of development etc., and you do not get water-power without cost of development, as Mr. Longwell seems to think, if you let somebody else develop the water-power, and pay him for the cost of development in the price of water bought of him. The economic considerations from the station switchboard to the light radiated from the customer's lamps, are equally important, as Mr. Longwell states, but they are not the subject of my paper.

I am sorry that Mr. Longwell thinks the kilojoule a new unit: the kilojoule, or kilowatt-second, is, and has been for some decades, the international unit of energy, used throughout the world, not only by the electrical engineers, but also by other engineers, such chemists as Ostwald, and others, with the exception of that rapidly dwindling group of mechanical engineers in the two English speaking countries, who still prefer to measure distances by what in the earlier middle ages was considered the proper length of a king's foot, and to measure temperatures by a scale using as zero the lowest winter temperature of Northern Germany, and as 100 degrees an erroneous measurement of blood heat (Fahrenheit). While the two English speaking nations have not yet reached the courage of completely freeing themselves of the curse of the ancient "English system" (so-called because all the other civilized nations have long ago discarded it), with its seven or more incompatible units of length* and other curios, I would not impose such a system on the American Institute of Electrical Engineers which already years ago has gone on record as endorsing the metric system, by a ballot vote of over 90 per cent majority.

It is true, that in the English system, when expressing the energy of fuel in British thermal units, the output of the boilers in pounds of steam per hour, the indicated energy of the engine in foot-pounds, and the output at the switchboard in kilojoules, the numerical values are not so disturbing to the self-satisfaction of the engineer as when throughout the entire transformation, the energy is expressed by the same units, and the inefficiency of the performance thereby shown in all its nakedness.

* Foot, mile, chain, rod, fathom, knot, yard, etc.

But, whether such a hiding of the inefficiency is to the advantage of the art, is questionable.

In regard to Mr. Longwell's objections to my statement that electric energy is a secondary form of energy, I shall be glad to withdraw this statement, as soon as Mr. Longwell will have succeeded in harnessing the thunder cloud, and in lighting the cities from nature's stores of electricity, from lightning, or from the earth currents, or whatever other of nature's stores of electricity he has in mind for industrial use.

A paper presented at the 235th meeting of the American Institute of Electrical Engineers, New York, March 12, 1909.

THE INDUSTRIAL APPLICATION OF THE ELECTRIC MOTOR, AS ILLUSTRATED IN THE GARY PLANT OF THE INDIANA STEEL COMPANY*

BY B. R. SHOVER

Introduction and historical. Twenty years ago electricity, except for lighting purposes, was virtually unknown to the iron and steel industries, while to-day in all the steel works in this country it is used as a motive power for most of the auxiliary machinery, from the ore docks to the loading beds. The electric drive is utilized in ore unloaders, ore bridges, car dumpers, bin-filling cars, scale larries, blast-furnace skips, hot-metal mixers, electric cranes of all sizes and descriptions, open-hearth charging machines, ingot buggies, gas producers, roller tables, lifting tables, transfers, hot-bed apparatus—in short, a multitude of machines too numerous to mention here. This development has been such that it would probably be no exaggeration to say that the steel industry in this country would never have reached its present proportions without the use of electric motors. As a further gauge of progress, it is significant that the larger electric manufacturing companies have been forced to establish special departments to look after the business of the steel industries.

Development of the direct-current motor. The first application of electric power to the steel-mill industry was by means of the

* The magnitude of the work in this plant has been such that without the hearty coöperation of the staffs of all the electrical manufacturing companies (practically all of whom have contributed to the work) it would have been impossible to obtain the results herein outlined. The author is, therefore, glad of this opportunity publicly to acknowledge the valuable assistance which they have given him and his company in this work.

direct-current motor. In a historical way the following remarks may be of interest, but the absolute accuracy of the dates given the author does not wish to be held entirely responsible for, though all possible care has been taken to verify them.

In 1882 the first electrical installation in any steel plant in the United States was made at the works of the Edgar Thomson Steel Co., at Bessemer, Pa. It consisted of a two-light Brush arc machine.

In 1889 an electric traveling crane was installed at the works of the Pencoyd Steel Co. near Philadelphia.

In 1891 a 50-h.p., 250-volt, compound-wound generator was installed in the Edgar Thomson Steel Co. works. This machine was used to supply current for an experimental crane over the heating furnaces at the blooming mill.

In February, 1893, the first motor-driven roller table was put in operation at the Homestead works of the Carnegie Steel Co.

On February 9, 1895, the first mill having all its roller tables motor-operated was started by the Ohio Steel Co., of Youngstown, Ohio.

In the early days belt-type and street-railway motors rewound for 250 volts were all that were available, but the severe service to which this apparatus was subjected led the manufacturers to strengthen it and to design new motors to meet the requirements; so that the steel industries both assisted in and profited by the general improvement of electrical machinery. One result of these requirements has been the development of the so-called mill motor. This motor has a heavy cast-steel frame, a large armature shaft, which can be replaced without disturbing the winding, the minimum number of bolts, ample air-gap, and fire-proof winding throughout. So successful has this type of motor proved that it is now being used almost exclusively, even on traveling cranes, regardless of the fact that the entire crane and runway must be made much heavier in order to carry the additional weight.

In order to appreciate the importance of such apparatus to the steel manufacturer it must be understood that the real efficiency of any machinery used in the industry is indicated by the amount of steel shipped per day or year. A motor of higher engineering efficiency that causes shutdowns, and consequent decrease of profit, is not nearly so efficient as a motor that operates 24 hours a day and 365 days a year, if necessary, practically without stopping. This feature is considered so

important that in one plant the superintendent of the electrical department requires a special report from his mill foremen whenever the mill is delayed more than 3 minutes by changing the armature of a table motor. The steel works electrical engineer has long since recognized that the mills are built to manufacture steel, and to be successful electrical apparatus must meet the requirements, no matter how severe. To the credit of the engineers be it said that the number of absolute failures to operate any of the apparatus has been very small indeed.

Development of the alternating-current motor. The coming of the induction motor found the steel mills equipped with direct-current apparatus, but the induction motor is so well suited for much of this work that it has been gradually introduced into the field, until to-day it forms a considerable part of the aggregate horse power used in the steel industry. The following historical notes represent the more important steps in this development.

In 1900 the Joliet works of the Illinois Steel Co. installed a 250-volt direct-current synchronous converter operating from a 2200-volt, three-phase supply, the power being furnished by an independent power company.

In 1902 the Pennsylvania Steel Co., of Harrisburg, Pa., installed an 11,000-volt transmission system.

In 1903 the first induction motor used in the steel industry was installed at the Joliet works of the Illinois Steel Co.

In June, 1904, the South works of the Illinois Steel Co. installed a synchronous converter and step-up transformer to operate a 2200-volt transmission line, and in September of the same year a 4000-kw. power plant, generating at this voltage, was installed. About the same time this voltage was stepped up to 22,000 volts for transmission to the cement works at Buffington, 12 miles away.

In December, 1904, the first installation of 6600 volts for interworks use was installed by the Lorain Steel Co. This voltage now seems to be the favorite for use in the entire steel industry, as it has been employed in many of the late installations.

The application of electric motors to the roll trains. While the United States has led in the application of electric drive to auxiliary machinery, Europe has pioneered the way in its use for driving roll trains. In the different European steel plants

there are to-day about 230 motors with a normal capacity of 19,000 h.p. and a maximum capacity of 41,000 h.p. used for electric drive of non-reversing roll trains. In addition, one noteworthy installation is that of a 10,500-h.p. reversing outfit at the Hildegardehütte mine.

The first application of motors for driving roll trains in this country was made at the Edgar Thomson works of the Carnegie Steel Co. in October, 1905, where two three-high roll trains for rolling small rails were operated by 1500-h.p., 220-volt, direct-current motors. The speed of these motors is varied by shunt-field resistance from 100 to 125 rev. per min. The success obtained by this installation has stimulated the installation of similar outfits elsewhere.

In June, 1907, the Illinois Steel Co. put in the first and only reversing-mill drive that has been installed in this country. The mill, a 30-in. universal plate, is direct-coupled to two 2000-h.p., 150-maximum rev. per min., 575-volt, shunt-wound motors mounted on one shaft; 2200-volt, three-phase, 25-cycle, alternating-current power being used to drive a motor-generator set consisting of a 1300-h.p. motor and a 3000-kw., 600-volt, direct-current generator. On the same shaft is mounted a flywheel 100 tons in weight, 13 ft. 2 in. in diameter, the whole making 375 synchronous rev. per min.

In August, 1907, at the same works, a rail mill for rolling small rails similar to those rolled at the Edgar Thomson works, was put in operation. The roll trains here, however, were driven by 2200-volt concatenated motors. The primary motor has a capacity of 1200 h.p., and runs at 120 rev. per min.; the secondary motor is 600 h.p., and runs at 82 rev. per min. In ordinary operation the resistance in the secondary motor is so adjusted that the combined speed of the two varies between 60 and 80 rev. per min. according to the character of material rolled.

The use of a storage battery. Another electrical device which has only recently been used in the steel industry is the storage battery. On March 27, 1904, the first installation of this nature was made at the Ohio works of the Carnegie Steel Co., Youngstown, Ohio. This battery had a capacity of 1600 ampere-hours and was used for regulating the load on a direct-current station. On April 29, 1905, the capacity of this battery was increased 50 per cent. The success of this installation was so marked that batteries of considerable size were installed at the Lukens

Iron & Steel Co., Coatesville, Pa. Three large batteries were also installed at the Illinois Steel Co., South Chicago; one at the Carrie furnaces of the Carnegie Steel Co., Rankin, Pa.; one at the Duquesne works of the Carnegie Steel Co., and the largest of all at the Edgar Thomson works of the Carnegie Steel Co., Bessemer, Pa.

Inception of the Gary plant. For several years the United States Steel Corporation had recognized the necessity of supplying the increased demand for its product throughout the West. To give an idea of the rapidity with which the demand for these products had grown, the total product of steel in the United States in 1894 was 4,000,000 tons; in 1895 this product

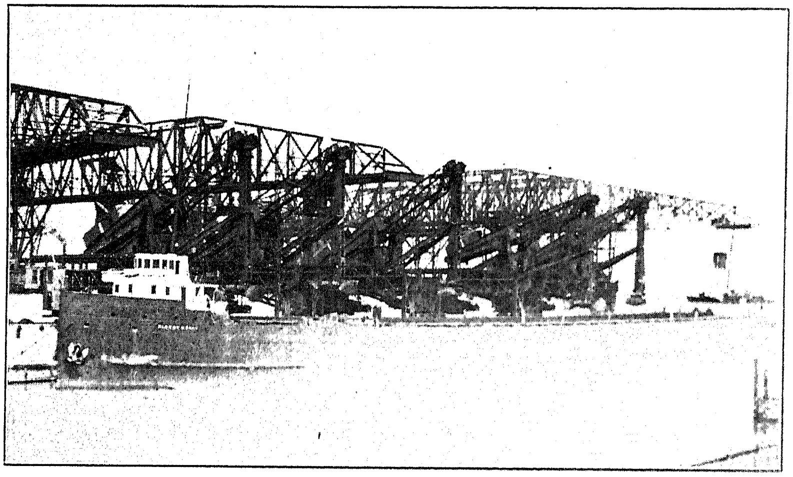


FIG. 1.—The Elbert H. Gary anchored for the first time under the unloaders.

had increased to 6,000,000 tons; in 1900 to 10,000,000, and in 1906 to 23,000,000 tons.

The constantly increasing proportion of this material used throughout the West necessitated the location of additional manufacturing facilities closer to the point of consumption. As none of the existing plants had ground enough on which to instal the required increase, it was decided to build an entirely new plant, to be designed so as to embody all the features that would ensure economical production. The site chosen is at the extreme south end of Lake Michigan, about 26 miles from the center of Chicago. It consisted of about 10,000 acres of sand dunes absolutely uninhabited, and covered by a growth

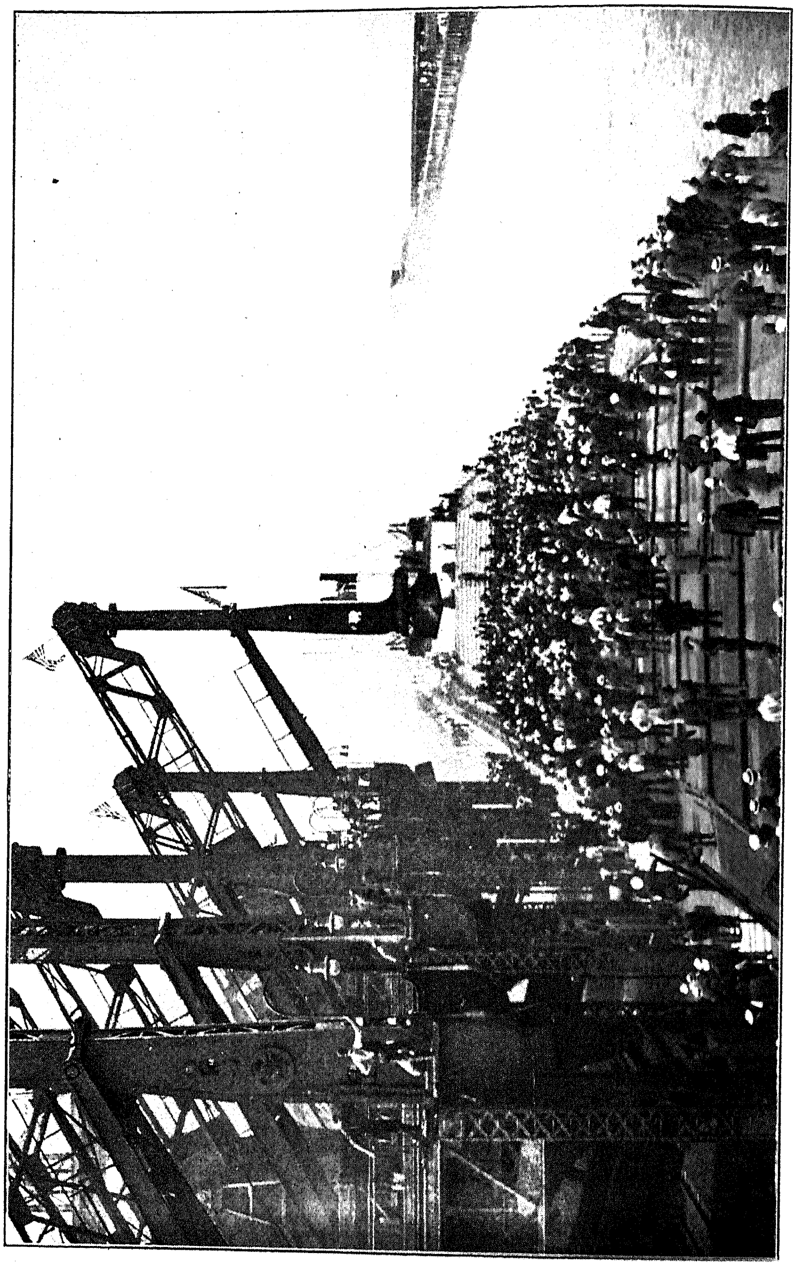


FIG. 2.—The first bucket of ore removed from the Elbert H. Gary

of scrub oak, pine, and cactus. The advantages of this location consist of lake transportation, splendid railroad facilities, and cheap land.

The governing feature in the design of the plant was economy. To this end the plant was planned so as to get the best possible transportation facilities for the material during its various stages of manufacture throughout the entire works. To add to the economy the design included the use of blast-furnace gas in gas engines, driving electric generators to furnish electric power for use wherever its application was practicable.

In the general design all features were incorporated which had proved satisfactory elsewhere, it being the intention to exploit only the new applications and schemes which were absolutely necessary to the construction of such a works and thus minimize, as far as possible, experimental apparatus.

Layout of the plant. The layout of the plant has been so well described in several recent publications that detailed description need not be given here.

Present condition. Of the plant as planned, the slip and docks are complete; one-half of the ore yard and its machinery, including the ore bins, is finished; also one group of blast furnaces—of which three are in blast—with its complement of blowing engines and gas-washing plant, and the central pumping station. The pig-casting machines have all been in operation for some time. Four more blast furnaces, with their auxiliary buildings, are about 75 per cent completed. The No. 3 electric power station and storage-battery are ready for operation; the turbines in No. 2 power station have been in operation since July, 1908, and the remainder of the station is rapidly being completed. One-half of one open-hearth plant is in operation, a second plant is practically complete, and foundations are ready for a third. The entire shop group has been in operation for more than a year. The rail mill has been tried out. Most of the machinery has been installed in the billet mills. The foundations are laid and part of the structural material of the merchant mills is already erected.

In the part of the works now complete there are 110 electric traveling cranes with an aggregate lifting capacity of 3812 tons, 22,025 h.p. in direct current, 5312 h.p. in 440-volt alternating current; 27,000 h.p. in 6600-volt alternating-current motors have already been operated. About an equal aggregate number of horse power will be required for the operation of

that part of the plant now under construction, and still more for parts which are at present being designed.

Of the transmission line 31,000 ft. has been completed, 300 tons of copper wire being strung over the 281 supports. In addition to this, a total of 500 tons of copper wire and cable of all sizes is used in the distribution of current to motors, lights, etc., 19.5 tons of copper bar on the main distribution towers directly outside of the station and 100 tons of bar and cable inside the station.

Electric power station. For this part of the plant it is intended to use the gas available from eight blast furnaces. On account

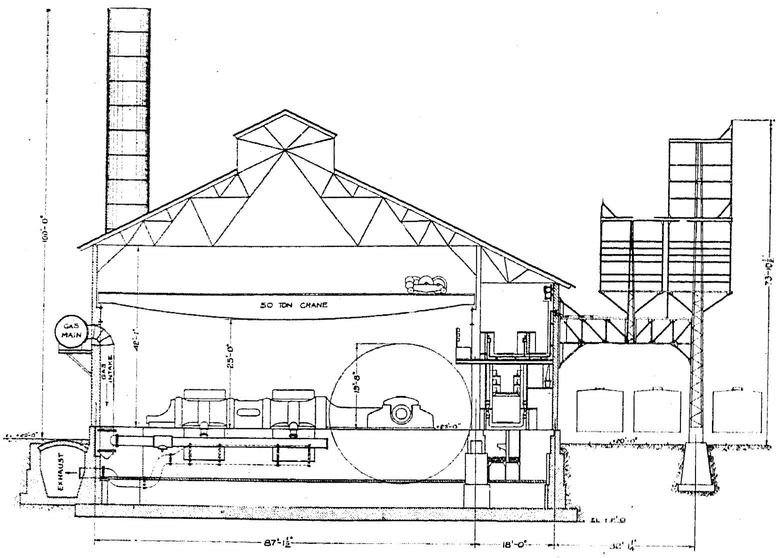
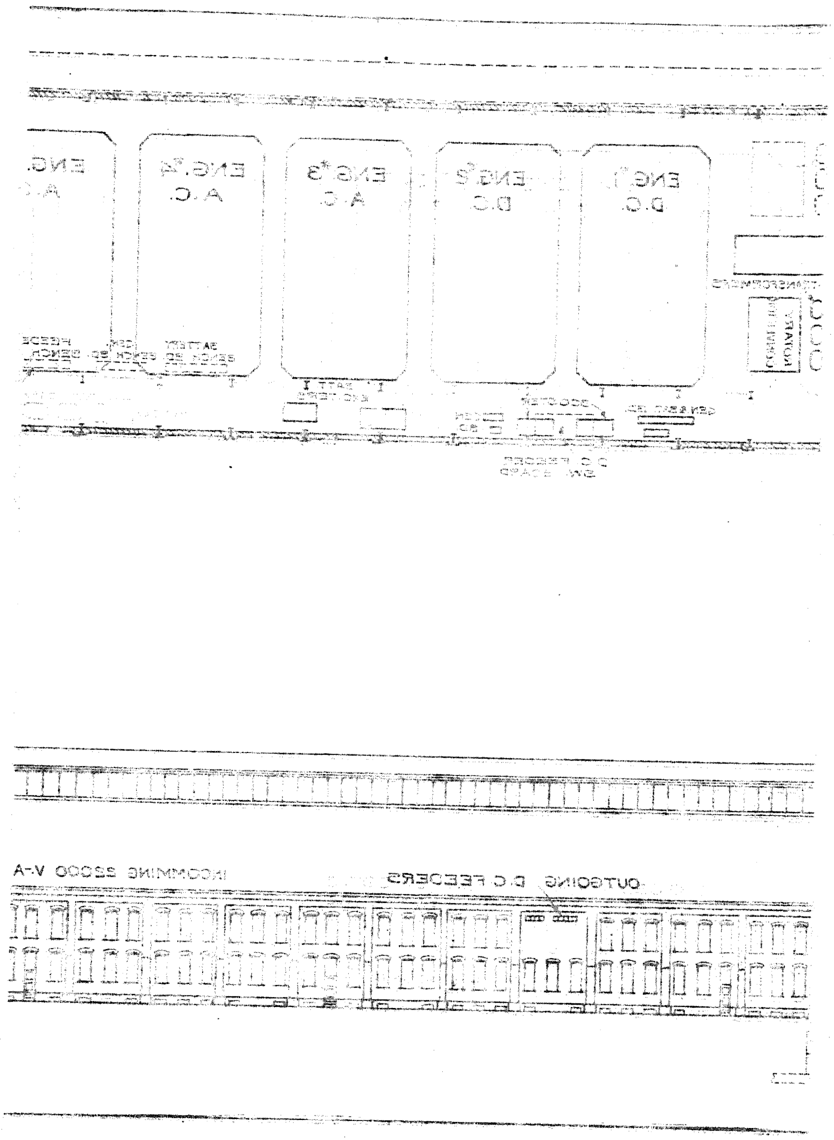
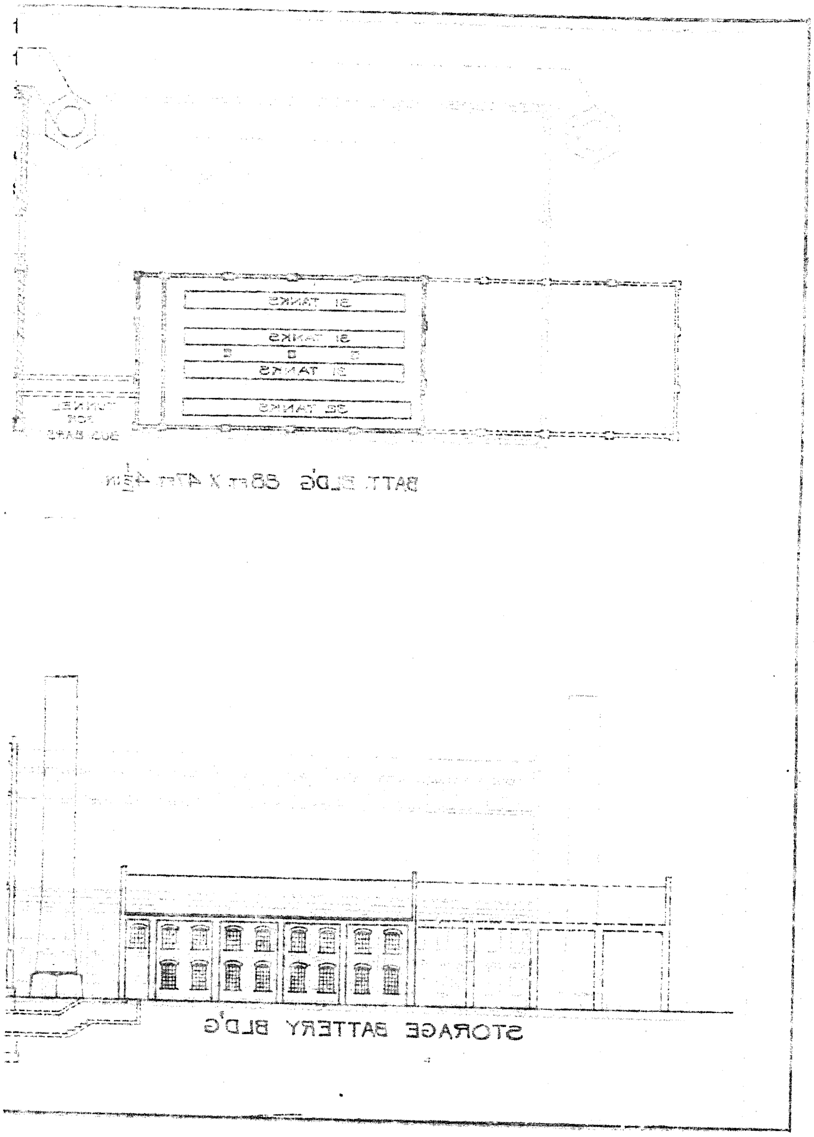


FIG. 4.—Cross-section of power station.

of the large amount of current, the especially large number of circuits and units, and also to make the operation more reliable, this plant is divided into two sections, which are called power houses No. 2 and No. 3, respectively.

The eight blast furnaces producing 3600 tons of pig iron per 24 hours will give a total of 22,450,000 cu. ft. of gas per hour. Thirty per cent, or 6,750,000 cu. ft. of this gas is used for heating stoves; 7.5 per cent, or 1,700,000 cu. ft. is used under the boilers to furnish steam for spare steam engines, pumps, and miscellaneous heating; 2.5 per cent, or 600,000 cu. ft. is necessary for operating the gas washers; 12.5 per cent, or 2,800,-




31 TANKS
31 TANKS
31 TANKS
32 TANKS
TUNNEL FOR BUS BARS
BATT. BLD'G 28 ft X 47 ft 4½ in
STORAGE BATTERY BLD'G

000 cu. ft. is used by the blast-furnace blowing engines; 2.5 per cent, or 600,000 cu. ft. for auxiliary use in connection with these engines. The remaining 45 per cent, or 10,000,000 cu. ft. per hour, is available for power purposes.

If estimated at 90 B.t.u. per cubic foot and 10,000 B.t.u. per brake horse power, this gas is equal to 110,000 brake horse power in gas engines.

There are installed in No. 2 and No. 3 electric power stations a total of 17 gas engines each rated at 3000 h.p., but capable of about 50 per cent overload. It will therefore be seen that only approximately 50 per cent of the available power as calculated will be used in this station. This allotment will make allowance for furnaces out of blast, and for shortages of gas due to troubles that are liable to occur in furnaces during operation.

The electrical equipment of power houses No. 2 and No. 3 comprises fifteen 2000-kw., 6600 volt, three-phase, 25-cycle alternating-current units, two 2000-kw., 250-volt direct-current units—all driven by gas engines—and also two 2000-kw. alternating-current turbine units.

The gas engines. The gas engines are horizontal twin tandem, double-acting, running at 83⅓ rev. per min. The cylinders are 44 in. in diameter by 54 in. stroke. The floor space occupied by each engine and generator is 74 ft. by 39 ft., with total approximate weight of 1,700,000 lb. The largest piece in the engine is the bed plate, which weighs from 90 to 95 tons. The flywheel is 23 ft. in diameter and weighs 200,000 lb.

Turbines. The two turbines are rated at 2000 kw. each, and were installed primarily for use in the construction of the plant and to furnish power for starting up. It is also expected that they will assist materially in the regulation of the station by taking care, to a certain extent, of the sudden peak loads.

The switching apparatus and station wiring were designed for simplicity of control, reliability, and ease of repairs. All apparatus, including that for storage-battery regulation, is controlled at the benchboard. A 250-volt excitation current is used and tie-switches are provided, so that in case of failure of exciter sets, or for starting up after a shutdown, current can be supplied direct from the direct-current station bus-bar.

The direct-current switchboards are all of the single-potential type, with ample space between switchboards of opposite polarity. Individual generator switchboards are used so as to

get the protective apparatus as close as possible to the generator.

The alternating-current switchboard is of the double bus-bar type, each bus-bar being protected by an electrolytic lightning arrester. In addition to this protection, each feeder has a multigap arrester.

The 22,000-volt line to the South Chicago works is protected by both electrolytic and multigap arresters and has an automatic oil-switch on both sides of the transformers.

Storage-battery. The installation of the storage-battery was

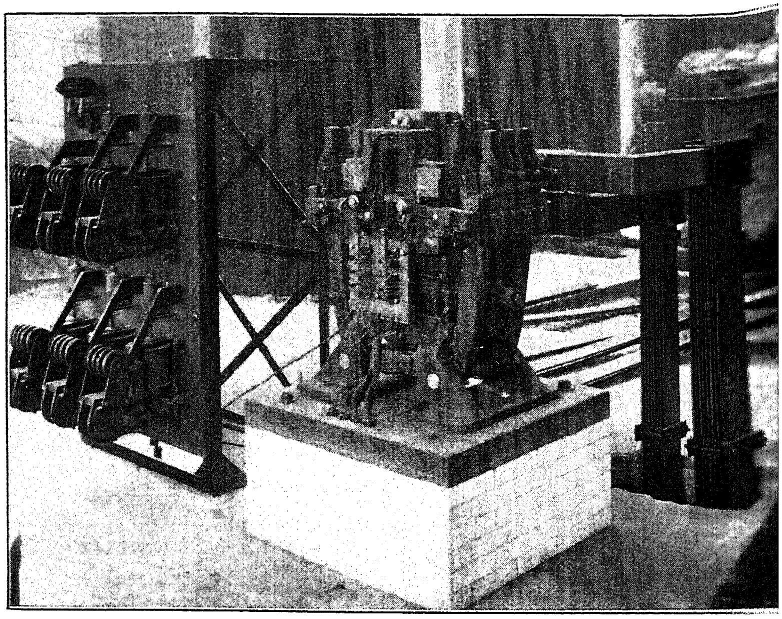


FIG. 5.—10,000-ampere remote-control switch.

for the purpose of minimizing the fluctuation of load on the generating station. The storage-battery consists of two separate batteries of 125 cells, 73 plates per cell, each battery having a rating of 4320 amperes, with a momentary rating of from two to three times that amount. They are installed in a two-story building located directly north of the power station, the connection between the two buildings being through a tunnel.

The direct-current regulation is accomplished by means of two 2500-ampere, 35-volt boosters. The motors and generators of this booster are of the interpole type, controlled by a carbon

pile regulator acting through a motor-driven exciter. The alternating-current regulation is accomplished by means of special 2000-kw. split-pole converters. The regulating current for this converter is supplied by a series transformer in the leg of each generator lead. In turn, these transformers are connected to a totalizing transformer of the compensator type which supplies current to a synchronous motor-driven special

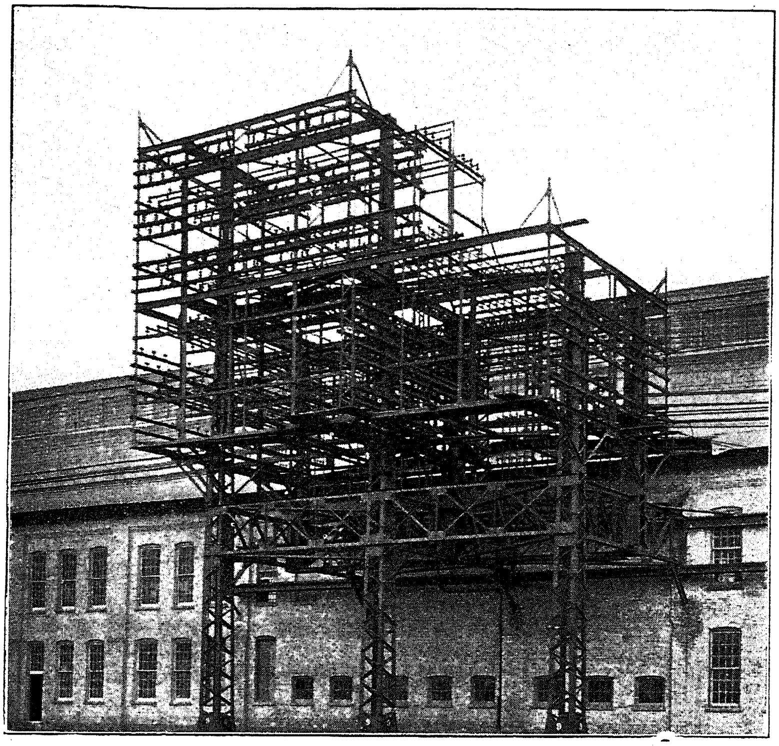


Fig. 6.—One of the special alternating-current wiring towers.

synchronous converter. From the direct-current side of this converter is taken the regulating current for the split-pole converter. A special 10,000-ampere remote-control switch, shown in Fig. 5, short-circuits the starting resistance.

The generator oil-switches are arranged so that the regulating transformer of each generator is short-circuited when that generator is not in service. Connections are also made so that when stations No. 2 and No. 3 are tied together the battery

will regulate on both, but should either tie-switch be thrown out, thus disconnecting the two stations, no regulation is possible on station No. 2 and the current transformer in the legs of the generators in that station are at the same time short-circuited. In respect to the average load on the station the

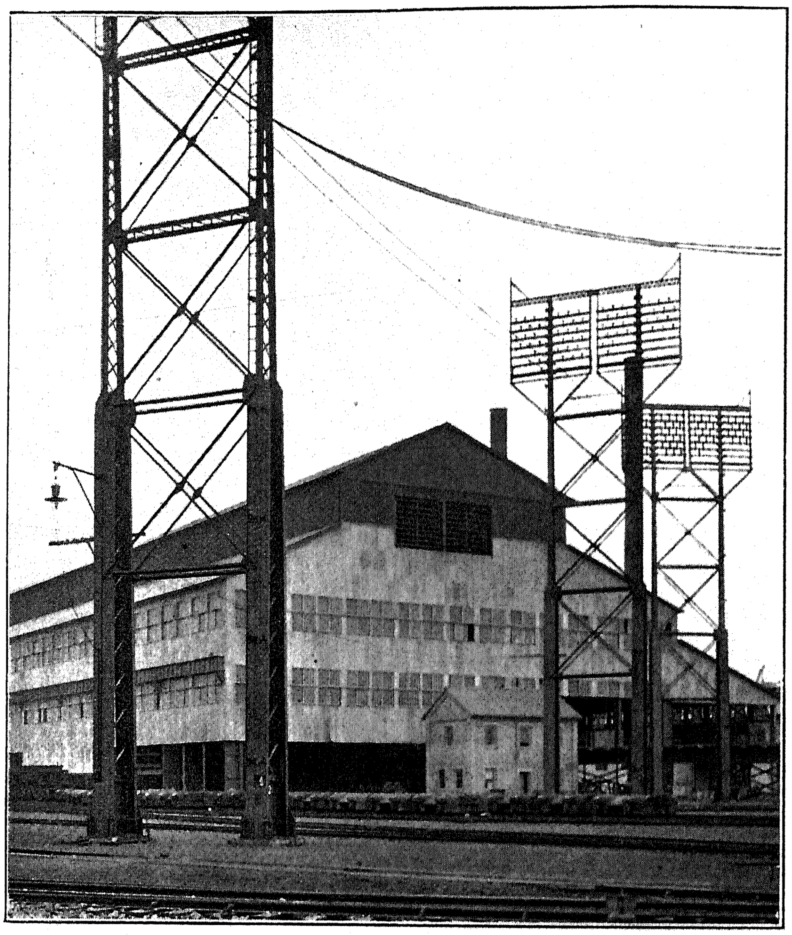


FIG. 7.—The standard 75-ft. straight-line towers.

regulation of both direct current and alternating current is by remote control from the benchboard.

The transmission system is in duplicate, each section having sufficient capacity to carry the entire load in case of accident to the other sections. The lines are supported upon a steel

tower construction made exceptionally heavy on account of the great height of the towers and heavy complement of feeders.

No holes are bored in the cross-arms, which are clipped to the poles, the special malleable pins being clipped to the arms.

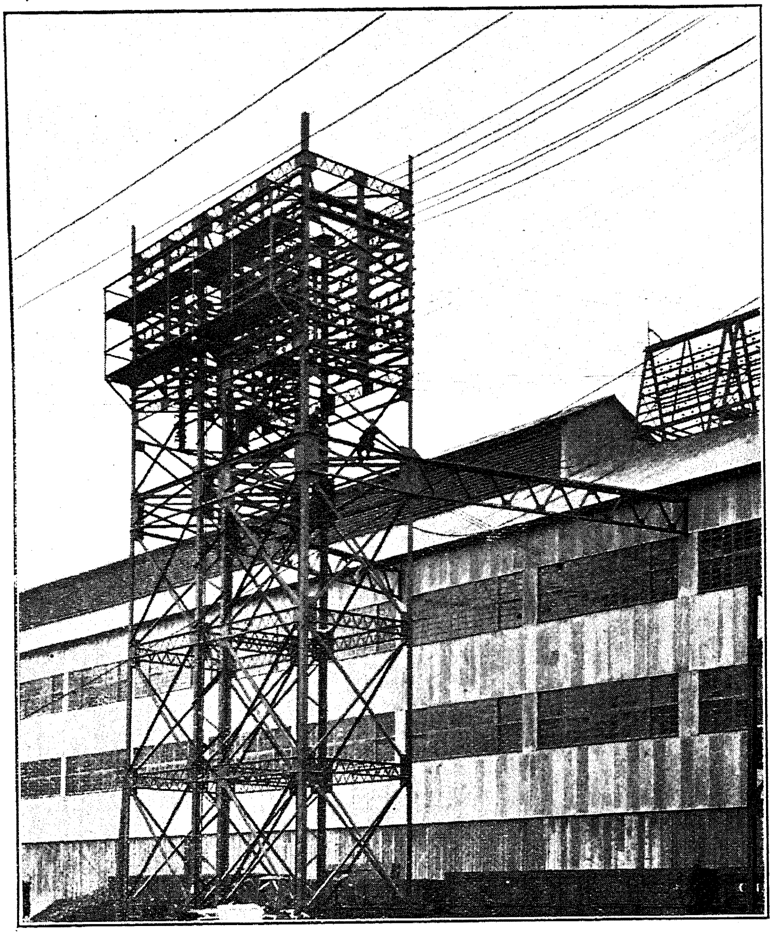


FIG. 8.—Dead-end tower between the rail and the billet mill.

The alternating-current insulators, which are designed for 17,000 volts working pressure, are of three colors to distinguish the phases over the entire works. As will be noted in Fig. 7 and 8, extensions above the pole proper are provided for carrying $\frac{3}{8}$ in. galvanized steel ground cables. Each pole is also well grounded.

Sub-stations. There are three sub-stations located as shown on the plan. Sub-station No. 1 is located in the rail mill and consists of four 500-kw., 6600-volt synchronous motors direct coupled to 250-volt direct-current generators. This sub-station

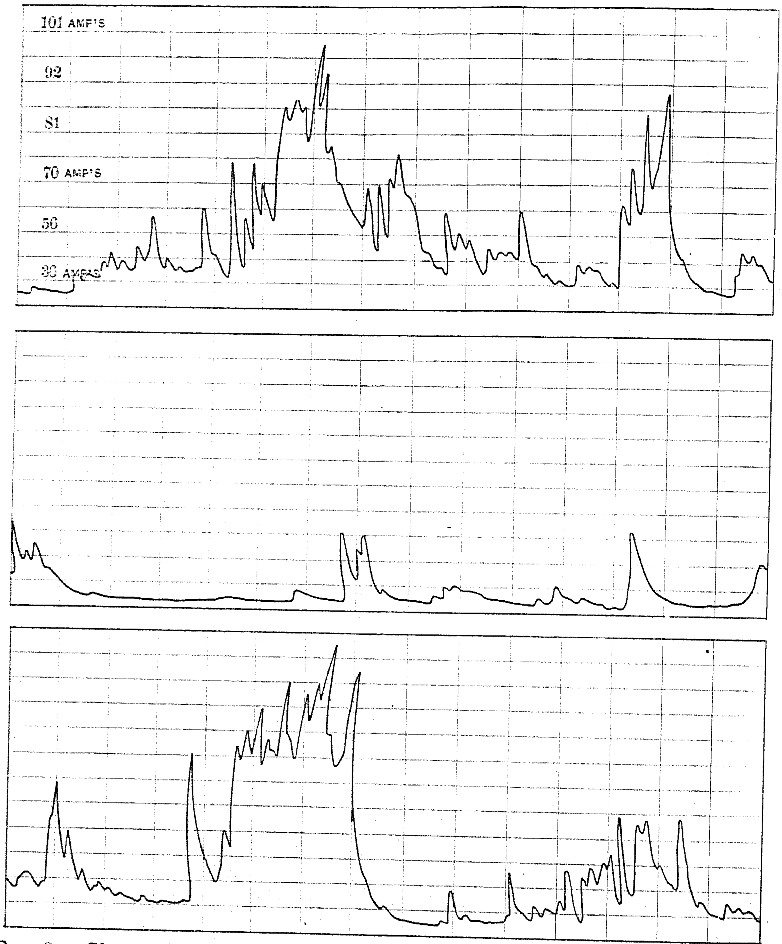


FIG. 9.—Chart showing fluctuating load on a 500 kw. sub-station motor generator set.—To be read from left to right.

normally supplies current for all the direct-current apparatus in the shop group, rail mill, and billet mill.

Sub-stations No. 2 and No. 3 each have two units, duplicates of those in sub-station No. 1, and normally supply current to the ore unloaders and bridges.

ent power furnished from the two 2000-kw.
units in the power station is used to supply
motors for the blast furnaces and open-hearth
e mills are not in operation and only the
cranes are needed, it will be possible to shut
ion and furnish power direct from the power
thod of operating will result in considerable
g expense.

ie character of the load on this sub-station
in service. A study of this chart shows the
be such that a storage-battery installed at the
be of great advantage in its operation. This
bly be installed in the near future.

tions. There are located at various parts of
stallations of transformers for supplying 440-
alternating current. In these stations there
ine 800-kilovolt-ampere, oil-insulated, water-
rs, and twenty-seven 100-kilovolt-ampere, oil-
ling transformers.

PLANT POWER

g station. All the water for the works is
ke at one point. There are two 10-ft. tunnels
west from the middle of the slip, feeding a
ping station located midway between the gas-
blast furnaces 8 and 9.

re are installed four centrifugal pumps, each
lons capacity per 24 hours, pumping against
t. These pumps are driven by three-phase,
per min. induction motors.

ake this installation as free as possible from
to electrical troubles, there are two main
station, one coming from power station No. 2,
ower station No. 3. Each feeder has its own
three 800-kilovolt-ampere transformers, and
-driven pumps are operated from each set of
ie bus-bars are provided with disconnecting
either set or both sets of transformers can be
her or both main feeders.

ant. In the tabulation of the use of blast-
is an allowance of 2.5 per cent for auxiliary
auxiliary machinery is used in the gas-washing
one such plant for every group of four blast

furnaces. Each gas-washing plant consists of eight washers, driven by 150-h.p., 440-volt, 375 rev. per min. induction motors. These motors are fed from one of the transformer installations,

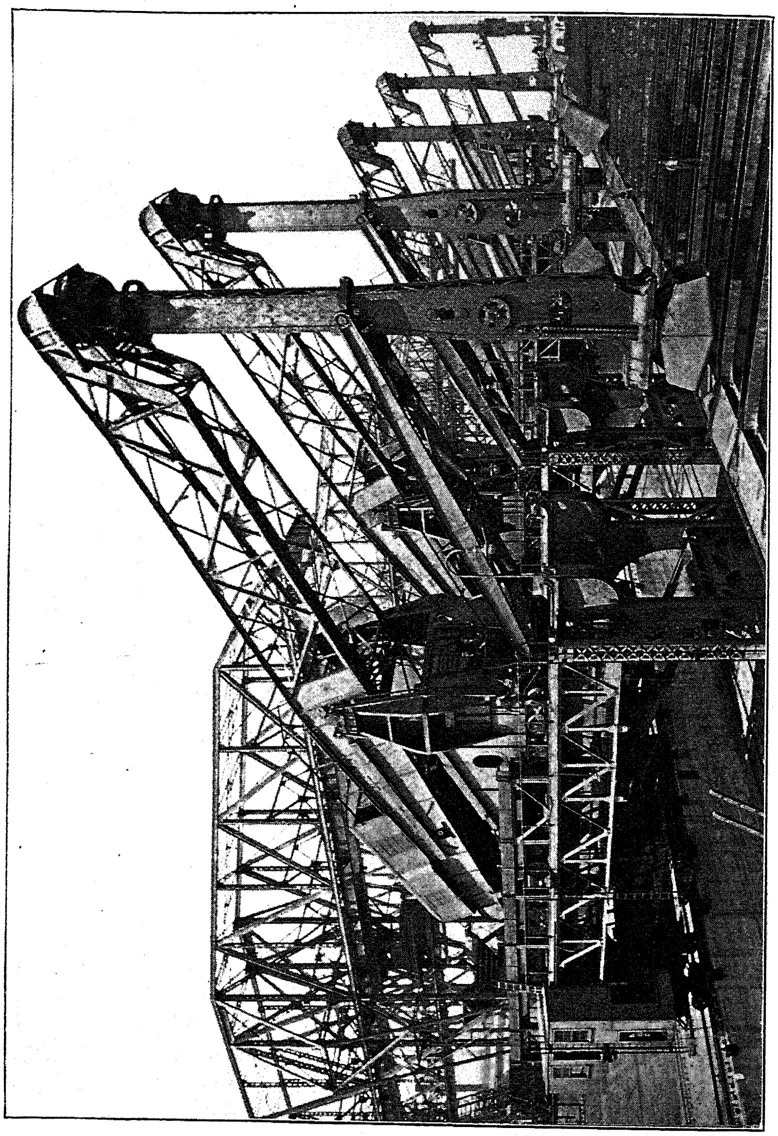


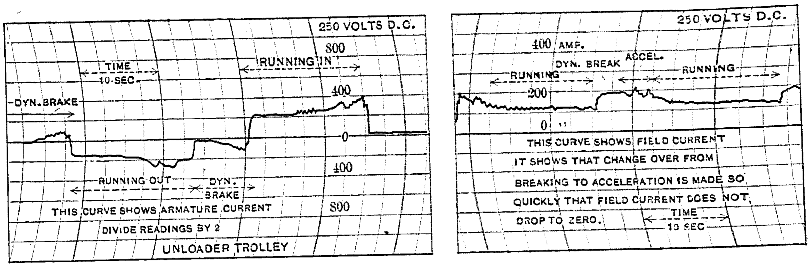
Fig. 10.—Electrically operated ore unloaders.

but the secondaries of this transformer station can also be tied in parallel with the transformers in the pumping station, running both as a unit. In case of the failure of one of the sta-

tions, a part of the pumps and the gas washers can be operated from the other station.

Repair shops. To the south of the rail mill on both sides of the main roadway are located the various repair shops, consisting of machine shops, boiler shop, blacksmith shop, foundry, electric repair shop, carpenter and pattern shop, pattern storage, general store house, brick shed, stable, roll shop, and locomotive house. These shops are operated solely for the repairs necessary to the apparatus throughout the plant, and when in normal operation very little material is purchased from the outside in the finished state, practically all the work being done within the plant. A total motor capacity of 2440 h.p. is installed for operating these shops.

The following description is given in the same order as that in which the raw material is carried through the various processes, until it comes out as finished steel.



FIGS. 11 and 12.—Charts showing the fluctuating load on the ore unloaders. To be read from right to left.

Ore unloaders. Located on the edge of the slip are five ore unloaders, shown in Fig. 10, of 10 tons capacity each. The total weight of each machine is 890,000 lb. The motor equipment of these unloaders is given in the following table:

TABLE A.—ORE UNLOADER

	Horse power	Total horse power	Service	Type
2	25	50	Bucket rotate	Series direct-current
1	50	50	Trolley	Series direct-current
1	85	85	Bucket close	Compound direct-current 75 per cent shunt 25 per cent series
1	100	100	Br. & Car	Series direct-current
1	150	150	Hoist	Compound direct-current 60 per cent shunt 40 per cent series
		435 total		

Total weight 890,000 lb.

The character of the load due to the various motions of these machines is given in the following. Fig. 11 shows the load on the trolley motor on which is used a dynamic brake for stopping. Fig. 12 shows that the change from driving to braking is made so quickly that no record whatever is made of this interruption by the recording instrument. Fig. 13 shows very clearly the power required for hoisting and lowering. With the exception of the car haulage and bridge motion, all motions of the unloader are controlled through remote-control automatic magnetic torque-limit controllers. The master controllers for operating these motors are located in the leg of the unloader, so that the operator who rides with the bucket not only has a clear view of everything he is doing, but has absolute control of all motions of the machine.

Ore bridges. The ore taken out of the boats is deposited in

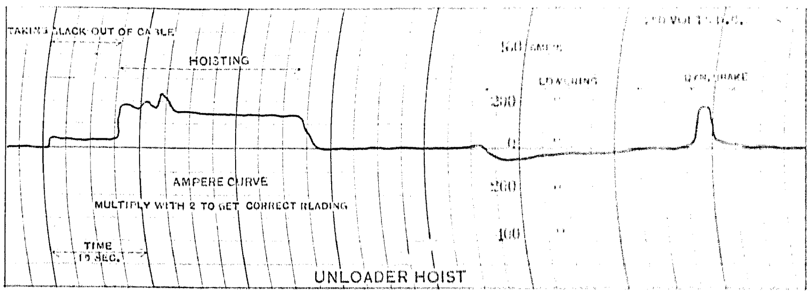


FIG. 13.—Chart showing the fluctuating load on the ore unloaders. To be read from right to left.

a huge concrete trough directly in the rear of the unloaders. From this point it is picked up by the ore bridges, of which there are five. These bridges are of the cantilever type, having a total over-all length of 459 ft. The lifting capacity of this bridge is 13 tons of ore, in addition to the weight of the bucket. The total weight of the bridge is 238,000 lb. The rear end of the bridge is considerably longer than is necessary to take ore out of the trough; the bridge was made this way so as to be used as a traveling crane for repairing the unloaders.

The motor equipment of the bridges is given in the following table. The trolley and hoist motions are controlled by automatic magnetic control. The secondary motions, which are seldom used, are controlled by hand. Fig. 14 and 15 represent clearly the character of load on the hoist and trolley motor of the bridge.

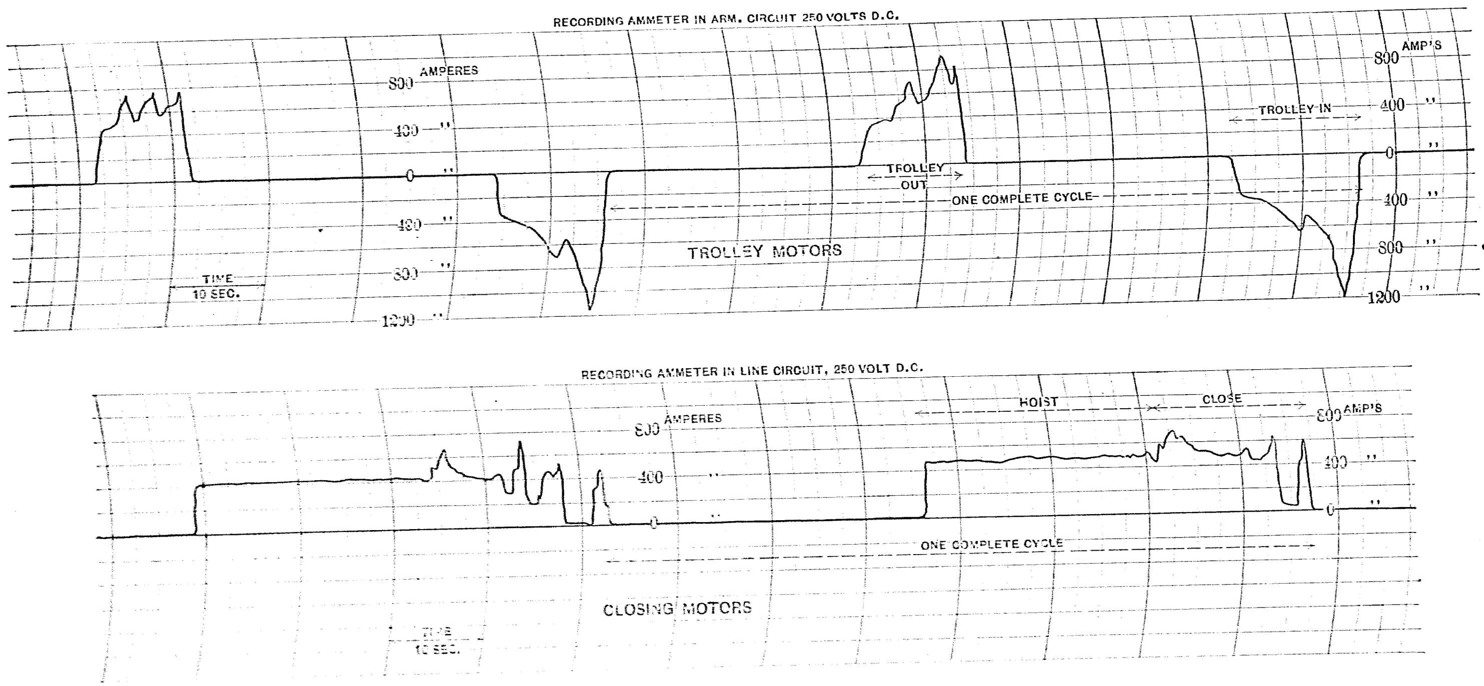


FIG. 14.—Charts showing the varying load on the ore bridges.
To be read from right to left.

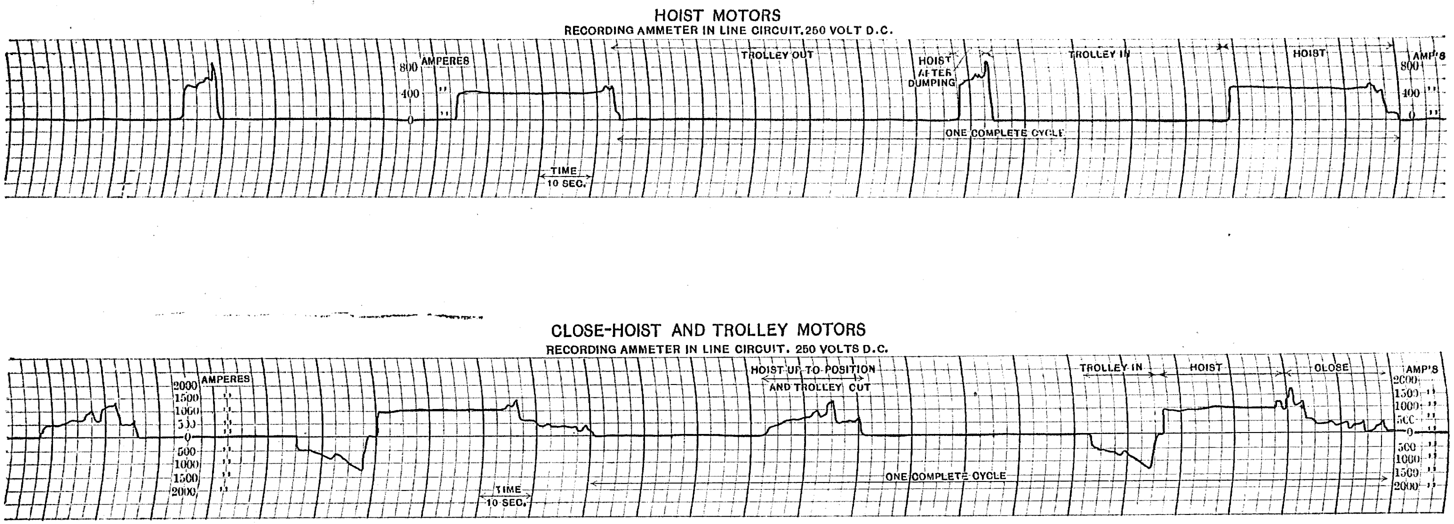


FIG. 15.—Chart showing the varying load on the ore bridges.
To be read from right to left.

TABLE B.—ORE BRIDGE

	Horse power	Total horse power	Service	Type
4	30	120	Bridge	Series direct-current
4	40	160	Trolley	Series direct-current
4	80	320	Hoist	Series direct-current
1	10	10	Ore pocket	Shunt direct-current
1	8.5	8.5	Air compressor	Series direct-current
		Total h.p. 618.5		

Weight of bridge	238,000 lb.
Capacity of hopper	60,000 lb.
Total	298,000 lb.

Bins with transfer car. Between the order yard and the line of blast furnaces there is located a long steel trestle-like structure which contains the ore, coke, and limestone bins. The ore picked from stock by the bridge is deposited in a transfer car of 100,000 lb. capacity, the electrical equipment of which consists of two 50-h.p. motors operated by means of an ordinary street-car controller, and an air compresser supplying air for opening and closing the doors and for air brakes. This car is used for transferring the ore from the bridge to the appropriate bin.

Scale larries. Underneath the bins are the scale larries, driven by one 21-h.p. direct-current series motor, and carrying a similar motor for opening and closing the door of the bin. This larry is used for transferring ore, coke, and limestone from the various bins in the appropriate quantities and depositing it in the blast-furnace skip.

Skip cars. The material is carried from this stock house and dumped into the top of the furnace by means of a double skip, driven by a 150-h.p. compound-wound motor. The control of this skip is absolutely automatic; the operator simply starts it, after which the accelerating, running, retarding, and stopping are entirely automatic.

The total travel of this skip is 163 ft. up an incline at 60 degrees to the horizontal. The time of making one trip is 60 seconds. The loads carried are approximately as follows:

Ore	7000 lb.
Coke	3600 "
Limestone	6000 "

In spite of the variable weights carried this skip stops auto-

matically within about 4 in. of the same place under all conditions. In the cast-house of each furnace is located a 15-ton crane for handling runners, scrap, etc.

Pig-casting machine. For taking care of the material from the blast furnaces on Sundays, when the open-hearth plant is not in operation, a pig-casting machine is provided. There is a main building in which are located two double-trolley ladle cranes; the main hoist is 75 tons, and the auxiliary hoist 15 tons. Six strings of moulds and conveyors are provided for disposing of the cast material. The ladle is handled by a 5-ton electrically operated jib-crane. The moulds are driven by 40-h.p., 750-

FIG. 16.—Low type open-hearth charging machine.

rev. per min. induction motors and conveyed by 30-h.p., 750-rev. per min. induction motors.

Open hearths. Each open-hearth plant consists of fourteen 60-ton furnaces. The molten metal from the blast furnaces is poured into a 300-ton mixer by a 75-ton electric ladle crane of the same design as that in the pig-machine building.

The metal is carried from the mixer to a point in front of the various furnaces by an electrically operated hot-metal car whose equipment consists of two 25-h.p., 250-volt, 290-rev. per min. series motors, operated by a standard series-parallel controller. The ladle from this car is handled and the contents poured directly into the open-hearth furnaces by another 75-ton ladle crane.

The cold scrap for charging the furnace is stored and handled in the stock-yard, which is entirely covered by two 5-ton electric traveling cranes. The stock is handled by means of electro-magnets, each one capable of handling from 1000 lb. to 2000 lb. of scrap, according to the character of the material, or of lifting 10,000 lb. in large pieces. After this scrap is loaded, the charging boxes are shifted in front of the open-hearth furnaces and the contents put in by means of a low-type charging machine, as shown in Fig. 16. The equipment consists of: a main hoist, with a 30-h.p. series motor; bridge travel, two 30-h.p. series motors; trolley, 20-h.p. series motor; tilting, 11-h.p. series motor; rotate, 3.25-h.p. series motor.

The steel is handled in 60-ton ladles by three 125-ton ladle cranes. The electric equipment is as follows: main hoist, 110 h.p.; first auxiliary hoist, 50 h.p.; second auxiliary hoist, 30 h.p.; main trolley, 30 h.p.; auxiliary trolley, 11 h.p.; bridge travel, two 50-h.p. motors. The controllers for this apparatus, except that for the main hoist on all ladle cranes, (which is automatic magnetic control) are of the ordinary hand-operated lever type.

The electrical equipment of cranes, etc., in these open hearth plants is practically the same as that in common use at other plants.

The electric equipment of the coal-handling apparatus in connection with the gas producers for these furnaces consists of a coal crusher driven by a 25-h.p. direct-current motor; a double-skip coal elevator driven by a motor similar to that on the coal crusher; and four hopper cranes, each driven by 7.5-h.p. series motors. The hopper cranes are used for distributing crushed coal in overhead bins directly into the producers.

The control of the skip is entirely automatic in its action. As the descending car approaches the bottom point of its travel, it opens the door of the coal bin; in stopping it rests on a counter-weighted lever. When the skip is nearly filled with coal the counterweight on this lever is overbalanced, the skip dropping down about 3 in. more. This actuates the switch which starts the skip on its upward travel, its first action being to close the door of the coal bin. The operation is repeated by the second skip and continues, unless stopped, until the coal bin is entirely empty.

In connection with the open-hearth plant are two stripper buildings, one of which contains one, and the other, two, 200-ton electrically operated stripper cranes. These cranes are equipped

with 100-h.p., 375-rev. per min. series motors on the main hoist; a duplicate to the above on the stripping hoist; 30-h.p., 500 rev. per min. trolley motors, and one 50-h.p. 480-rev. per min. motor on the bridge.

In the soaking pits are three 7.5-ton soaking-pit or ingot cranes. Each of these cranes has a 5-ton, high-speed auxiliary hoist on the trolley for repairing the pits. The electrical equipment is as follows: main hoist, 50 h.p.; auxiliary hoist, 30 h.p.; trolley, 7.5 h.p.; bridge, two 50 h.p.; tongs, turning and opening, 5 h.p.

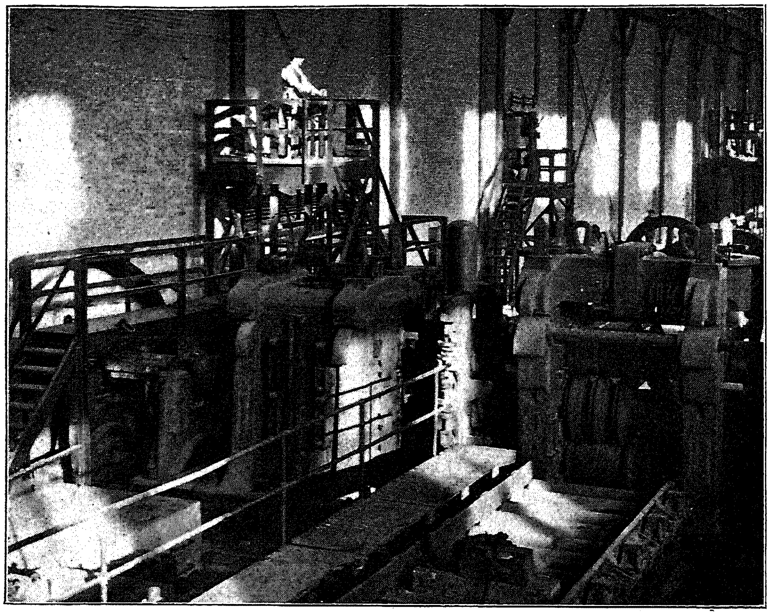


FIG. 17.—Control pulpits for rail mill.

For delivering the ingots from the pit to the first table in the rolling mill two ingot buggies are used, for which a special control has been designed. Each buggy is intended to take care of six rows of four pits which are located respectively north and south of the table. To prevent the two buggies colliding, should both happen to be coming toward the table at the same time, the trolley bars, through which the current is transmitted to the motors on the buggies, are in three sections, one section in front of the table and one long section on each end

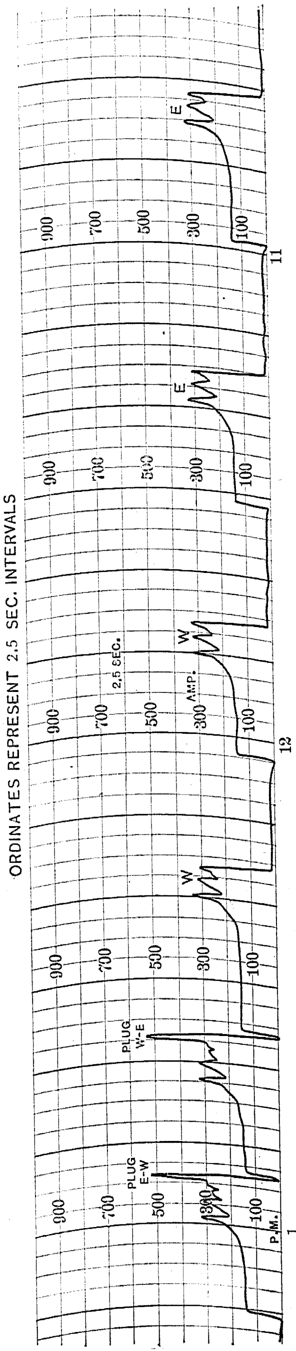


FIG. 18.—Chart showing current variation on two 25-h.p. motors, operating in parallel on the same shaft, driving a mill table. To be read from right to left.

of the first section. Each controller has six points corresponding to the six rows of pits, one point corresponding to the table and one to the off position. The operator can throw his controller to the point representing the row of pits to which he desires to run, and when the buggy reaches this point it is automatically stopped. Return motion with the controller on the table position will bring the buggy up to the table, where, however, on account of the great accuracy required, the buggy is not stopped automatically but at the will of the operator. When one buggy is at the table it is impossible for the other one to reach that point,

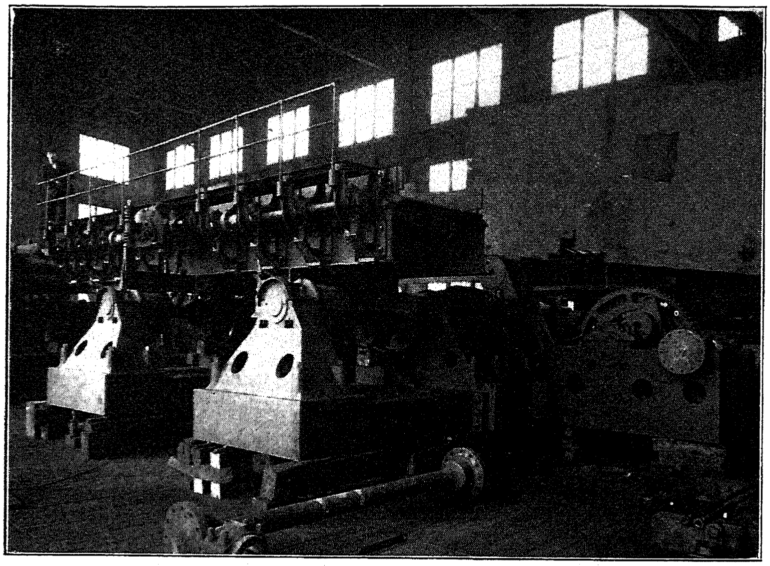


Fig. 19 —Lifting table for the 40-inch blooming mill.

as the two controllers are so interlocked as to prevent this occurrence.

Table and transfer motors. In the rolling mill proper there are 21 tables and two transfers, all of which are controlled by automatic magnetic torque-limiting control.

The eight operators stand on elevated platforms or pulpits, shown in Fig. 17. Some of the controllers are mounted on the operating pulpits and others on special platforms located near the motors which they control.

These tables and transfer motors are all of the so-called mill type. Tables running constantly are equipped with series-shunt

motors. Those doing a considerable amount of reversing or frequent starting and stopping have series-wound motors. Fig. 18 gives the ampere chart of one of these tables.

The transfers have a simple chain drive, having two sets of dogs for moving the rail sidewise, and the control is entirely automatic. When the operator throws his controller to the forward position, the transfer chain makes one-half revolution, depositing the rail in the proper place. Then the motor stops automatically. The next motion, for transferring the following rail, is made

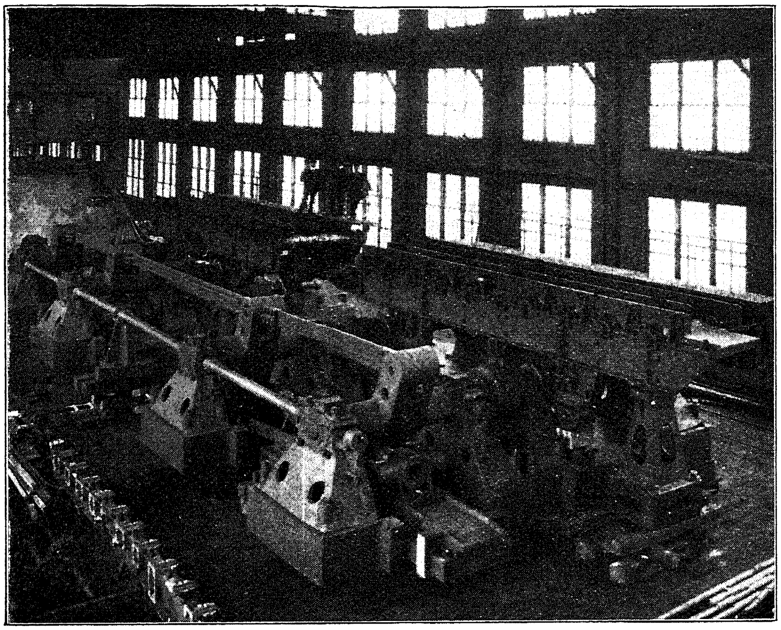


FIG. 20.—Lifting table for the 40-inch blooming mill.

by the operator putting his controller to the "off" position and again throwing it to the "on" position.

An interesting feature of the auxiliary drive is that of the lifting table at the three-high 40-in. blooming mill, Fig. 19, 20 and 21. This table has a total weight of 256,000 lb., partly counterbalanced by means of a cylinder containing air at 500 lb. per square inch pressure, and is lifted or lowered 40 in. in 3 sec. The operating motor is 250-h.p. 250-volt direct-current, 100-rev. per min. compound wound. The control, like that of the table motors, is entirely automatic and is manipulated by the

operator simply throwing his master controller to the up or down position, the accelerating, running, retarding, and stopping being taken care of automatically.

There is also a tilting table at the three-high roughing rolls similar to the lifting table, but of considerably less weight. This table gives one complete stroke of 26 in. either up or down in 1.5 sec. It is driven by a 150-h.p., 250-volt, 100-rev. per min., compound-wound motor and is controlled like the lifting table.

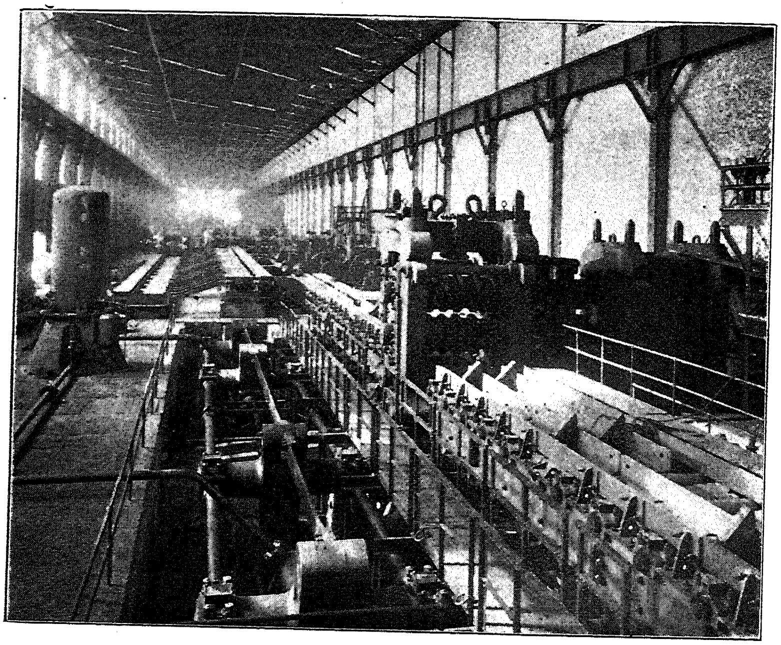


FIG. 21.—Lifting table for the-40-inch blooming mill.

The hot saws, Fig. 23, five in number, are driven individually by 40-h.p., 750-rev. per min. induction motors. The saws are 40 in. in diameter and make 1440 rev. per min. All five are raised and lowered at one time by a crank shaft, operated by a 25-h.p. series shunt-wound mill-type motor. The control is similar to that of the lifting table, except that if the operator throws his control to the first point the saws lower and stop in that position. It is then necessary for him to bring his control to the first point on the reverse in order to raise the saws; then the latter come up to the top and again stop automatically. In

regular operation, however, the operator throws to the second point; upon which the saws lower, cut the rail and again rise, stopping automatically in the upper position.

The cambering machine is driven by a 40-h.p., 750-rev. per min. induction motor. On the hot beds, four in number, there is a push-up for shoving the rails off the delivery table onto the beds, and a pull-up for dragging the rails from the beds onto the receiving table in the finishing mill. These "push-ups" and "pull-ups" are driven by 75-h.p. series mill-type motors, the controllers of which are also automatic.

From the receiving table the rails are delivered in both directions to the finishing department by feed tables similar to those in the rail mill. Because of the continuous running of these feed tables the controllers are the ordinary hand-operated type.

For delivering the rails from this table to the straightening

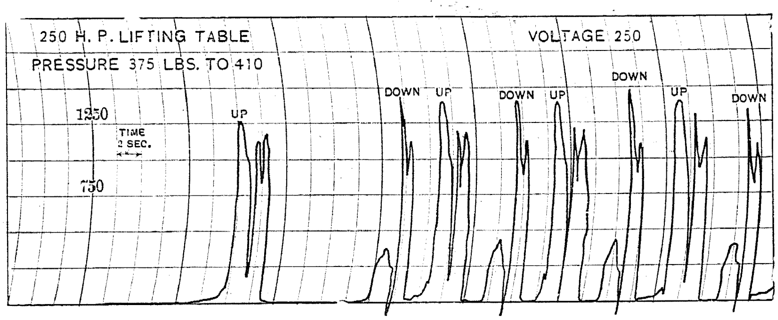


FIG. 22.—Chart showing load on lifting table.—To be read from right to left.

beds "kick-offs" are used. These are simply bars lying normally below the surface of the table but raised at an angle so that rails are lifted and then slid off by means of a cam on a motor-operated shaft. The straightening presses, 18 in number, are driven by 10-h.p., 750-rev. per min. high-resistance-rotor induction motors. Opposite the ends of the straightening beds for each straightening press is located a 24-in. headlight in which is used an ordinary 16-c.p. lamp to enable the straightener to see his rail clearly.

The drill presses, 18 in number, are driven by 10-h.p., 750-rev. per min. induction motors with ordinary squirrel-cage rotors.

All the induction motors in the finishing department are standard and are controlled by means of a triple-pole, double-throw switch which is connected to a five-wire system. The

two extra wires are taps taken off the main transformers, thus furnishing low voltage for starting. This system does away entirely with the complication and trouble accompanying individual starting compensators.

The loading beds outside are covered by two special electric cranes which transfer the rails from one part to the other or load them in cars, the rails being handled by means of large special magnets.

On the south side of the runway of these cranes is also a small monorail crane, whose sole purpose is to deliver rails to the cold-saw bed.

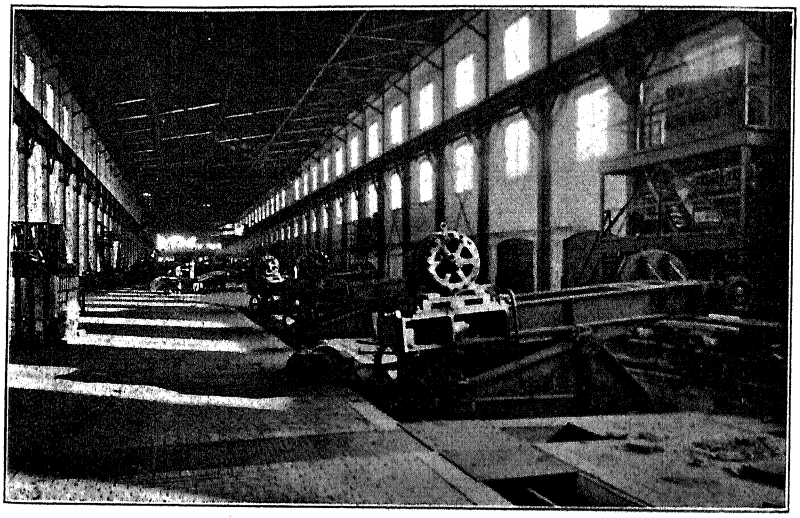


FIG. 23.—The hot saws.

The cold saw, a tooth saw 42 in. in diameter making 1800 rev. per min., is driven by a 100-h.p., 750-rev. per min. induction motor. This saw is capable of cutting an 85-lb. rail in 22 seconds, the load varying between 50 and 100 h.p. according to the part of the rail that is being cut.

Motors for driving the rolls. As outlined earlier in the paper, although electric motors have been used for some time to drive rolls, the motors used in this plant are several times larger than any motors of their type previously built. Their use for this purpose marks a new era in the industrial application of electric power, and a short description of some of the principal eatures in the design of these motors is not out of place.

Horse power:					
Normal continuous rating 40° cent rise	2000	2000	6000	6000	6000
25% overload continuous 50° cent. rise	2500	2500	7500	7500	7500
50% overload for 1 hr. 60° cent. rise	3000	3000	9000	9000	9000
Speed:					
Synchronous	214	68	83.3	88	75
Full load					
Maximum slip, %					
Performance specification number					
Outline drawing number					
Bearings:					
FLYWHEEL SIDE.					
Diameter	24	26	30	30	30
Length	60	65	70	70	70
COLLECTOR SIDE.					
Diameter	24	26	30	30	30
Length	60	65	70	70	70
Thrust collar area	575	910	1370	1370	1370
Revolving part of motor:					
Diameter	8 ft. 8 in.	21 ft. 0 in.	21 ft. 0 in.	21 ft 0 in.	21 ft. 0 in.
Flywheel effect, lb. at 1 ft. radius	354,000	8,950.000	11,600,000	11,600,000	14,100,000
Stationary part of motor:					
Outside diam. of frame	13 ft. 4 in.	28 ft. 0 in.	28 ft. 0 in.	28 ft. 0 in.	28 ft. 0 in
Width of frame	3 ft. 10 in.	3 ft. 8 in.	4 ft. 11 in.	4 ft. 11 in.	4 ft. 11 in
Air-gap, total	0.28	0.40	0.40	0.40	0.40
Induction motor:					
Poles	14	36	44	34	40
Horse power	2000	2000	6000	6000	6000
Rev. per min	214	68	83	88	75
Volts	6600	6600	6600	6600	6600
Efficiency at full load	95%	93%	95.5%	95.5%	95.5%
Power factor at full load	89%	80%	87%	88%	88%
Break-down torque	6,800h.p.	5,100h.p.	18,500h.p.	20,600h.p.	16,400h.p.
Flywheel:					
Material	Laminate	d steel	—	—	—
Number of sections					
Diameter	17 ft.	—	—	—	—
Outline dimensions:					
Height of machine above bottom of base	12 ft. 2 in.	21 ft. 0 in.	21 ft. 0 in	21 ft. 0 in.	21 ft. 0 in
Width of bed plate	20 ft. 4 in.	32 ft. 8 in.	32 ft. 8 in.	32 ft. 8 in.	32 ft. 8 in.
Length of bed plate	20 ft. 2 in.	18ft 11½in	21 ft. 9 in.	21 ft. 9 in.	21 ft. 9 in.
Total length of shaft	22 ft.4¾in.	21 ft.3¾in.	24 ft.6¾in.	24 ft.6¾in.	24 ft.6¾in.
Center line of shaft to bottom of base.	5 ft. 6 in.	7 ft. 0 in.	7 ft. 0 in.	7 ft. 0 in.	7 ft. 0 in
Weights:					
Stationary part of motor	42,500	126,000	175,000	175,000	175,000
Rotating part of motor without shaft	30,800	148,000	176,000	176,000	209,000
Shaft (hollow)	33,400	34,300	50,000	50,000	50,000
Pillow-blocks with bearings	72,300	105,500	138,000	138,000	138,000
Bedplate	100,000	150,000	185,500	185,500	185,500
Flywheel	100,000	—	—	—	—
TOTAL.					
Without flywheel	296,000	—	—	—	—
Complete	396,000	578,000	749,000	749,000	783,000

FIG. 24

The main rolls of the mill are driven by six induction motors having a combined capacity of 24,000 h.p. made up of the following units:

Two	2000 h.p. at	214	rev. per min.
One	2000 h.p. at	68	"
One	6000 h.p. at	88	"
One	6000 h.p. at	83	"
One	6000 h.p. at	75	"

In the construction of these motors the parts were made extremely heavy and rigid, following out as far as possible the practice which has proved successful in the construction of

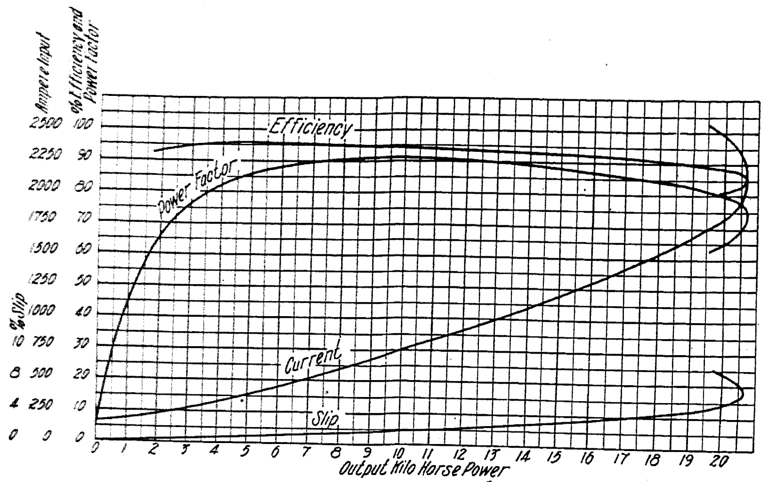


FIG. 25.—Characteristic curves of the 6000-h.p., 88 rev. per min. induction motor.

steam engines for similar duty. The stator frame is of the box-type construction, and is split into four sections for ease in handling and transportation. The rotor spider is of cast steel and is made up of four sections with two arms per section. The sections are bolted to disc hubs which are pressed on the shaft.

As the flywheel effect of the rotors necessary to overcome the excessive loads in rolling can only be determined accurately by actual trial, it was deemed advisable where possible to construct the motors so that the flywheel effect could be altered after the motors had been put in operation. This was accomplished in the 6000-h.p. and the 2000-h.p. motors at 68 rev. per min.

by attaching to the rotor spiders heavy cast-steel rims which could easily be removed and exchanged for rims of different weights.

On account of their high speed the two 2000-h.p., 214-rev. per min. motors have separate flywheels weighing 100,000 lb. each. These flywheels are built up of riveted boiler plates, which do not permit of alteration.

The end-thrust which may result from a diagonal fracture of a spindle or roll is frequently sufficient to wreck either the mill or the motor unless special precautions are taken. This problem, which is extremely difficult to solve when an engine is used for

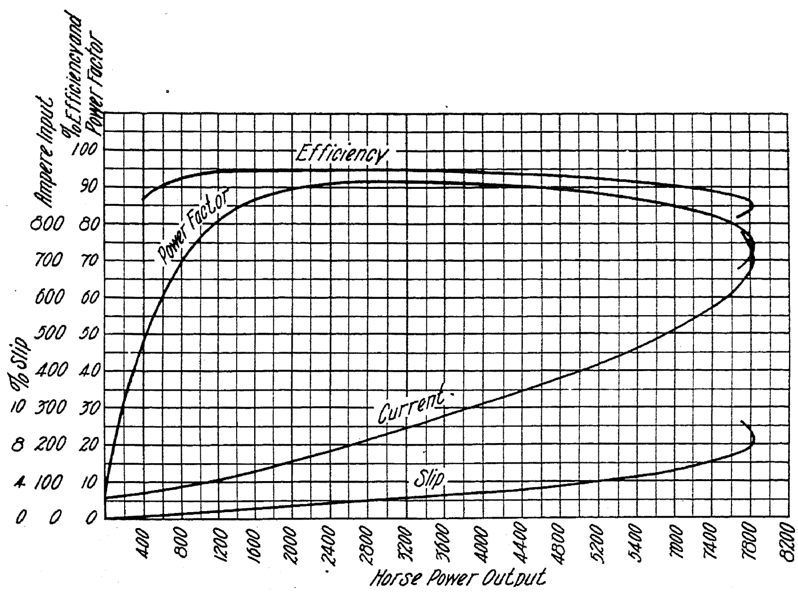


FIG. 26.—Characteristic curves of the 2000-h.p. 214-rev. per min. induction motor.

driving the rolls, is very easily solved when electric motors are used. A device termed a mechanical fuse is attached to the pedestal by two breakable rods. These rods are so proportional that they will break only when the end-thrust exceeds 150 tons. When the rods give way under this pressure, the rotor is free to move longitudinally away from the rolls, thereby relieving the thrust. To prevent injury to the brush rigging, it is so arranged as to move freely with the brushes, always maintaining their proper position on the collector rings.

The bearings are self-aligning, with oil-ring lubrication.

They can also be lubricated from an independent oiling system. Provision is made for water cooling in case of emergency.

The coils, which are assembled in open slots, are very rigid. In order, however, to prevent any possible vibration due to excessive fluctuations of the current, the coils are also firmly laced to a rigid supporting ring.

The electrical characteristics of the motors are shown by Fig. 25 and 26, which represent the results of tests of the 2000-h.p. motor at 214 rev. per min. and the 6000-h.p. motor at 88 rev. per min.

Reference to these curves shows that the power-factor and efficiency are near their maximum at the rated output of the motors, and that high values are maintained throughout the complete operating range.

The two 2000-h.p. 214-rev. per min. motors were received at the Gary works complete, except the flywheels which were built up and turned after the installation of the motors. The three 6000-h.p. and one 2000-h.p., 68-rev. per min. motors were assembled and wound in position. The 2000-h.p., 214-rev. per min. motor and its controllers were tested before shipment, but the controllers for the remaining four motors were first assembled at the works.

Only two minor troubles have developed so far in the entire installation. On starting up one of the 6000-h.p. motors, one section of the rotor resistance overheated, but investigation proved this trouble to be due to a stray piece of arc-lamp carbon. In starting up another of these motors trouble developed due to a broken grid in the rotor resistance.

Control for roll-train motors. In designing the equipment for these 2000-h.p., and 6000-h.p., 6600-volt induction motors, not only were the sizes of the motors to be controlled beyond anything previously attempted, but the specifications presented many novel features.

For most of the motors the service required a very large flywheel effect. Because of the well-known characteristics of the induction motor, it was clearly recognized that there would necessarily be large fluctuations in current, even though the flywheels were very large, unless some means were employed for automatically introducing resistance into the motor circuit whenever the load was sufficient to cause even as small a change of speed as two or three per cent. It was desirable that the automatic features be adjusted so as to operate continuously, regu-

lating the current taken by the motors so that the demands on the source of supply for any one motor would be uniform—the motor and its flywheel, meanwhile, accelerating and decelerating at a point just below synchronism to meet the power demands of the rail mill which it was driving.

The large current in the secondary circuits of the motors necessitated separate contactors. Rheostats were arranged in multiple delta, and the successive value of the resistance was made such that as each contactor closed, the rush of current through it would be approximately one-fourth of the intake of the motor. This arrangement prevents the rheostats or contactors being called upon to carry anywhere near their full rated current, and gives the whole control system a conservative continuous rating.

The control apparatus for each motor consists of:

A. Preliminary control apparatus, which includes the oil switches, reversing switches, series transformers, primary relays, etc.

B. Secondary control, comprising the rheostats, contactors, regulating panel, etc.

C. Master controller.

Reference to the plan of the plant shows that the motors and the controlling apparatus are placed in a room separated from the rolling mill proper by a partition. The master controller, however, is in the rolling mill. Under these conditions, therefore, the operator must depend chiefly on the automatic features of the control, which are so far away that he must judge of their correct working by the behavior of the rolls while the steel is passing through them. In case of emergency provision is made for the motor attendant to shut down the motors independently of the master controller.

The 6600-volt circuit enters through the triple-pole main oil-switch; thence it passes through a number of oil-switches which determine the direction of rotation. These switches are interlocked, making it impossible for them to be operated unless the main switch is open. Additional interlocks prevent them from being closed simultaneously.

This group of preliminary-control apparatus also includes the necessary relays to open the circuit in case of overload and to prevent the secondary control from being operated in an injurious manner.

The arrangement of the secondary-control rheostats in multiple

delta is shown in Fig. 30. Direct current at 250 volts adopted for the control of this secondary apparatus, and voltages were so arranged that each contactor coil is subject a pressure of only about 15 volts. This pressure is so small the arcing is insignificant. To compensate for the different nu of contactors in circuit at different speeds, a balancing resist equaling the resistance of the contactor is cut in or out o cuit inversely as the contactor. This keeps the current ir contactor circuit approximately constant at about 5 amp and results in a simple arrangement of contacts on the

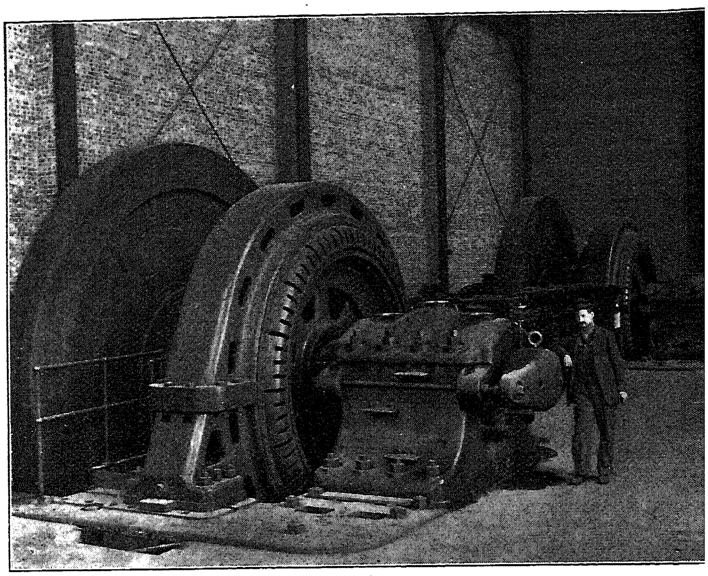


FIG. 27.—The 2000-h.p., 214-rev. per min. induction motor, which passes 1, 2, 3, and 4.

lating device. The regulating device consists of two conc groups of buttons, one connected to the contactor coils an other group to the contactor resistances, the two groups cross-connected, step by step, by a simple arm cont automatically. Should the 250-volt supply fail, a w attached to the grooved pulley on this arm serves to r the mechanism to the off position. The master controller operates both the preliminary-control and secondary-contr paratus has two handles; one is a reversing handle, anc other is for applying and shutting off the power.

The rail mill. The rail mill has a capacity of 4000 tons of finished rails per 24 hours. It is not only the largest, but also the only motor-driven mill in the world rolling rails directly from the ingot without reheating.

There are nine passes in the blooming mill. The first two passes are two-high rolls of 42 in. pitch diameter running at 6 rev. per min., and are connected to one of the 2000-h.p., 214-rev. per min. motors through gear reductions. The next two passes

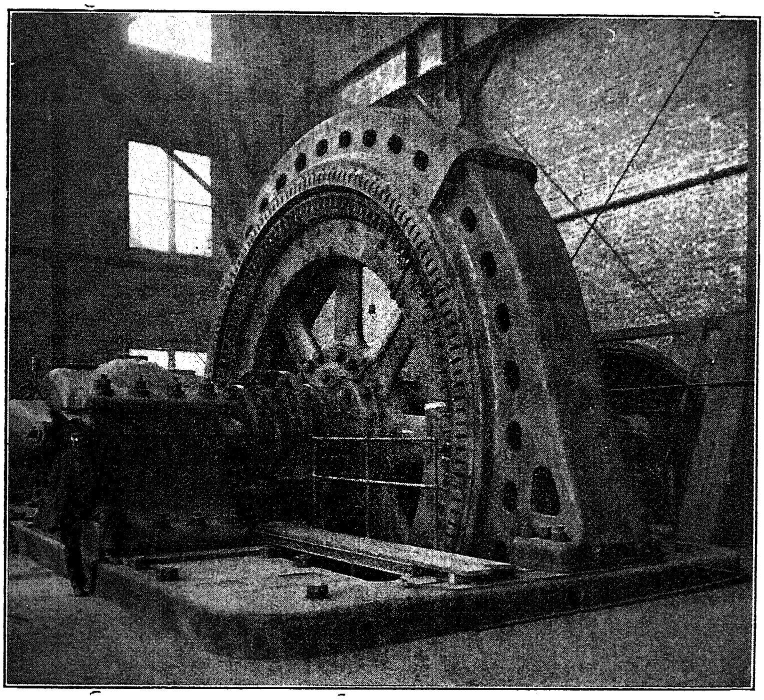


FIG. 28.—The 6000-h.p., 75-rev. per min. induction motor, which drives passes, 5, 6, 7, 8, and 9.

are identical with passes one and two, except that the rolls are 40 in. in diameter and make 10 rev. per min. and are driven in the same manner. The next five passes are made in a 40-in. three-high train direct-connected to a 6000-h.p., 75-rev. per min. motor. The lifting table for this train has already been described.

The bloom as delivered from pass 9 is then cut in two by means of the bloom shears, operated by a 75-h.p. induction motor.

The next train of rolls, which comprises a three-high roughing mill with passes 10, 11 and 12, is operated by means of the tilting table, the second edger or pass 16 and leader or pass 17. This train is 28-in., pitch diameter and direct-connected to a 6000-h.p. motor running at $83\frac{1}{3}$ rev. per min.

The next pass is the 28-in. two-high former, direct connected to a 2000-h.p., 68-rev. per min. motor.

The third roll train consists of the dummy, or pass 14; the first edger, or pass 15, and the finisher, or pass 18. These rolls also are of 28-in. pitch diameter and the train is direct-connected to a 6000-h.p. 88-rev. per min. motor.

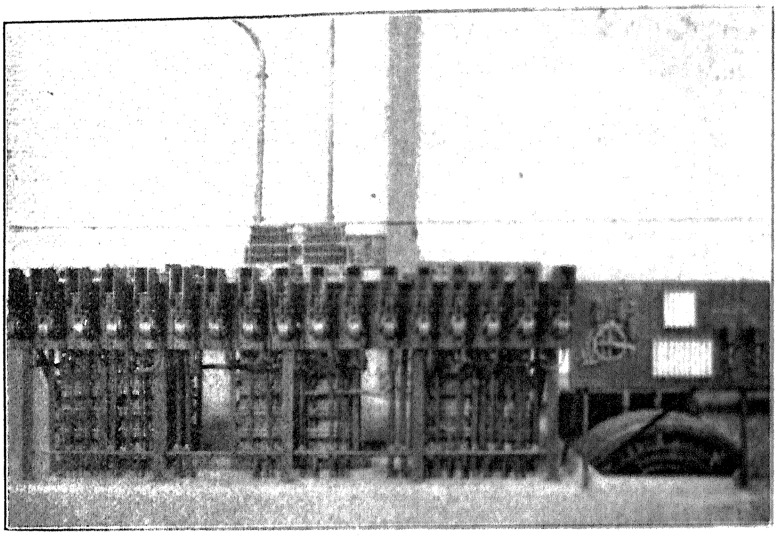


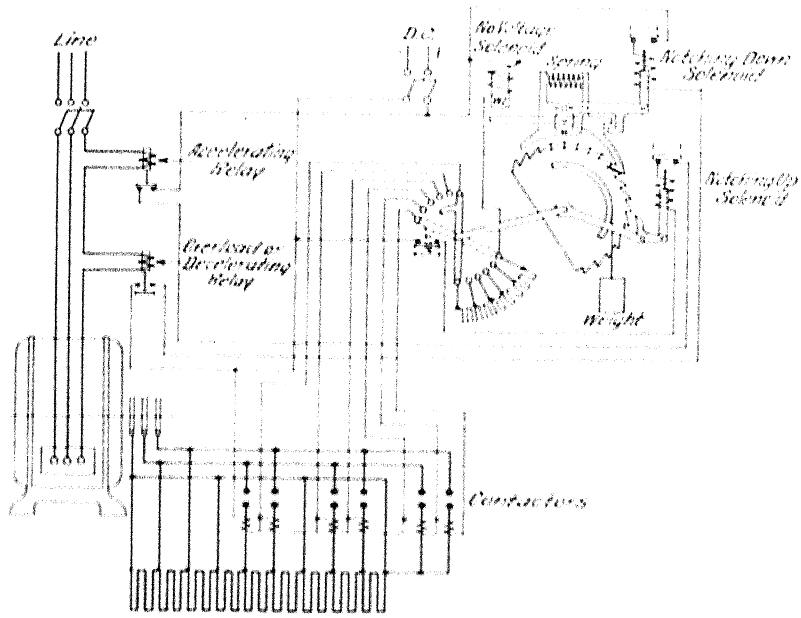
FIG. 29.—Secondary control for the 6000 h.p., 75 rev. per min. motor.

A diagram of these passes, together with the approximate shapes of the pieces leaving them, is shown in Fig. 31 and 32, the former also showing the table giving the size of the pieces leaving the various passes, the area in square inches, the weight per linear foot, percentage of reduction, length of piece, and horse power per pass—all as observed in the rolling of the first rail, which was an 80-lb., 33-ft. rail.

To supply power to the train motors of this mill there are two circuits, each of 10,000-kw. normal capacity at 6600 volts. One circuit feeds the three blooming-mill motors; that is two 2000-h.p. and one 6000-h.p. The other circuit feeds the other three motors; that is, two 6000-h.p. and one 2000-h.p.

The estimated combined load with the mill working at full capacity, with voltage and amperage of each of the circuits, is shown in Fig. 33.

In originally designing the mill careful calculations were made of the time and horse power required to operate each pass. These data, with the calculated interval between passes, are shown clearly in Fig. 35. It will be noted on the right-hand side of this curve that there are both solid and dotted areas. The dotted area represents the second piece of the ingot after



Simplified Control Connections for 6000 HP Induction Motor

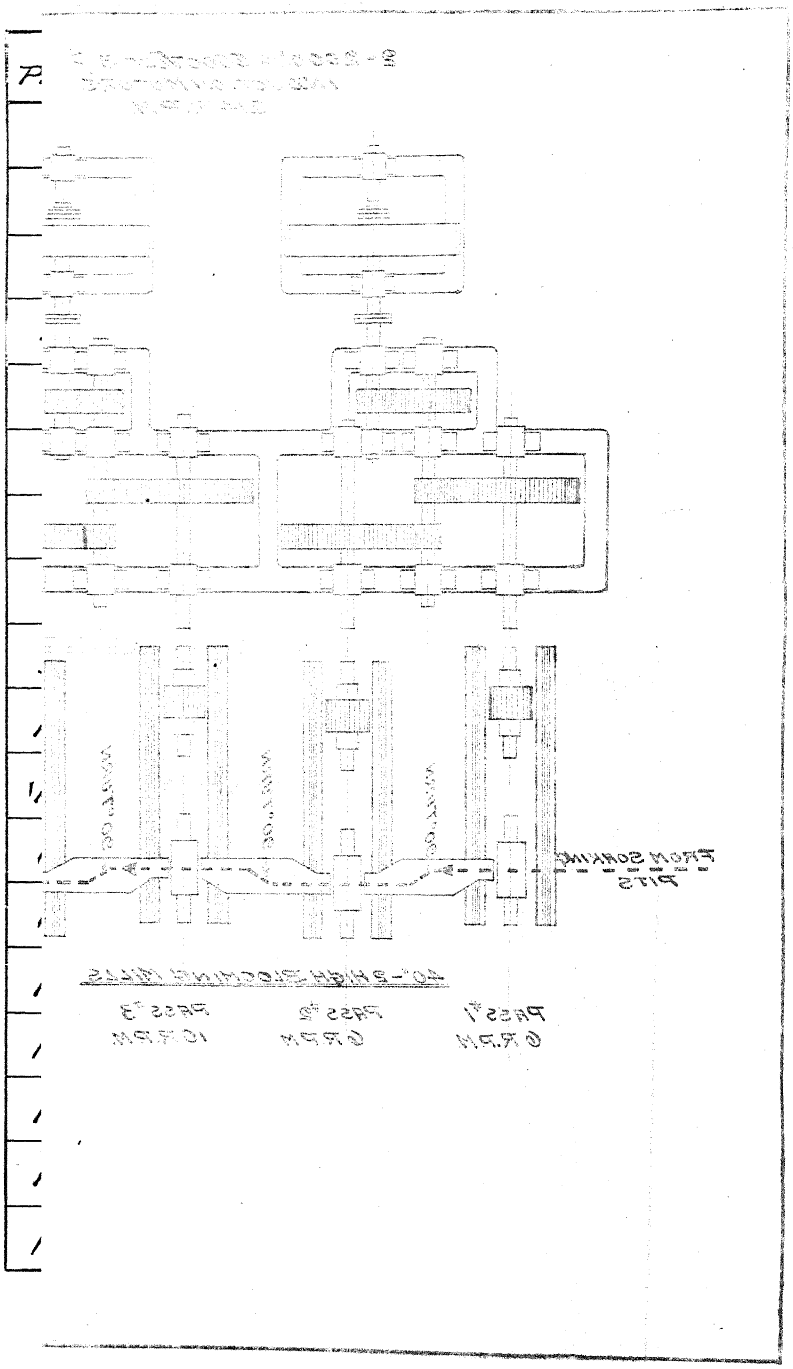
FIG. 30.—Diagram of secondary control of the rail-mill motors.

it had been cut at the bloom shears between passes 9 and 10, and the load estimated with the second piece in pass 10, while the first piece is yet in pass 12. Though it is improbable that such conditions will exist except in cases of the most rapid rolling, it was thought advisable to use them in the calculations of the mill. These calculations determined not only the total time consumed by the ingot from start to finish, but also the shortest possible time between ingots, the limiting time being in the three-high roughing mill.

PASSES	SIZE OUT	AREA OUT	WEIGHT PER FOOT	% RED.	LENGTH OF PIECE	HP ON ROLLS
1	20" × 19"	376.56	1224	21.20	6.7	1000
2	19" × 15"	282.37	918	25.00	9.0	1260
3	15 × 14½"	214.87	698	23.90	11.8	1600
4	14½" × 11½"	164.82	536	23.30	15.3	14.50
5	11½" × 11½"	130.32	424	20.90	19.4	8350
6	11½" × 9½"	107.90	351	17.20	23.5	7650
7	9½" × 9½"	88.90	289	17.60	28.5	6700
8	9½" × 7½"	70.76	230	20.40	35.8	6100
9	7¾" × 8"	58.91	191	16.74	43.1	4500
10		44.50	145	24.40	25.0	5500
11		31.25	102	29.10	35.6	5100
12		21.80	71	29.90	51.1	6200
13		18.50	60	15.50	60.4	2850
14		16.00	52	13.50	69.7	5650
15		12.80	42	20.00	86.3	
16		10.07	33	21.33	100.0	1500
17		8.62	28	14.40	130.5	980
18	RAIL	8.095	26.6	6.09	137.87	720

NOTE: TABLE FOR 80 lbs 33'-0" RAILS.
INGOT BUTT 20" × 24"
TOP 18¾" × 22¾" - 65½" WT. 8256 lbs

Fig. 31


40"-2 HIGH BLOOMING MILLS
PASS #3 10 R.P.M
PASS #2 6 R.P.M
PASS #1 6 R.P.M
FROM SOAKING PITS

These curves were then superimposed on each other at this interval, which was 31.89 sec., until the number of ingots was increased to the maximum load on the mill. Adding the ordinates under this condition Fig. 34 was produced, showing the integrated load carried by the motors. The shaded portion indicates motor and line losses. and the upper line of the curve shows the character of the load on the power station.

This cycle, which is 31.89 sec. indicates an exceedingly variable load, the total variations being from a minimum of 4300 h.p. to a maximum of 19,010 h.p. with an average of 12,025 h.p.,

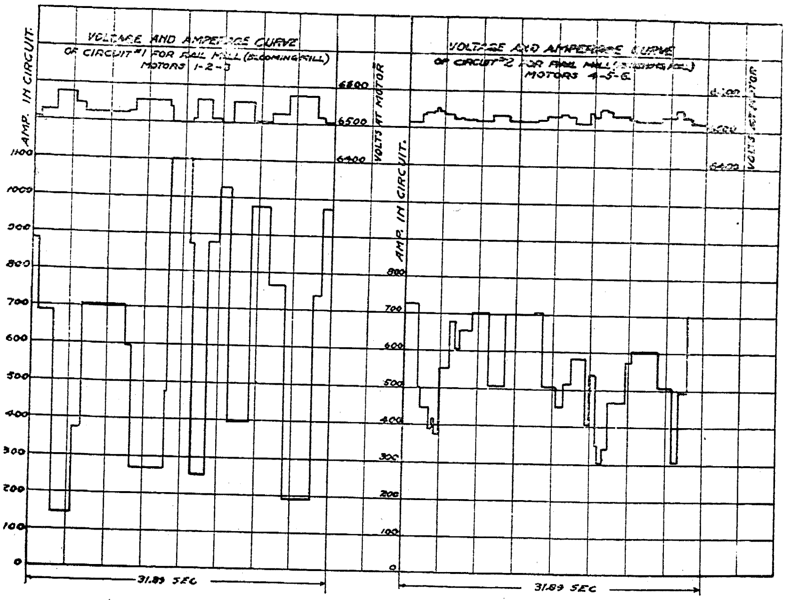


FIG. 33
To be read from left to right.

which makes the load-factor on the six-train motors almost exactly 50 per cent. The curve was developed to provide a basis for estimating the size of the storage-battery necessary to take care of the fluctuations and keep a constant load on the generating station.

Conclusions. One of the most satisfactory conclusions, so far as the Indiana Steel Co. is concerned, that can be drawn at the present date is that so far all the apparatus described in this paper, which has been tried out, has been practically a

perfect success. Nothing has occurred to indicate that there is any necessity of changing anything to a radical extent. As an example of this, the ore-handling machinery was started on

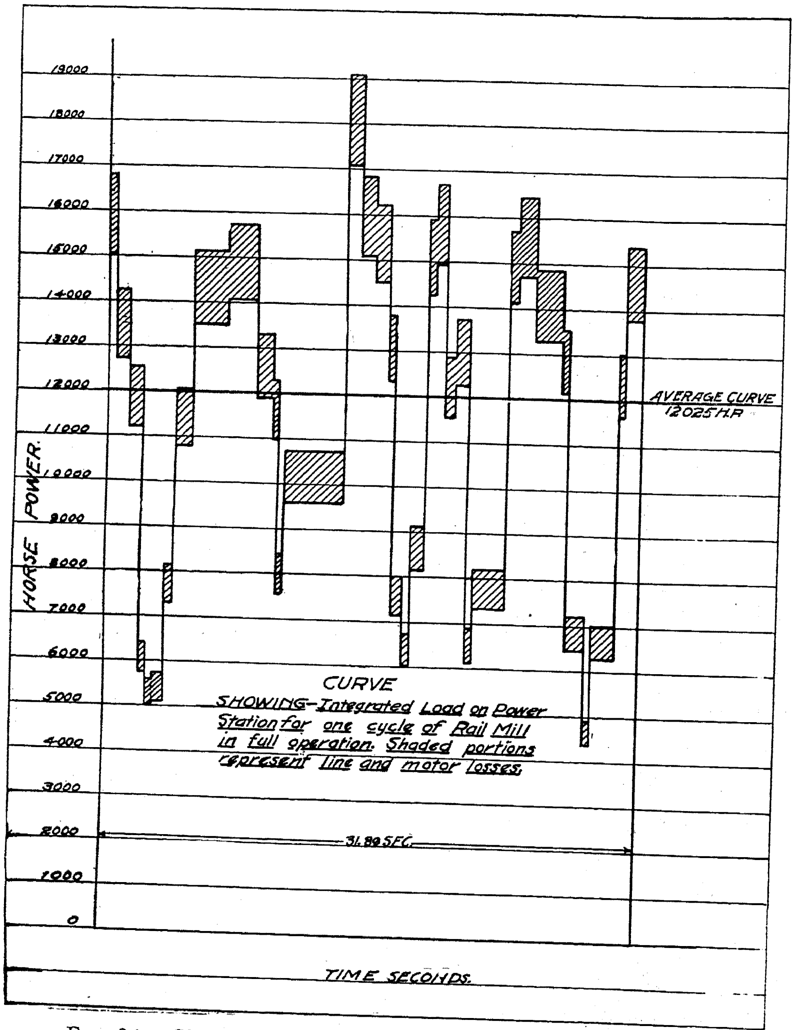


FIG 34.—Chart showing estimated combined load on the mill. To be read from left to right.

July 23, 1908, and by Nov. 1, three-quarters of a million tons of ore were stored in the yard, with virtually no more trouble from the machinery than would have been experienced at a works which had been in operation for several years.

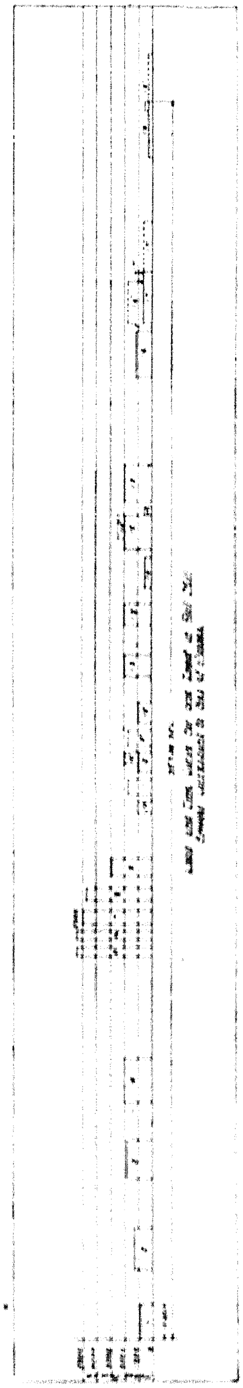


Fig. 35.—Chart showing calculated time and horse power per pass.

In the original design of the plant considerable attention was given to the question of underground versus overhead transmission. In view of some later developments although it might have been possible to put in underground service, which would not have greatly exceeded the cost of overhead construction, yet the latter has given no trouble. On the night of Jan. 29, during one of the severest wind storms ever experienced in this section, the velocity of the wind being estimated at over 70 miles per hour, not a single case of trouble occurred on the lines within the plant.

As stated in the description, all the 6600-volt feeders were protected by multigap lightning-arresters, while the 22,000-volt

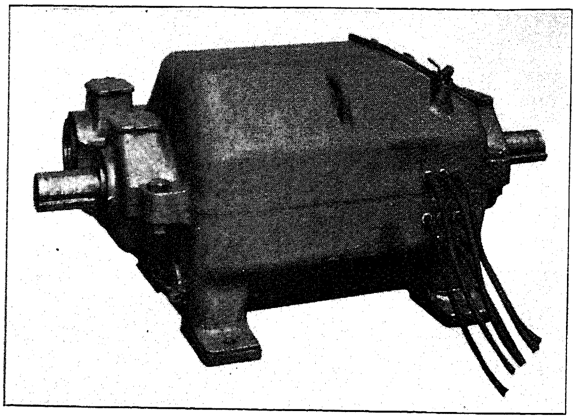


FIG. 36.—Mill type induction motor.

line has both multigap and electrolytic lightning-arresters. The latter were not installed until after the former had been operated for some time, and since the installation of the electrolytic lightning-arresters there has been no discharge over the multigap, nor have there been any bad effects from lightning either in the plant of the Indiana Steel Co. or at other points on the line, which are protected in like manner.

In open-hearth plants No. 1 and No. 2 one feature not adopted in No. 3 and No. 4 will be the use of a large number of electric motors for operating the gas and air valves and furnace doors which had previously been worked by hydraulic pressure.

A comparison between the estimated horse power and the observed horse power required for the various passes in the rail

mill shows some discrepancy; but because the steel rolled was colder than it would have been in actual practice, since all of the machinery is new and is not operating so quickly as it should, and because also of the lack of adjustment of rolls, it is believed that the power required will be very little, if any, in excess of the original calculations. After the roll-train motors had been started it was discovered that the stopping of them was an important feature. The 2000-h.p., 214-rev. per min. motor when disconnected from the rolls required 2 hr. to come to rest, while

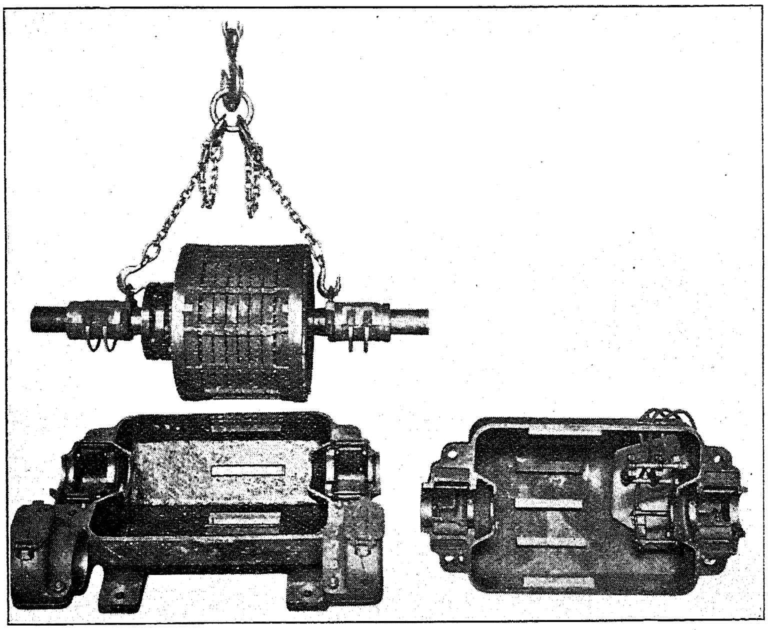


Fig. 37.—Disassembled view of mill-type induction motor.

the 6000-h.p., $83\frac{1}{3}$ rev. per min. motor required 1 hr. and 37 min. to stop. This time consumed would mean corresponding delays in case of breaking of the main spindle, which, of course could not be countenanced. In order to stop these motors within a reasonable length of time, direct current at 250 volts was introduced into one phase of the winding through an external resistance after the motor had been disconnected from the 6600-volt line. By this device the 2000-h.p., 214-rev. per min. motor was stopped in 2 min. and 55 sec., and the 6000-h.p. motor in 1 min. and 42 sec. During this time the first section only of the resist-

ance of the rotor was closed. This device is being put in permanently, and a 6600-volt switch connected to one phase, the other side of which will be connected to the 250-volt line through a permanent resistance, and this switch interlocked with the main 6600-volt oil-switch so that both cannot be thrown in at the same time.

Probably no industrial application of electricity has been the result of more careful study on the part of the engineers in charge, or has marked a more general adoption of electric power than the one just described. Although many of the motor applications are not new, this plant is unique in respect to the number and variety of the applications and the size of many of its units. The rail mill now in operation, driven by induction motors with a combined capacity of 24,000 h.p., and having a normal output of 4000 tons of steel rails per 24-hr. day, is without a rival. The operation of the plant will therefore be watched with more than usual interest, both by steel-mill engineers and electrical engineers. Its success will greatly accelerate the application of the electric motor in this industrial field.

Results thus far obtained indicate that not only have the greatest expectations of the engineers been realized, but another stronghold of the steam engine has been carried, and that in the near future the rolling-mill engine is destined to give way to its successor—the electric motor.

DISCUSSION ON "THE INDUSTRIAL APPLICATION OF THE ELECTRIC MOTOR, AS ILLUSTRATED IN THE GARY PLANT OF THE INDIANA STEEL COMPANY." NEW YORK, MARCH 12, 1909.

B. A. Behrend: The subject of Mr. Shover's paper is not altogether confined to its title. Not only has he referred to the application of the electric motor in steel mills, but he has also described the usefulness of the storage battery in connection with steel mill problems. The application of blast-furnace gas in the power plant at Gary has also been mentioned. It is carried out there on the most gigantic scale. As the paper itself does not rigidly conform to its title, I wish to make a few remarks concerning the gas-engine generating plant which is illustrated in Fig. 3 and 4.

As the designer of the 17 generators constituting the 40,000 kw. gas-engine power plant at Gary, I have been particularly interested in the paper, and I wish to take this opportunity to add a few words regarding some special problems which arose in connection with the generating plant, which will contribute to a better understanding of the fundamental conditions which must be met in power plants utilizing alternating-current generators for direct connection to gas engines, a problem of the greatest importance in steel mills. The choice of the right flywheel effect is intimately bound up with this question, as the necessary moment of inertia of the flywheel can be determined intelligently only by the designer of the alternating-current generators.

The gas-engine power plant at Gary consists of 17 four-cycle, double-acting, cross-tandem gas-engine units of 2500 kw. capacity each. Two of these engines, No. 1 and 2, are direct connected to two 2000-kw., 250-volt, direct-current generators, while the remaining 15 engines are direct connected to alternating-current generators of 3000 kilovolt-amperes each at 6600 volts and 25 cycles. The engine speed is 83⅓ rev. per min.

As the load on this power plant is composed chiefly of induction motors of large capacity and slow speed, the amount of lagging current under average operating conditions is likely to be equal to, or even greater than, the amount of watt current required by the motors. The power factor in this plant is not likely to exceed 70 per cent, and the generators, therefore, were designed for a large ampere rating at low power-factor.

Electric gas-engine-driven units must operate successfully in parallel if the gas engine is to be an important factor in steel mills. This problem resolves itself into an investigation of the periodic disturbing forces, produced by the engine, and the natural frequency of the generating unit. The periods of the former depend upon the type of engine used, if we neglect the period of the governor. As the gas engines at Gary are four-cycle double-acting, cross-tandem engines, there are introduced two periodic disturbing forces, due to inequalities in the force of explosions, adjustment of valve gear, leakages, etc., the frequencies of these forces being one per revolution, and one-half per revolution.

of the engine. It is the latter frequency which is absent in steam engines, and which introduces a complex factor of the greatest importance into the problem of parallel operation of gas engines.

Following the terminology used in dynamics, we call the results of these periodic disturbing forces, "forced oscillations." For the Gary units the frequencies of the forced oscillations, which are likely to be dangerous, are 1.39 and 0.695 cycles per second.

It was shown by André Blondel and Boucherot in 1893, that synchronous generators and motors, when displaced from a condition of equilibrium, perform a series of swings which constitute the "natural frequency" of the oscillation of the generator or motor, and which can be dampened out just like other natural oscillations. The damping circuits in and between the pole faces, which were suggested at the time by Maurice Leblanc, perform this function.

The determination of the natural frequency by calculation, as carried out by Blondel, Boucherot, and Kapp, ten years ago, was based on a knowledge of the angular displacement of the revolving field per unit torque. The short-circuit current was used for this determination, but experimental investigations which I carried out with my former assistant, Mr. A. B. Field, have demonstrated that other methods must be resorted to if serious error is to be avoided.

Supposing that we know the natural frequency of the generating units when operating on bus-bars of constant frequency, then we can readily obtain, by the methods of dynamics, the increase in the mechanical displacement between the forced and the natural frequencies produced by resonance, or the "magnifying factor", as it has been called by Messrs. Goerges and Rosenberg who have done such excellent work in the theory of parallel operation. This magnifying factor is infinite when the natural frequency equals the forced frequency, provided there is no damping. Under this condition, even with damping, however powerful, it is not possible in practice to obtain satisfactory parallel operation.

Turning now our attention to the Gary units, it was evident that if any measure of success was to accompany the construction of this plant, great forethought had to be exercised lest the successful parallel operation should be dependent upon a lucky guess on the part of the gas-engine builders. Therefore, before calculating the moment of inertia of the units, or the flywheel effect, extensive experiments were carried on to determine the natural frequency of oscillation of generators of similar design. The generators were run in multiple when the circuit-breaker was opened and closed again immediately. The number of complete oscillations per second could be counted and the natural frequency was thus actually measured.

These investigations, carried on with a number of different machines, over the entire range of the saturation, made it possible

to determine beforehand the natural frequencies at all excitations of the Gary generators, and, therefore, under all conditions of load and power-factor. The curve in Fig. 1, marked " natural frequency ", shows the average result. The straight lines show the two forced frequencies of the gas engines. The points of intersection, *C* and *D*, mark conditions of instability, the range between them marking the range of stability. The operating range of the generators on the saturation and regulation curves, lies between *A* and *B*; and, therefore, is well within the range of stability.

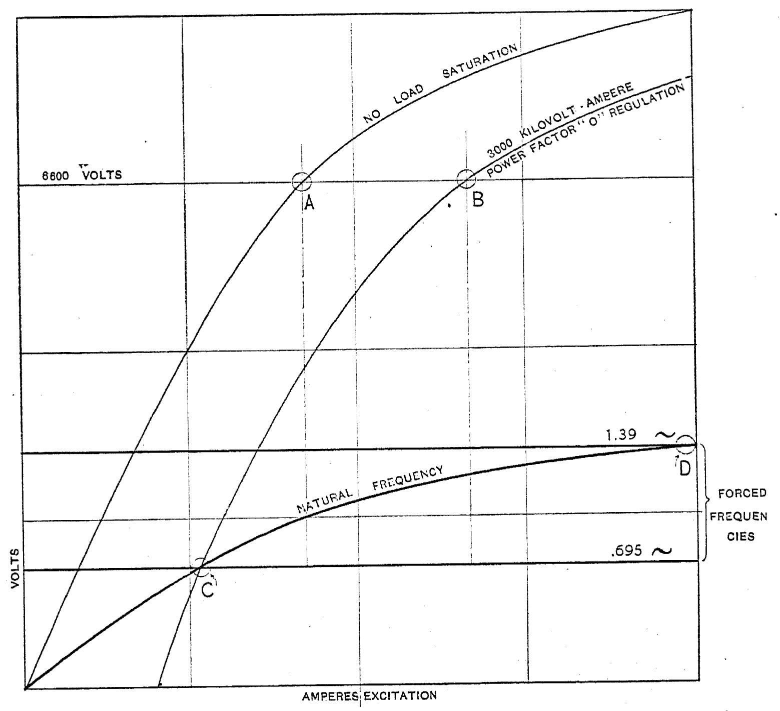


FIG. 1

Very light wheels had to be used to comply with the condition described. With light wheels, dampers are efficient and they have been provided on the Gary generators.

A flywheel effect of four times the flywheel effect used would have reduced the natural frequency below that of the lowest forced frequency, and the conditions would thus have been similar to those obtaining in steam engines. Such large flywheel effects on gas engines require the use of steel wheels of large diameter, which are not at all impracticable. In point of fact, the question was carefully considered whether large wheels, reducing the

natural frequency, should not be installed. It was, however, finally decided to bear up with the greater fluctuations, and to install light wheels working between the two forced frequencies.

These brief remarks may suffice to indicate that the problem of multiple operation of gas engines has been reduced to an accurate scientific method based on careful experimental and practical data. While the details of the method, and the experimental data obtained, are beyond the scope of my present remarks, it should be mentioned that these methods have been applied to the plants of the Carnegie Steel Company at Rankin, Pa., where three 2000-kw. units are operating in multiple, to the plant of the Illinois Steel Company at South Chicago, where four 2000-kw. units are operating in multiple, and to the plant of the American Steel and Wire Company at Cleveland, Ohio, where four 1200-kw. generators are operating in multiple, and in all cases reasonably satisfactory parallel operation has been obtained. These plants are using blast-furnace gas and the generators are of my design. In another plant, generators of my design direct-connected to gas engines are operating on producer gas, using flywheel effects large enough to reduce the natural frequency below the lowest forced frequency.

Gano Dunn: My admiration for the work done at Gary is not second to that of anyone here. I feel without the slightest exaggeration that we are really considering one of the wonders of the world. I am reminded that the whole character of the Gary design is determined by the choice of prime movers, and that when we put our finger upon the quarter of a mile of gas engines we have placed it upon the heart of the plant. I also feel that, as Mr. Edison once expressed it, we are playing with blue chips, or as Mr. Carnegie once said, we have put all our eggs into one basket and are watching that basket. I believe so much attention has been put upon the gas-engine feature of the Gary plant that it simply cannot fail, and that the same intelligent engineering judgment which it is evident has been applied to other features of the plant, will be found to have been applied to the gas-engine equipment; and in the end, although possibly after some trouble, we shall have a wonderfully successful plant which will set a pace to be followed all over the country, and that gas will be used for the fundamental equipment of steel mills.

In the early days when direct-current motors were applied to steel mills, their duty was regarded as intermittent. Their continuous capacity then was low, but their intermittent capacity was extremely high. So great has been the progress in the evolution of steel mills, that this type of motor, originally designed for very intermittent service, has come to be the modern mill motor, subjected to a service that is more severe than continuous service, a service in which through rapid reversal and the encountering of enormous inertias, the duty on the motor is even greater than that corresponding to constant continuous load. Of course the adoption of very large units for roll driving

meant alternating-current motors. Had direct-current motors been desirable for the great rolls they could not have been used, because the sizes are beyond the limits of direct-current design. The general judgment of everyone is, that the adoption of alternating current in the Gary plant for the major part of the distribution, has been a success. The battle may wage a little about the choice of current for auxiliaries and small distributions, and in this connection I am particularly interested in what Mr. Shover said of the desirability of an induction mill motor. He wanted an induction mill motor that would be as much better than the ordinary induction motor as the direct-current mill motor was better than the ordinary direct-current motor.

If he wants this earnestly enough I have no doubt he will get it. The same intelligent coöperation between the manufacturing companies and the consumer, shown at Gary in the production of the great induction motors for driving the rolls, undoubtedly will be shown again in the production of a good induction mill motor, but I question if in the long run Mr. Shover will want such a motor earnestly enough. An induction mill motor that is as much better than the ordinary induction motor as the direct-current mill motor is better than the ordinary direct-current motor, is still not as good as the direct-current motor. I am reminded of the remark made by Dr. Steinmetz on this platform not so very long ago, that if the frequency of a single-phase alternating current motor were reduced, the characteristics of the motor were improved, and were best when the frequency was reduced to zero; that is, when direct current was used.

It is the advantages of generation and distribution that have brought the induction mill motor to the front, not good qualities in the motor itself. The low power-factors, the necessarily small air-gaps, the complexity attending the use of three phases with at least two sets of controlling apparatus, the great reduction in capacity for a given weight of motor; all in the end are bound to indicate that the problems of distribution ought not to be controlling to too great an extent, and ought not to cause the locating at every point around the mill where power is used, of the complications that attend an induction motor. They should rather lead to the collecting of such complications into a substation or power house where a motor-generator taking power from the main alternating distributing system will transform it into simple and powerful direct current to be sent out for local service.

It is my opinion that in the end our steel mills will not be supplied throughout with alternating current but will find themselves equipped with mixed systems of distribution—alternating current for the great powers and big motors, and direct current for the cranes, reversing motors, and other motors of similar service.

When the mill men first started electrical equipment, they were not in touch with the electrical men, and many mistakes

were made. They were accustomed to steam engines and the measurement of power by indicators. On account of the difficulty of doing it, no one thought of measuring the power absorbed in accelerating. When motors were first used, acceleration was not yet a vital item, and many of the mistakes and failures of electric motors in steel mills have been due to the neglecting of the vast amount of power absorbed by the inertia of moving masses. The railroad men went through all this and developed speed-time curves. It was what Mr. Sprague, whom we heard last night at the annual dinner so correctly referred to as the "hero of Richmond", did not know when he put two 7.5 h.p. motors on a trolley car to do the work of two horses. Some of the most beautiful features of the Gary equipment are due to the appreciation on the part of the engineers of the characteristics of acceleration loads.

When I see how completely the steel industry has capitulated to the electrical engineer in the Gary plant I am filled with joy, remembering the attitude of the steel men toward electric motors not long ago, even on such innocent things as roll tables. Successfully to drive a roll table by motors was impossible. As for motors on the rolls themselves, you could not get a power station big enough to supply the current even if you could make the proper motor, which was doubtful. From such an attitude there has been victory after victory, until to-day we see one of the wonders of the earth completely electrically equipped. If the steel industry has capitulated to the electrical engineer, it is also true that the electrical industry has been developed by the requirements of the steel men, and has received the biggest kind of dividends from this development. It used to be said that electric railroad duty was the severest known to electric motors. It does not begin to approach in severity steel mill duty, and the modern mill motor, together with the great induction motors driving the rolls at Gary, indicates how the electrical engineer has responded to the steel mill demands.

It is a great night, gentlemen. It means much for the future, and we shall look back to this time, signalized by Mr. Shover's paper, as commemorating the solution of one of the greatest problems that electrical engineers have solved.

William T. Dean: Mr. Shover spoke very briefly on some of the problems which have been met; that is, whether the mill shall be two high or three high. A three-high mill is one having three rolls, each of which rotates constantly in one direction, that is, the upper and lower roll rotate in the same direction; while the middle roll rotates in the opposite direction, as indicated in Fig. 1.

A piece of steel, for instance, passes through the lower set of rolls at *A* to the right, the table is then raised and it passes through the upper set at *B* to the left. On the two-high mill where the steel must pass back and forth through the rolls, as indicated in Fig. 2, it is obviously necessary to reverse the

direction of the rolls, and therein comes the difficult problem in electrical drive, especially when large motors are used. The mill must reverse quickly to prevent delay, and in this, very serious problems are encountered.

Mr. Dunn mentioned the use of direct-current motors on the rolls which, in some instances, may be as large as 6000 h.p. Such a motor would be extremely difficult to design, but if it could be designed it would be a very expensive machine. A plan worked out in Europe, where direct-current motors of large size are used on reversing rolls, is to put three armatures on a common shaft, thus reducing the inertia, on account of the

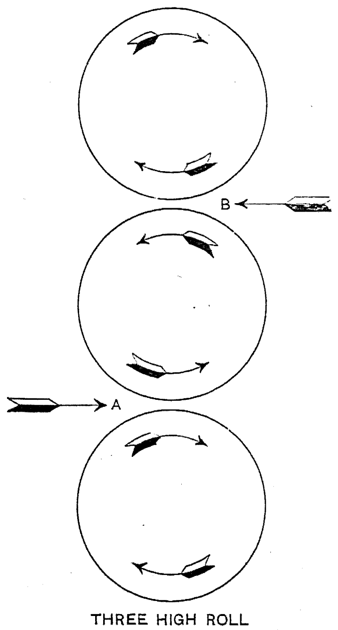


FIG. 1

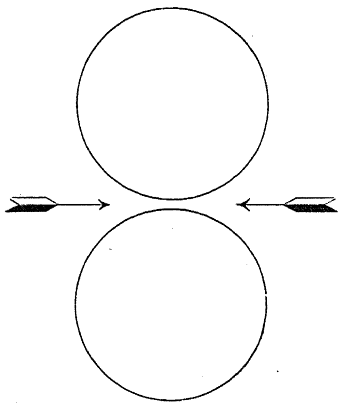


FIG. 2

diameters being smaller than if only one armature of large diameter were used.

Fig. 3 represents three direct-current motors, the armatures of which are mounted on a common shaft. As the mill reverses, it is impossible to take advantage of the mechanical value of flywheels; the enormous peak loads on reversal, therefore, are reflected back in the direct-current system, and to overcome this difficulty the three direct-current motors are electrically connected with two or three generators which are driven by an induction motor with a heavy flywheel, the generator armatures, motor armature and flywheel being on a common shaft, as shown in Fig. 4.

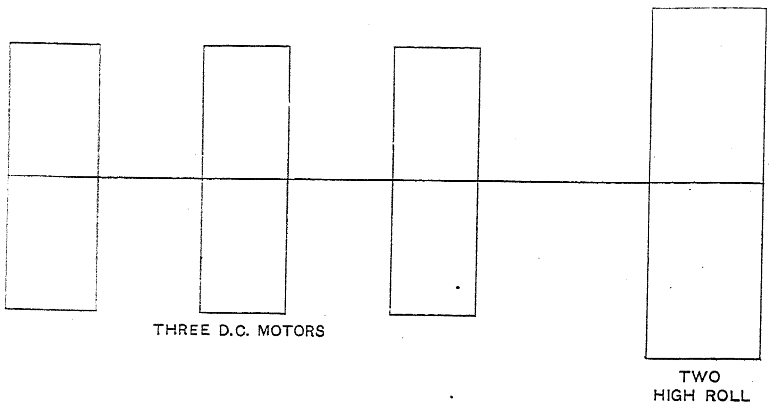

THREE D.C. MOTORS
TWO
HIGH ROLL

FIG. 3

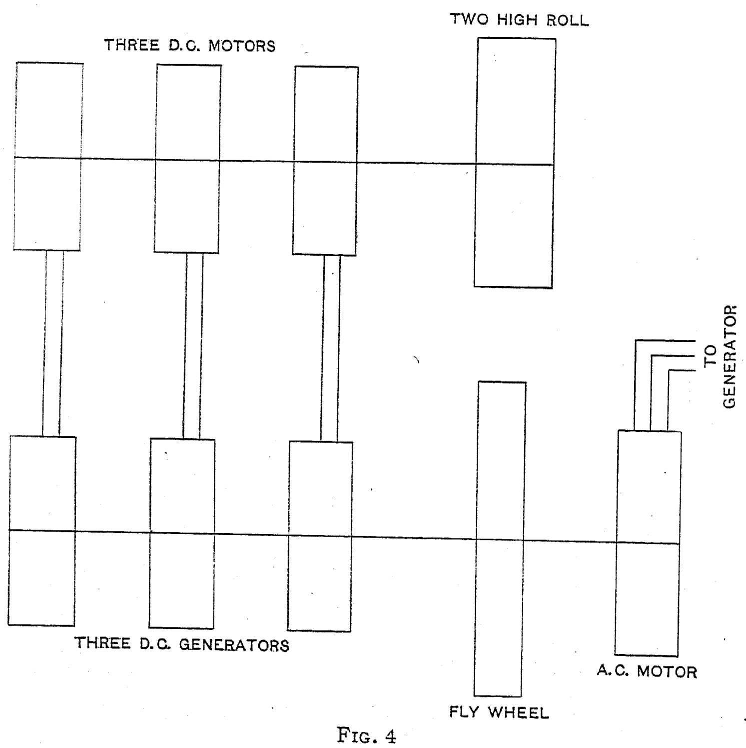

THREE D.C. MOTORS
TWO HIGH ROLL
TO
GENERATOR
THREE D.C. GENERATORS
A.C. MOTOR
FLY WHEEL

FIG. 4

If now it were possible to use a three-high mill instead of the two-high mill, the system becomes considerably simplified, inasmuch as the direct-current motors and generators can be omitted, and the induction motor can be directly connected to the rolls, as indicated in Fig. 5. The induction motor with its flywheel will do the work on any equipment, and the difference in first cost is just about one to three.

This is one of the problems which I think has been solved, in connection with the study of the Gary plant, it being first proposed to install a two-high mill, but after careful study it was found that the three-high mill could be used at about one-third the cost of the two-high equipment and with a large elimination of troublesome features.

It may not be apparent why a mill should have ever been made two-high. The steel business has been a gradual growth,

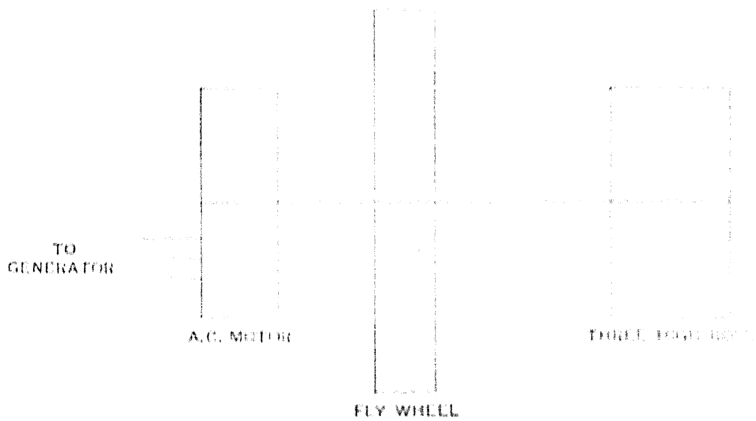


Fig. 5

the first steel being run through rolls driven by steam engines. The very first plan was to use small engines which were reversed at each pass, this practice having gradually grown to the larger work. Some of the older steel men do not believe, for instance, that universal plates can be made in any other than a two-high reversing mill, but some plants have demonstrated that they can be made in three-high mills, and by so doing, have greatly reduced the problems of electrical drive.

On the subject of small alternating-current motors versus direct-current motors for driving auxiliary machinery in steel plants, I will refer to Fig. 6 which represents a small direct-current motor driving a roll table. In a system such as is used at Gary, this direct-current motor receives its power through a motor-generator set, the generator being driven by an alternating-current motor, which in turn receives its power from the alter-

nating-current generators in the power station. If now an alternating-current motor were used on the roll table instead of the direct-current motor, the motor-generator set could be omitted by connecting the alternating-current motor direct to the power station through transformers, if such are necessary. After all, simplicity of general construction, as well as a general cheapening in the cost of the construction, is a highly important feature.

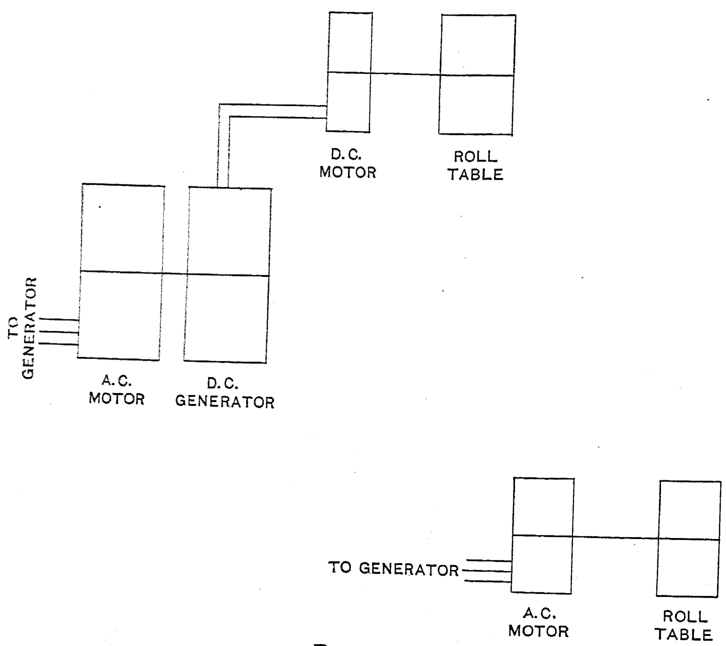


Fig. 6

Brent Wiley: Regarding the question of surplus gas power over and above that required by the furnace auxiliaries, a review of data obtained from several of the principal blast-furnace-plants abroad shows that the capacity installed per 100 tons of pig iron produced in 24 hours does not exceed 1500 h.p. The average pressure is approximately 8 lb. and the amount of coke burned per ton of iron produced is approximately 2400 lb. The author gives a surplus of 3000 h.p. per 100 tons of pig iron produced per day. I would like to have his opinion of a practical figure to use in this relation for comparatively small plants of more than two, and less than six, furnaces of modern type and capacity. My idea would be an average of the two values given above, that is, 2000 h.p.

Referring to the subjects of ore unloader and ore bridge, I

wish to submit data on power requirements of machines which are similar to those described, and which are located at the Lorain works of the National Tube Company. Two of the unloaders have a total of 475 h.p., two machines a total of 525 h.p. and a capacity of 10 tons; the bridge has a total of 560 h.p., its capacity being 12 tons.

1 Unloader, maximum current 1200 amperes. Average current 500 amperes.
2 Unloaders and 1 ore bridge, maximum current 3300 amperes. Average current 1520 amperes.
4 Unloaders and 1 ore bridge, maximum current 6000 amperes. Average current 3500 amperes.
The voltage is 220.

With the older methods of unloading ore, where hand labor is employed to fill the conveying buckets, the unloading price per ton ranges from 12 to 18 cents, and for the boats of modern design ranging from 8000 to 1200 tons burden it would require several days' docking. Machines of the above-mentioned type have reduced both the handling price and the holding time of the ore boats at the dock fully 75 per cent. This is a good illustration of the advance that has been made with the development of the auxiliary steel mill machinery; and motor application has been one of the prime features.

The principal point of the universal application of electric power in the steel industry is the commercial value, and this is made up of a number of items, some of which relate to the subject only indirectly. A summation of the advantages of the electric drive will bring out the questions that should be considered.

Advantages of electric drive.

1. More reliable, less breakage of couplings, pinions, etc., with a cor respondingly less maintenance cost.
2. Low power cost, with the following advantages:
   Centralizing of power equipment, low fuel cost, use of gas engines of medium size in power stations, uniform load conditions in power station engines by equalizing methods.
3. Less space on account of absence of boilers, pipes, etc.
4. Easy adaptation to mill machinery.
5. Easy distribution by means of cables, instead of steam or gas pipes.
6. Easy and simple handling with reliable protective features of control.
7. Less labor.
8. Less oil and other stores.
9. Ease of obtaining power indications and records.
10. Constant and regular torque.
11. Increased output due to higher and more uniform speeds.

As to item No. 9, economical conditions can be worked out and modifications made with a knowledge of results based on exact information, having available power indications and records which do not exist with the use of steam engines. Again, power indications are of valuable assistance in locating faults of machinery, a timely discovery of which prevents loss of time and material, and probably damage to the mill. The disadvantages mentioned by some engineers are high capital outlay, and loss of

efficiency due to two intermediate machines between source of power and point of application.

Taking all the above items into consideration, however, the results are decidedly favorable to the electric drive, especially for new plants; and it will be found that there are comparatively few cases of existing plants, where the conversion from steam to electric drive is not worthy of the fullest investigation. This point is emphasized by the very active interest taken in the exhaust-steam-turbine proposition by a majority of the large steel companies. The exhaust steam from the existing mill steam engines is used to drive a low-pressure turbine connected to a generator which in turn furnishes electric power for the mill motors. Within the past two years the steel companies of the United States have installed more than 200,000 h.p. of gas engines and turbine electric station equipments, to drive approximately fifteen motor units aggregating 20,000 h.p., exclusive of thirteen units of the Indiana Steel Company now operating and on order and aggregating 51,700 h.p.

Within a year or so the data obtained from the above installations should give a conclusive analysis of the electric drive from both an actual and comparative cost standpoint, and while it is scarcely probable that this information will be available in detailed form, the electric companies are certainly justified in requesting that a review of the summary of the compiled data be made. It is to be hoped that attention will be given to the various phases of the question, bearing on such items as are mentioned under the heading "Advantages of the electric drive," in order that the full significance of the subject may be recognized.

The universal application of electric power at the Indiana steel plant, makes this subject of electricity versus steam a particularly fascinating one. In no other case has work of such magnitude or of such completeness been considered. Each mill proposition has been analyzed with great care and skill by the steel companies' engineers and they have outlined their conditions and requirements in a very definite and conclusive manner. The electric companies have appreciated this excellent work and as a valuable adjunct to his paper I would suggest that Mr. Shover add a copy of the specification, load diagram and data sheet items as furnished to the various electric companies for the rail mill proposition. This may appear to be an unusual suggestion, but a clear conception of this method of presenting a rolling mill proposition would further the development of the art to a greater degree of perfection, and this particular point is one of mutual interest and benefit to the steel and electric interests.

Robert Hull: The gas engine will have more and more to do in the manufacture of steel. An important question in the case of gas engines operating alternators is that of hunting, and I would like to have Mr. Shover give us his experience on any

trouble with hunting he has had, and also some information as to the power-factor.

David B. Rushmore: It is my pleasure to-night to call attention to the fact that this is the beginning of a new era in the work of the Institute, and one which will be very closely associated with the administration of President Ferguson. It is interesting to note that for an illustration of the industrial application of the electric motor, it was necessary to go quite near to his own home. We have for years been familiar with the application of electricity in the telegraph, in the telephone, in lighting, in electric railway service, etc., but it has been only in machines of small size, and only in installations of comparatively small total capacity, that we have heard of the industrial use of the electric motor.

But we have in this installation the beginning of a new era in electrical work, where the application of electricity to industrial power more than takes an equal footing with the earlier application in other fields. In mill work, especially in this plant, we have very unusual conditions.

Owing to the limited time, it is desirable to confine my remarks to one phase of the subject. In this plant, with its many varied applications of electricity, two features stand out distinctly considering the scale on which they are practised in the manufacture of steel in this country. I refer, first, to the gas-engine power station. While such stations have been installed in other places on a smaller scale, it is the first time in this country that the complete power plant of a steel mill has been electrically operated. Secondly, I wish to refer to the drive of the rolling mill by electric motors. Smaller installations have been put in before, but nothing of this size, and nothing in the world has ever been attempted with machines of this capacity.

In rolling mills we have a number of varied problems, and it is very pertinent to repeat the remarks of the previous speaker to the extent that under no other conditions of service are the requirements of the electric motor as severe as they are in this work. The load curves we saw this evening on the auxiliary as well as on the main roll drive, indicate a load condition not met with elsewhere. This necessitates in the motor a very unusually strong mechanical construction and an interesting mechanical device mentioned by the author; it necessitates a comparatively large air-gap in the large machines, and also in smaller machines, making the design of the machinery, electrically and otherwise, with the idea of reliability first.

The author has very interestingly brought out the point that what a steel mill desires is nothing but the maximum output of steel, and the incidental features of the plant must all be designed with this in view. That means that the electric motors have a considerably larger air-gap than was necessary from any other consideration; that the factor of safety, electrical and mechanical, very much exceeds the figures that would be used

for any other class of service, and as the author saw, in regard to the overload capacity of these motors, that no variation in temperature of ingots should be able at any time to stall them. This is illustrated in the new type, where all the mills operate continuously in the same operation, and all the motors are equipped with flywheels.

The design of a flywheel and the motor, with the automatic control, in order to have the motor properly function with its variation in speed, to draw the proper amount of energy from the flywheel and throw back on the power station the desired load, has been one of the very interesting problems of this work. Other engineers connected with the Steel Corporation have been studying, and have worked out with a very considerable degree of refinement the load thrown on the rolling mill by the reduction of the steel passing through it. This has been done to a large extent by the indicating steam engine, and now with the beautiful accuracy possible with the electric motor, a much more refined study of this problem will be possible.

The question of the alternating-current motors is a very vital one, and I wish to speak of it in connection with another engineer who has done a large amount of pioneer work for the Illinois Steel Company. This engineer is going to be the pioneer in introducing through the system which he is operating, high-speed rotary condensers, which entirely take care of the power-factor and the effect of the peak on the generating station. In this plant and in the other plants in which they will be used, these rotary condensers will give all the advantages of the electric motor, with the power-factor obtainable with zero frequency, as we have just heard.

President Ferguson: Before I call on Mr. Shover to close the discussion, I want to say a word which I thought of mentioning before, but as the hour was so late I omitted it. This subject of industrial power, not only as applied to steel mills, but to all practice, is really very important. It is not quite so pyrotechnic as some of the other things we have discussed, like the electrification of railways, for instance, but there is a great deal of work which is being done very quietly in the application of electricity for power purposes, but not nearly as much as should be done. It was with the idea of bringing out the possibilities of the use of electricity for power purposes generally that this year I appointed a commitee on industrial power, of which Mr. Rushmore is the chairman. It was hoped at this meeting that we would be able to have a discussion of more than one feature of the subject, but when we looked into the matter we found the application of electricity to power purposes in steel mills was so important, and involved such large quantities with such a great variety of applications, that we felt it was worthy of a whole evening, and I think we all feel that opinion is justified by the results of listening to Mr. Shover's interesting paper and the discussion which has followed.

We have arranged that the annual convention, if possible, should devote a session to the general subject of industrial power. We will then have a symposium on the subject, and discuss industrial power in all its phases, and we hope you will all be very much interested. I desire to let you know that this is only the beginning of what we think is a new and important work.

B. R. Shover: Mr. Hull's references to hunting and power-factor are important enough to require some comment. We have had some slight difficulty in paralleling the gas engines, but only at times of light load. The operation of the alternators in parallel under three-quarter load or more, is in the main as steady as that of any reciprocating steam-engine-driven alternators. On lighter loads there is always a tendency to hunt, due to inherent characteristics of the gas engine; for one cylinder is always a little stronger than the others, and one cylinder is a little weaker than the others. Therefore at light load there may be a point where the weak cylinder misses fire; this will be indicated by the needle on the ammeter, and the strokes of the engine can be counted at that time. If the weak cylinder and the strong cylinder act exactly opposite to each other there will be more or less trouble with the engine. I have never seen the engines thrown out of step, even under the very worst conditions, but sometimes I have thought that they would be. I do not think it is necessary to take up here the reason why it seems almost impossible to overcome this inherent characteristic of the gas engine.

The matter of power-factor was held up to us as a bugbear for a long time, so long in fact that we ceased to bother about it. We did not know what our power-factor was going to be; we had no way of finding out nor did we care what it was: there was nothing for us to do but to put in the plant, operate it, determine the power-factor, and then ascertain if changes were necessary. We have not yet reached the point where we can say what we are going to do, because the heavier the load on our plant the better the power conditions. When we get the mill full of steel we will know what the power-factor is going to be: at present it varies, according to the conditions in the mill, from 35 to 85 per cent.

A paper read at a joint meeting of the American Society of Civil Engineers, American Society of Mechanical Engineers, American Institute of Mining Engineers, American Institute of Electrical Engineers, New York, March 24, 1909.

ELECTRICITY AND THE CONSERVATION OF ENERGY

BY LEWIS B. STILLWELL

In any problem, accurate and, so far as practicable, concise statement is essential to proper consideration and correct solution. The economic problems which present themselves when the complex and far-reaching subject, Conservation of Natural Resources, is considered, can be approached best by first stating and defining them with reference solely to physical and economic facts and relations without reference to political boundaries or limitations. To approach the subject by first considering real or supposed difficulties imposed by the respective rights and duties of states and of the nation, is to discuss method of treatment before diagnosis. We should first consider the problem as if there were no such thing as states within the Union, assuming, for the time being, the existence of one central and absolute authority within the federal boundaries. The question what is economically desirable upon this assumption, is that which the engineering profession should first agree upon and, if possible, state in a manner which will be understood by the general public.

Conservation as applied to our natural energy resources means utilization without unnecessary waste. In a broader sense it means also development along lines which will not only utilize but increase those resources; for example, as regards water-powers it has relation to the maintenance and renewal of forests affecting variation in stream-flow, and the construction of storage reservoirs which, properly used, are capable of adding greatly to that part of the run-off which can be used for industrial purposes and navigation.

Much has been uttered recently with reference to these relations which can not be expected to hold good in the light of that clearer knowledge which will result from further study and experience—much that is erroneous and misleading even when examined critically in the light of facts now ascertained and determined. General statements from sources commanding the attention and arousing the interest of the public are necessary first steps in turning a nation from reckless waste and almost unrestricted appropriation of natural resources by individuals, to a policy of wise conservation, having due regard to the common interest now and in the future. Those first steps have been taken, on the whole, in an admirable manner. Public attention has been arrested. Public interest has been aroused. Public and legislative opinions are forming. Obviously, it is of the utmost importance that our engineering societies should take an immediate and active part in working out the complex problems of conservation and, if possible, in directing the formation of public opinion along lines that will result in the enactment of just and wise laws.

The economic utilization of our natural resources is the fundamental problem of all engineering. If the President of the United States were to summon a conference of governors at the White House for the purpose of considering and promoting reforms in current medical practice, the medical profession undoubtedly would be greatly interested, and would manifest its interest by assuming proper and unchallenged prominence in discussing the questions raised. If such a conference were to assemble for the avowed purpose of initiating reforms in the machinery and methods for the administration of justice, it is not to be doubted that the lawyers would manifest their vital interest not only by exposition and discussion, but also by actual leadership.

The conference of governors in May, 1908, called by President Roosevelt to consider and advise regarding conservation of the natural resources of the United States, raised questions in respect of which the engineer occupies a position closely analogous to that which the medical doctor holds in respect of medical practice and the lawyer in respect of legal procedure and administration.

The analogy is not perfect, nor does responsibility for final decision rest exclusively upon the engineer, but it is peculiarly the patriotic duty of the engineering profession to enlighten the public by unbiased consideration and accurate exposition of essential pertinent facts, physical and economic.

I propose in this paper: 1. To illustrate the function of electricity in the conservation of natural resources. 2. To summarize statistically the present power requirements of the United States and present certain data (necessarily far from complete) relative to water power available. 3. To point out certain economic bearings of the plan which proposes the imposition of a tax on water powers, and the use of the proceeds for improvement and construction of inland waterways.

The Function of Electricity in Conservation

The part which electricity is destined to play in the conservation of our energy resources is demonstrated clearly by what it already has accomplished. Three typical illustrations will suffice: 1. The saving of coal by the utilization of water power, as illustrated by the plants of the Niagara Falls Power Company. 2. The saving of coal by the substitution of large and highly efficient steam plants for smaller and less efficient plants, as in the case of the Newcastle-upon-Tyne Electric Supply Company. 3. The saving of coal used for transportation purposes by the substitution of large and highly efficient engine units for comparatively small and inefficient locomotive units, as accomplished, for example, by the Interborough Rapid Transit Company of New York.

In each case the economy is due primarily to the fact that we can now use for transmitting and distributing power, the electricity produced in dynamos, distributed by conductors, and utilized by motors—all of remarkably high efficiency.

The Niagara Falls Power Company. During the year 1908, the plants of the Niagara Falls Power Company delivered an output of 560,000,000 kw-hr. Had this output been generated by large modern central stations using steam power, their consumption of coal would have approximated 2000 tons per day. Were the users of Niagara power dependent to-day upon their own individual steam plants, they would use in the aggregate not less than 3000 tons of coal per day; in other words, more than 1,000,000 tons per annum. If this power were replacing steam, as used under average conditions in our manufacturing cities, instead of being used for the most part in supplying power to a comparatively small number of customers using large blocks of power, it would replace and save nearly 2,000,000 tons of coal per annum.

Important as is the saving of coal from the standpoint

of conservation of our natural resources, perhaps the most striking feature of the Niagara power enterprise is the demonstration which it affords of the great industrial value of cheap power, the greater part of the output of the plants being utilized to-day by electrochemical industries of great value to the community, all of which have been stimulated and some of which owe their very existence to their ability to secure power at very low cost.

The North-East Coast Power System. The North-East Coast Power System, supplying electric power to the great industrial district in and about Newcastle-upon-Tyne, effects a very important economy in coal consumption. In a paper presented at the Middlesbrough meeting of the British Iron and Steel Institute in 1908, Charles H. Merz, the engineer under whose able direction this large enterprise has been carried out, shows that the economies resulting from centralization of power development and electric distribution have led to the construction of plants now in operation, aggregating 102,000 h.p. installed; that additional plant aggregating 31,600 h.p. is under construction; that during the last 4 years the demand has increased at a rate averaging 20,000 h.p. per annum; that to-day every shipyard on the north bank of the Tyne is purchasing practically all of its power supply in the form of electricity; that the system is now

> responsible for the supply of current to eighty (80) miles of electrified railway, four tramway systems, the lighting in towns having populations aggregating over 700,000, motive power to the extent of 85,000 horse power and electrochemical works of over 12,000 horse power.

Not only has Mr. Merz established highly successful steam-driven power plants in a district where the cost of coal ranges from 7s. to 9s. per ton, but he has demonstrated that important economy of coal consumption results from the supply of electric power to the collieries for their mining operations.

Referring to this very interesting feature of the development, Mr. Merz says:

> The output of coal from Northumberland and Durham in 1906 was over 52,000,000 tons and, according to the Report of the Royal Commission on Coal Supplies, between six and eight per cent of the total coal brought to bank is used by the collieries for the purpose of power generation. From the make of coke * * * it appears that about one-fifth of the coal mined on the northeast coast is converted into coke. Making a liberal allowance, therefore, for the power at present used from the surplus heat resulting from the coking process, the collieries of Northum-

berland and Durham must burn for their power requirements some 2,500,000 tons of coal per annum. As the almost invariable rule is to work non-condensing, as the steam piping is usually long, and as a large portion of the load is intermittent, it is certain, and is proved by experience in this district, that the same power can be provided electrically in a large central power station by the consumption of less than a quarter of this coal. Apart, therefore, from the efficient utilization of waste heat, apart from the saving of coal in ship-building and engineering works, and apart from the saving resulting from the electrification of railways, the application of electricity to coal-mines in this district, when as complete as that to the Tyne shipyards, will render available for outside sale over $1\frac{3}{4}$ millions of tons of coal, equivalent to, say, over half a million sterling per annum.

The power plants of the Interborough Rapid Transit Company of New York. The output of the power houses of the Interborough Rapid Transit Company, New York, for the year 1908, was 409,000,000 kw-hr. The consumption of coal was 494,000 tons. In a paper presented at the 214th meeting of the American Institute of Electrical Engineers, by the writer and H. St. Clair Putnam,* comparison was made from the company's operating records of the fuel consumption upon the Manhattan elevated lines during the year ending June 30, 1901, when steam locomotives were employed, and during the year ending June 30, 1904, when electricity was used. I quote from this paper:

> Referring to the period first mentioned, one pound of coal produced 2.23 ton-miles, if the weight of the locomotive be included, and 1.5 ton-miles, if the weight of the cars only be considered.
>
> During the latter period (electric traction), one pound of coal burned at the power house produced 3.85 ton-miles; excluding weight of locomotives, therefore, the ratio of ton-mileage per pound of coal in favor of electric operation was 2.57 to 1. Including weight of locomotive it was 1.72 to 1.
>
> The average speed under electric operation was approximately 2 miles an hour greater than that attained by steam, and if correction be made for this difference, the ratio of ton-mileage per pound of coal, excluding weight of locomotives, is approximately 3 to 1, and, including locomotives, 2 to 1 in favor of electric traction.

If, therefore, we can conceive the possibility of operating today, by locomotives, the entire service of the elevated and subway lines of the Interborough Company, it appears that the saving in coal consumption effected amounts to not less than 988,000 tons of coal per annum.

In each of the three typical cases cited, it will be noted that

* TRANSACTIONS A. I. E. E., 1907. Vol. xxvi, p. 31.

electricity results in a radical economy of coal. That it also results in a material saving of household and other property which, in industrial communities using large numbers of small steam plants, suffer rapid deterioration by the effects of smoke and dust; that it tends strongly to improve the appearance of our cities and conditions which affect comfort and health, are facts not all of which are strictly pertinent to a consideration of the subject before us, but may nevertheless be mentioned in this connection.

Where steam is used to generate power in large plants from which electricity conveys it to users, economy results not only from the employment of comparatively large power generating units, but also from the introduction of plant economies and a degree of skill not attainable in smaller plants.

Industrial Use of Power.

From the latest available census returns, the following tabulation of aggregate capacity of prime movers used in the United States at the dates and in the respective industries mentioned, is compiled:

	Installed horse power
Manufactures, census 1905	12,765,594
Mines and quarries, census 1902	2,753,555
Street railways, census 1902	1,359,289
Electric light and power stations, census 1902	1,845,048
Custom flour, grist- and saw-mills, census 1900, (omitted from census 1905)	883,685
Telephones, telegraph and fire-alarm systems, census 1902	3,148

The United States census reports since 1870 afford data from which interesting conclusions in regard to the respective rates of increase of power used for various industrial purposes, the relative proportions of steam and water power now in service and the enormous growth of electric motor applications may be investigated. Such a study has been made by H. St. Clair Putnam, of New York, and the results set forth in an interesting paper on "Conservation of Power Resources,"* presented by him at the conference on the Conservation of Natural Resources, held at the White House, May 13–15, 1908. Mr. Putnam employed the method of constructing curves based upon census statistics beginning with the year 1870, and projecting the resulting respective curves from the dates of the

* Proceedings A. I. E. E. Vol. xxvii, p. 1397.

latest census figures to the year 1910. Making due allowance for the check to our industrial progress, which began in 1907, it is safe to say that at the present time the aggregate horse power of prime movers installed for industrial use in the United States, exclusive of steam railways, approximates 25,000,000.

In round numbers, 50,000 steam locomotives are owned by our railway systems. Based upon maximum drawbar pull, these locomotives would be capable of developing about 30,000,000 h.p., but the average power actually developed on a 24-hr. basis, when averaged over the entire year, approximates only about 2,000,000 h.p.

Were all the railways of the United States operated by electricity generated in large and properly located power plants, the aggregate installed capacity of these plants would approximate 4,000,000 h.p.

Of the grand total of 25,000,000 h.p. installed for industrial purposes, exclusive of steam railway operation, water motors represent, in round numbers, 5,000,000 h.p., and gas engines and oil engines about 800,000 h.p.

I quote from Mr. Putnam's paper:

> Prior to 1870, the use of water power in manufactures exceeded that of steam power. Water power expressed in percentage of the total power employed has since steadily declined, falling from 48.3% in 1870 to 11.2% in 1905. During the corresponding period, steam power increased from 51.8% in 1870, to 78.2% in 1900. The census of 1900 showed a marked falling off in the rate of increase in the percentage of steam power used as compared with the rate prior to 1890, and this was accentuated in the census of 1905, when the percentage of steam power fell to 73.6% of the total. This check to the ascendency of directly applied steam power was due to the introduction of electric power. In 1890 electric power was negligible. In 1900 it constituted 4.8% of the total. In 1905 this had increased to 11.8%—a marvelously rapid growth when the aggregate increase of over 1,000,000 h.p. in five years is considered. If the present rate of increase prevails until 1910, electric power will have reached 18 per cent of the total and steam power will have dropped to 68%.

The facts to which I have called attention point unquestionably to further and rapid progress in the work of substituting electric motors for small steam engines. While this development has made a highly significant beginning in the field of transportation, in the replacement of steam locomotives for elevated and subway service and for terminal operation, it has already covered a substantial portion of the entire field of stationary operation. Its economies and their advantages are fast becoming matters

of common knowledge. The cost of electric apparatus in general has decreased materially in the last decade, and the day appears not far distant when the isolated steam-engine plant as used for general industrial purposes will be practically banished from our cities.

Our ability to utilize in the near future a large proportion of our water-powers depends primarily upon the distance across which power can be electrically transmitted at practicable costs. This in turn depends upon the limits of potential against which transmission lines can be insulated in a manner which secures reasonable continuity of service. No precise limit of practicable distance can be fixed, nor is it necessary here to enter into a detailed discussion of the subject. It is sufficient to point out that power to-day is being transmitted from Niagara Falls to Syracuse, a distance of 160 miles. In California, it has been transmitted successfully a distance exceeding 200 miles. While 200-mile transmission does not bring every water-power of the country within reach of an adequate market, it does obviously suffice as regards a large proportion of our hydraulic resources not as yet utilized.

Available Water Power

From a purely physical standpoint, an estimate of horse power available in the case of a given stream requires accurate topographical survey and careful measurement of flow extending over a considerable period of years. Much work of very great value has been accomplished in these directions by the United States Geological Survey, but much still remains to be done. Systematic prosecution of this work is essential to the ultimate solution of our problems of power conservation, and the influence of the engineering profession should be strongly exerted to insure the effective extension and continuance of these surveys and measurements.

The majority of estimates of "water power available in the United States" which have been made, do not and cannot pretend to be exact. Obviously, there is a vast difference between the aggregate horse power which may be considered available, if the problem be looked at from a purely physical standpoint without reference to cost of development, and the amount which is available if cost of development and transmission to an adequate market is considered.

From the economic standpoint, determination of the aggregate,

the horse power which a stream is capable of developing, involves a step-by-step examination of its profile from its source to its mouth, the approximate location of sites where the gradient and other topographical features indicate the practicability of development within the limit of practicable cost as fixed by cost of competing steam power, and the summation of the powers thus located. Following this general method, M. O. Leighton, chief hydrographer, United States Geological Survey, estimates the available water power of the upper Mississippi river and tributaries at 2,000,000 h.p. and that of the southern Appalachian region at approximately 3,000,000 h.p. The aggregate available water power in the state of Washington has been estimated at 3,000,000 h.p. and that of northern California at 5,000,000 h.p.; but these figures are merely approximations and cannot be regarded as authoritative.

The report of the Inland Waterways Commission acting as the Section of Waters of the "National Conservation Commission," before the recent Joint Conservation Conference in Washington, states:

> The theoretical power of the streams is over 230,000,000 horse power; the amount now in use is 5,250,000 horse power. The amount available at a cost comparable with that of steam installation is estimated at 37,000,000 horse power, and the amount available at reasonable cost at 75,000,000 to 150,000,000 horse power.

The assumptions and facts upon which these estimates are based are not set forth in the report referred to with that precision which is essential to correct judgment of their value. The amount named as "available at a cost comparable with that of steam installation," namely, 37,000,000 h.p., exceeds the aggregate mechanical power now in use within the borders of the United States, and suggests the enormous saving in coal which would result from anything like a general development and utilization of our water-powers.

But whatever may be the aggregate amount which ultimately can be utilized, certain guiding facts are obvious and cannot be controverted. Among these are: 1. Under average conditions every hydraulic horse power utilized for industrial purposes in 10-hr. service saves at least 7.5 net tons of coal per annum. 2. Present knowledge does not permit us to obtain from coal burned for power purposes, even under conditions of best commercial practice, more than 10 per cent of the energy which it contains; under average conditions less than 5 per cent is utilized.

3. Electricity enables us to substitute a few and comparatively efficient steam plants for large numbers of small and relatively wasteful installations, thus effecting important economies not only in fuel consumption but also in other directions. It also enables us to transmit and utilize in available markets a very large proportion of the aggregate water power of our streams.

In view of these facts, it will be admitted that from the standpoint of conservation of our power resources, the attitude of the federal and state governments should be such as will hasten and not retard the development of our water-powers. Any policy which operates to retard this utilization is a danger to the community, not only because it tends to increase the average cost of power and, therefore, of transportation and the manifold products of our manufacturing industries, but also because it tends to prolong and even increase the, at present, necessarily wasteful utilization of coal supplies which can never be replaced. Such a policy, therefore, is on its face in direct contravention of the true principles of conservation.

Water-powers not hitherto appropriated under existing laws belong to the state and unquestionably should be utilized in a manner which will secure the utmost practicable advantage to the community. There is every reason why they should not be appropriated in perpetuity either by individuals or corporations. In permitting their appropriation and use for a limited period, the state undoubtedly should obtain the best terms possible, but the fact that prompt utilization means not only a saving of coal resources but a reduction in cost of manufacture and transportation is a consideration of the utmost weight. As compared with the direct revenue which can be expected to result from levying a direct tax upon water-powers, this consideration, from a broad economic standpoint, is in all probability controlling. Doubtless the state *can* tax water-powers and *can* devote the proceeds of such tax to any special purpose which it may elect; for example, the construction of inland waterways, as has been proposed; but correct determination of the wisdom or folly of such an arrangement requires careful consideration and at least approximate knowledge of the quantitative values of the economic results to be expected.

In a short speech before the Conference of Governors at the White House in May, 1908, referring to water-powers, President Roosevelt said:

My position has been simply that where a privilege, which may be of untold value in the future to the private individuals granted it, is asked from the Federal Government, that the Federal Government shall put on the grant a condition that it shall not be a grant in perpetuity. Make the term long enough so that the corporation shall have an ample material reward. The corporation deserves it. Give an ample reward to the captain of industry but not an indeterminate reward. Put in a provision that will enable our children at the end of a certain specified period to say what in their judgment should be done with that great natural value which is of use to the grantee only because the people as a whole allow him to use it. It is eminently right that he should be allowed to make ample profit from his development of it but make him pay something for the privilege, and make the grant for a fixed period, so that when the conditions change, as in all probability they will change, our children—the Nation of the future—shall have the right to determine the conditions under which that privilege shall then be enjoyed.

With the ideas thus vigorously expressed, every good American citizen must be in sympathy. To the admirable exposition of principles included in the preliminary report of the Inland Waterways Commission, dated February 26, 1908, few engineers will take exception. But the attitude of some of the speakers at the recent Conservation Conference in Washington, upon the occasion of the second gathering of the governors of the states, clearly evidenced a disposition to beg one of the fundamental questions which arises at the very threshold of consideration of the utilization of our streams; namely, the question of the real economic value of inland waterways. The minds of some who are taking an active part in the discussion of this great economic question apparently start with two assumptions: 1. That the economic value of a vast system of inland waterways is admittedly so great as to justify practically any expenditure in its development. 2. That the water-powers upon our streams are inexhaustible mines of wealth capable of yielding, under a general system of taxation, large revenues. From these premises, the conclusion that water-powers should be taxed to pay for waterways is easily deduced.

To the engineer it is evident at once that before any conclusion, involving expenditure of large amounts either of public money or private capital, is agreed to, both premises upon which that conclusion is based should be critically examined.

I do not propose in this paper to attempt anything purporting to be an exhaustive discussion of this complex subject. Personally I am frank to admit I have found it impossible from my present knowledge to form definite and final opinions as

respect of some of its phases. If our national engineering societies, during the coming year, will secure from their members best qualified by special knowledge to supply pertinent facts and suggestions, carefully considered papers discussing the comparative economics of transportation by rail and by inland waterways, much light will be thrown upon this very important and far-reaching question. At present data apparently essential to well-grounded judgment have not been collected and compared in a manner to justify formation of definite and final opinions which can be expressed in precise terms, opinions which may be expected to stand the test of time.

In calling attention to certain considerations which apparently tend to controvert the present popular impression that a radical improvement and extension of our inland waterways is the natural and proper solution of the great problem of freight transportation, I trust it will be understood that I am not approaching the subject from the standpoint of one whose interests are identified with railways. Such is not the case. The electrical engineer has everything to gain and probably nothing to lose from a policy which during the next 10 years may result in the appropriation of from $50,000,000 to $100,000,000 per annum for the improvement of our waterways and development of our water-powers. An engineer's first duty in a case of this kind is to bring any special knowledge which he possesses to bear upon the economic problem presented, to form and to state his opinions absolutely without bias. In our offices and in the field we may be retained properly to represent this or that special interest, but on the floors of our engineering societies our proper attitude is that of the man of science interested solely in the facts, their causes, relations, and consequences.

In considering the proposition to impose a tax upon water-powers and to devote the proceeds of this tax to the construction of a system of inland waterways, it is clear that the first effect of such a tax would be to retard the development of water-powers unless some compensating advantage were offered. It is also evident that such a tax would operate indirectly to stimulate the consumption of coal, and that if it be decided that we can afford to tax manufacturers indirectly in order to improve transportation facilities it would be wiser, from the standpoint of conservation of our power resources, to impose a tax upon coal used for power purposes. A tax of $3.00 per horse power

($4.00 per kilowatt) per annum is equivalent to a tax of 40c. a ton (2000 lb.) on coal used for power purposes in manufacturing, if we assume that 5 lb. of coal per h.p.-hr. are used. A proposal to impose any such tax on coal used for power purposes and use the proceeds for the construction of inland waterways probably would command little influential support, and yet a similar proposal, as applied to water powers, has received weighty endorsement and apparently is highly approved by those who are especially interested in the development and extension of inland waterways.

In the report of "The Inland Waterways Commission, acting as the Section of Waters of the National Conservation Committee before the last Joint Conservation Conference," I find the following referring to the uses of water power in our streams:

> The paramount use should be that of water supply; next should follow navigation in humid regions and irrigation in arid regions. The development of power on the navigable and source streams should be kept subordinate to the primary and secondary uses of the waters; though other things equal, the development of power should be encouraged, not only to reduce the drain on other resources, but because properly designed reservoirs and power plants retard the run-off and so aid in the control of the streams for navigation and other uses.

It would be interesting to know how the conclusion has been reached that in humid regions the development of power on our streams is less important than the improvement of navigation. To one familiar, for example, with power developments in the cotton-mill district of the Carolinas, and with the character of the streams there utilized so extensively and with such vast advantage to the community for power purposes, the idea that any conceivable use of these streams for purposes of navigation can be comparable to their value as producers of power is, to say the least, highly improbable.

Obviously, before any such general policy as that recommended is adopted, we should have assurance that the imposition of such a tax will not cost the community more than the resulting improvement of transportation facilities is worth. Cheap power is a factor of great importance, both in manufacturing and transportation. According to the census of manufactures taken in 1905, the gross output of our factories and mills had a value of $16,866,706,985. This product represented $1,152.00 per horse power installed. The wages paid amounted to $248.00 per horse power installed.

In the same year, the aggregate gross receipts of our rail-

ways were $2,325,765,167, or about 14 per cent of the value of manufactured products.

The object sought in constructing inland waterways is reduction in cost of transportation. The proposal to impose a tax which will operate to increase costs, in a business amounting to nearly $17,000,000,000 per annum, in order to attain an undefined advantage in reduction of cost in a business of less than one-seventh that amount calls for something further in the way of analysis than has yet come under my observation in this connection. The fact that manufacturing costs in America are in general much higher than in Europe, while our cost of transportation per ton-mile is now materially lower than can be found elsewhere, emphasizes the impression which results from a moment's consideration of the respective gross volumes of business in these allied fields of industry.

It would be impossible to suggest a more fruitful subject for unprejudiced analysis and illuminating exposition by competent members of our national engineering societies, in the immediate future, than the comparative economic advantages of railways and inland waterways. Facts in this field are urgently needed and should be supplied before public sentiment, unenlightened by unbiased competent advice, and influenced, perhaps, by prejudice and the clamor of local interests, shall crystallize in legislative enactment or executive rulings. Within the last two years, my firm, in considering problems presented by the substitution of electricity for steam in railway operation, has carefully studied the cost of operation of steam railways in the United States, using as the basis of this investigation not only the valuable reports of the Interstate Commerce Commission but also detailed operating cost-sheets confidentially furnished for the purpose by a number of the most important railways in the country. It also happens that within the same period we have had occasion to determine with great care the actual cost of operation of one important canal system, and in this connection we have secured considerable information bearing upon the general question of the economics of canal transportation. The conclusions which we have reached as a result of these comparative investigations do not support broadly and without material qualifications the popular impression that transportation of freight by inland waterways, in general, is less expensive than transportation by railways.

In engineering matters, general statements almost invariably

are subject to exceptions and I am not prepared to assert broadly that the construction of canals for transportation purposes is a mistake. I do assert, however, that in the majority of specific instances that have come under my observation the facts are far from justifying, from an economic standpoint, a propaganda aiming at the development of a general system of inland waterways beyond what may be attained by reasonable improvement of the channels of navigable streams with such comparatively short inter-connecting canals as may beyond reasonable doubt be justified by the results attained.

As regards the proposed "concurrent development of the streams and their sources for every useful purpose to which they may be put", as it is stated in the "declaration of principles" of the recent North American Conservation Conference, all engineers will agree that each stream should be studied with reference to its possibilities "for domestic and municipal supply, irrigation, navigation and power as inter-related public uses." But the development of a plan economically sound calls for unbiased consideration and fairly accurate knowledge of the economic value to the community of the resulting improvement of the stream regarded as a waterway for transportation. Obviously if we begin by assuming that because freight is carried across the Atlantic or through the Great Lakes at less cost per ton mile than it is carried by our railways, an increase in the depth of channel or improved regularity of flow of any given inland stream will secure corresponding results, we shall be misled and expenditures based upon any such assumption will be wholly or largely wasted. Each case should be studied, and competently studied, on its own merits. To tax water powers for the purpose of providing free waterways, from a broad economic standpoint, is a policy which, before adoption, should be compared carefully with the plan of imposing tolls upon all users of inland waterways and using the proceeds to develop water powers and secure cheaper power for our manufacturing industries.

The declaration of principles agreed upon by the recent North American Conservation Conference makes the following statement:

> We recognize the waters as a primary resource, and we regard their use for domestic and municipal supply, irrigation, navigation, and power, as interrelated public uses, and properly subject to public control. We, therefore, favor the complete and concurrent development of the streams and their sources for every useful purpose to which they may be put.

In the preliminary report of the Inland Waterways Commission, dated February 26, 1908, I find the following:

While navigation of the inland waterways declined with the increase in rail transportation during the later decades of the past century, it has become clear that the time is at hand for restoring and developing such inland navigation and water transportation as upon expert examination may *appear to confer a benefit commensurate with the cost*, to be utilized both independently and as a necessary adjunct to rail transportation. [The italics are mine.]

The fundamental facts here set forth will receive the unanimous and enthusiastic endorsement of the entire engineering profession. In determining, however, how the admirable and vastly important objects in view are to be attained, every engineer should use his best endeavors to prevent fundamental and far-reaching mistakes which easily may result from action based upon insufficient or inaccurate knowledge.

Discussion on "Electricity and the Conservation of Energy." New York, March 24, 1909

John Coffee Hays (by letter): In arousing public interest in the conservation of our natural resources the impression has been generally given in discussions and newspapers and magazine articles, that there was a very much larger waste going on than actually exists. It would seem, therefore, that the time is now at hand for engineers to discuss the different points of the question more in detail, and to show what work has already been done in the direction of conservation, and what in their opinion should be done in the future toward the use of our resources in the most economical manner. From the very nature of the work, engineering is more closely connected with the conservation of resources than is any other profession, and the engineer should lend every effort in his power to the general education of the public in this respect, and should discuss the various points of the subject practically and forcibly in line with his convictions and experience.

In the papers presented before the joint meeting of the four national engineering societies on the evening of March 24, it was clearly shown that very great work had been done in the past generation in respect to the economical use of our natural resources. The paper presented by Mr. Lewis B. Stillwell clearly pointed out the large part that electricity was accomplishing in this direction, and it is in line with this paper that I wish to discuss briefly the position of the water-power developments in the far West.

The writer in the capacity of consulting engineer to several of the largest hydroelectric projects of California has had the opportunity to make a close study of the subject on all sides, and particularly as to the policy of the federal government in connection with power projects in the forest reserves and national parks of California, and the effect that the present policy of the Department of the Interior has on the development of these projects which comprise practically all of the present available undeveloped water-powers in the state.

The water-powers have probably attracted more public attention in the last two years than any of the other natural resources, as they are the only ones which will not be exhausted by use. This fact makes it natural for the layman to assume that "the great water-power monopoly" which the press and the magazines frequently refer to, is a grave calamity facing the public, especially when the high officials of our government express similar opinions. The writer, of course, agrees that if in the distant future a monopoly were formed comprising all of the water-power in California provided such a monopoly could be formed over which the federal government and the local authorities had no control that it would be detrimental to the public interests when the natural

supply of fuel was exhausted. But even under the present laws of the state it does not seem possible that any extortion could be practised even were the water-power business controlled by one company; for the governing body of every community, be they supervisors or city trustees, has the power to regulate the rates of any water-power or any other public service corporation.

It is not my object to discuss whether or not a water-power monopoly threatens the people of California; what I wish to discuss is whether the present policy of the government is beneficial in checking a monopoly, or detrimental, by preventing the development of one inexhaustible natural resource and therefore necessitating a greater use of our exhaustible resources, such as oil and coal.

It should be clearly understood that the state and not the federal government owns the water in all of the streams available for power developments in California, and that the state has enacted laws which clearly set forth the manner in which this water is to be diverted and used. In the case of the forest reserves and the national parks, the government owns the land through which the streams flow, and therefore the only way that the government can control this development, is due to its ownership of the lands on which the conduits and power stations must be constructed. Where land along a stream is held by an individual, it may be condemned by a power company, the owner receiving compensation only to the amount of the actual damage done his property. As the land along these streams is useful only for grazing purposes, the damage is very slight, not greater than a few dollars per acre for a right-of-way 50 or 100 ft. wide, the entire length of the conduit; in fact, a private owner often grants this right-of-way to the power company free of charge. The riparian right, which entitles the land owner to the amount of water necessary for the irrigation of his lands, is his only other right; but in the mountain districts, owing to the character of the land, there are very few cases where the land is fit for agricultural purposes and therefore irrigation is not required.

The primary object of the forest reserves was to protect the timber thereon, and therefore should include only timber lands. This being the case, the majority of available power sites, which are at present embraced in the forest reserves, should not be affected, as the land on which these sites are located is in most cases below the timber line, and would consequently be outside of the reserves. However, the present forest reserves take in complete watersheds from the highest peaks several thousand feet above the timber line, to the edge of the valleys several thousand feet below the line of the marketable timber, as in these lower altitudes only the small trees and brush are native. It would appear therefore that the boundaries of the forest reserves were set with the idea of controlling the

water-powers. The forest department makes a point that owing to the protection of the forest they maintain the stream-flow and therefore should be entitled to regulate the use of the water in the streams so far as the streams lie within the reservation. It is generally accepted that the forests act as natural reservoirs in the regulation of streams; no doubt this is correct in respect to the eastern states where the mountains are comparatively low and heavily timbered, but in the far West we have a rather different condition as far as the water-power developments are concerned. I wish to venture the opinion that the forests on certain watersheds in parts of the Sierra Nevada mountains are of comparatively little advantage to the power company; for in these cases the timber line is far below the crest of the range. It is the melting snow on these barren rocky slopes above the timber line which gives the water during the lowest water period of the year, there being very little run-off from the forest portion during this time.

It is the aim of the federal government, the state, and all conservative men connected with water-power development to discourage the practice of irresponsible parties filing an appropriation notice on a stream, and then holding the right for speculative purposes. Under the state law, after posting a notice on a stream and filing a copy of the notice, the appropriator must within 60 days of the time of filing his notice begin work and continuously prosecute it to completion with due diligence. Where the stream lies within a forest reserve the state law has been amended to the effect that within 60 days after the filing of the notice, work must be started in the form of a survey for the preparation of an application to the Department of the Interior, and after receiving a permit from the Department of the Interior, work must be started within 60 days, as above, and pushed to completion within the time specified in the permit. The Department of the Interior allows one year for the making of the survey and the filing of the application, and as a considerable amount of time is necessary for investigating the project before an application is granted, it can mean from 18 months to two years before actual construction work is necessary. If it is a case of speculation, the speculator has really more time and is in a more comfortable position when his appropriation lies within the reserve than when it lies without. In the investigation by the Department of the Interior before granting a permit, it is the aim to ascertain the feasibility of the scheme as well as the financial responsibility of the appropriator, and were this investigation pursued thoroughly and with a view of developing a watershed of the highest efficiency, it would no doubt be of great advantage. At present the department is not in a position to do this, and the question of feasibility and financial responsibility is left largely to the judgment of the local forest ranger who is incompetent to judge of

these matters, owing to the fact that the majority of these rangers have been born and reared in the mountain sections; while they are usually admirably fitted for their position as far as the guarding and protecting of the forests are concerned, they have neither the education nor experience which would enable them to pass intelligently on a power project or anything relating thereto.

While in the case of the state's requirements the diligent prosecution of the work until completion is often interpreted to mean a very small amount of work by the speculator, it does not seem that the government requirements have bettered the matter to any extent. It would seem that the state at least had the advantage of simplicity, for should the appropriator not prosecute work with diligence, and not be in a position to prove that he had lived up to the requirements, he is in constant danger of having some one else make an identical filing, and push the construction of a plant to completion, thereby leaving him without anything; for no rights of any kind are obtained under the state law until the water is actually diverted and used, when the right is then acquired and dates back to the time of filing the notice of appropriation. While it is held by most of the water-power companies and many authorities that the government has no right to control the water-powers within the reserves simply because it has acquired land, it is my opinion that were the regulations and requirements of the forest department at all reasonable, they would meet with very little opposition from the power companies. It is further my opinion that practically all the opposition is caused by the revokable form of permit, and the so-called conservation charge made for it.

As to financing property and affording any protection to the stockholders of the power company, a form of permit which is revokable at the discretion of the Secretary of the Interior is worthless, especially when some of the absurd conditions which are imposed in the present form of permit are taken into consideration. The permits as now granted, mean that if at any time a company has not, in the opinion of the Secretary of the Interior, lived up to the conditions imposed and the rules and regulations of the department, the permit may be forfeited, and a plant representing the investment of millions of dollars be confiscated by the government. One of the absurd clauses in the permit is to the effect that the company shall not break any of the game laws, which means that should an employee catch a fish, or shoot a deer out of season, the company would be liable, and its permit could consequently be revoked. It is of course impossible for a company to be responsible for its men to this extent, and it is further absurd when it is considered that the breaking of a game law is a state and not a federal government offense, whether it be within a forest reserve or without.

While the conservation charge or the tax on the permit, is comparatively small at present, it is also subject to change at the discretion of the Secretary of the Interior within specified but broad limits. Aside from this it is open to considerable question as to whether or not the government has any right legally—it is manifest that it has no right morally to collect this tax; for by so doing it forces the power company to charge a higher rate for power, thereby placing it in the position of tax collector for the government to collect from the people of a comparatively small community, thereby throwing an additional tax on the citizens served. There has been considerable discussion as to what use this money could be put, and it is generally considered that it might be used for the improvement of inland waterways or applied toward the expenses of maintaining the forest reserves. Mr. Stillwell in his paper discussed the matter of taxing water-powers for the improvement of the inland waterways, and I will therefore pass over this point. However, as regards the application of this money toward the maintenance of the forest reserves, it would appear manifestly unjust from the simple fact that the forest reserves are created and controlled by the federal government and any expenses in the maintenance of them should be borne by all the people of the United States, and not by the people of a comparatively small community who derive little if any benefit from them. It is also impossible to state any fair rate of taxation which would be applicable to all water-powers, as each power is peculiar to itself, and the cost of construction varies within wide limits, whereas the market value for power is practically the same, so that a tax which would not be felt by one company might be ruinous to another.

This discussion is not intended in any way as an attack on the principle of government regulation of water-powers, and were this an entirely new industry in a new country having no laws governing the streams, or should the title to the waters of the streams pass to the government with the lands, it would be an entirely different proposition, and might be handled by the government to advantage. The fact is, however, that this is not a new industry, and that well-defined state laws have been enacted and in force for years. When it is taken further into consideration that the government lays absolutely no claim to the waters of the streams, it would seem impracticable and extremely unjust for the government in the form of a land owner to stifle the development of one of our most important industries, and the present policies of the Department of the Interior can have no other effect.

There is of course, a popular feeling and a natural one, that perpetual water rights should not be held by any individual or corporation for power purposes, still the state laws provide that this may be done, and it would seem that if a time and limit were to be put on a water-power, the only way to accomplish it would be by the changing of the state law and not by revokable

permit. A water-power is of necessity a monopoly to a certain extent; that is, to the extent that a street railroad franchise is a monopoly on the street during the life of its franchise. The power company's monopoly is confined simply to its position on the river and its right to use a certain amount, or all, of the water for power purposes at this point, but these rights do not give it a monopoly of the market, for should it charge its consumers a high rate for power it would immediately court competition, and a power company on an entirely different river could extend transmission lines into the territory served, for it must be remembered that electric power is being commercially transmitted hundreds of miles with an extremely small loss of energy.

The present condition could be relieved to a great extent, however, by a very simple means; that is by the granting of an irrevokable permit by the Department of the Interior to the water-power company for a certain definite period, and making a definite provision for the disposal of the property at the expiration of this grant. All the absurd conditions of the present form of permit should be omitted, and if there must be a conservation charge, a definite sum should be set for the entire period. In this way the power company would know exactly the position it was in, and the conditions which it would be necessary to meet. As to the disposition of the property at the expiration of the permit, it should be decided whether or not the power company should simply give up all its property at this time, whether it would be taken in by the government at a predetermined cost, or whether it would be sold at public auction. If the power company were to release all rights at the end of the period, it would of course necessitate laying aside a sinking fund to pay for the installation, which would throw a burden on the consumer, in that it would necessarily increase the cost of the power production.

As to the government control, during the term of the grant, to prevent extortion by the company, I am of the opinion that very few people aside from the government officials and magazine writers consider this seriously, as the state has laws which amply protect citizens from any extortion by power companies, and the government should therefore let the power company entirely alone during the term of its grant. At the expiration of the grant, the property would come again under the control of the government for its disposal, when it might make any changes which in the meantime had proved would be of advantage. In making any regulations, the fact cannot be overlooked that some incentive must be held out to capital in order that this power be developed, and these absurd difficulties which have been placed in the way of power companies by the Department of the Interior must be removed, before a hydro-electric project is considered in any other way than a most undesirable investment, as is the case to-day.

While it is the general impression that a monopoly of any kind is not beneficial to a community, it has also been proved that a large producer may operate more economically than a small one. This point is particularly noticeable in the development of the water-power in the far West, owing to the great distance of markets and the advantage of combining the output of several hydroelectric power stations and distributing current along the same transmission lines and so placing the power stations that the available water may be used several times, and in a more economical manner. It does not follow that all the water-powers in the state should be combined, but the writer believes it is absolutely necessary for an economical development of any stream, that all power installations in the watershed be under one management, so that the entire watershed may be treated as a unit and worked to its best advantage. In a case that I have in mind, approximately 50 per cent more power could be developed from the watershed were it held by one company than could be developed by two rival companies.

This is the case where the storage reservoir capacity would be on the small tributaries of the main river. With a generating station situated on the main river, the water from these tributaries could be stored during the high water season, when there will be plenty of water in the main river to operate the plant, and during the low water season the flow of the main river could be reinforced with the stored water in the reservoir, thereby keeping the plant up to full capacity. Now assume that one company owns a generating station on the main river and a rival company holds the reservoir sites on a tributary, and that no reservoir sites are available on the main river; the plant being supplied on the main river would of necessity be installed for a capacity equal to the minimum flow of the river during the low water season, while the plant on the tributary would be installed for the average yearly flow of that stream, as both stations must be operated at full capacity for 365 days a year. When it is taken into consideration that the minimum flow of the rivers of central California is not over 5 to 10 per cent of the average yearly flow, it will be clearly seen that without storage reservoirs there would be an enormous amount of water going to waste during the high water season.

The condition above assumed is the rule rather than the exception, in that it is a difficult matter to find storage reservoir sites on the large streams, and when available they are usually very expensive to develop, whereas on the smaller tributaries there are often mountain meadows, lakes, etc., which may be more practically converted into reservoirs by the construction of comparatively cheap dams, where almost the entire run-off of the tributary may be stored during each year, to be used later in reinforcing the main stream. The construction of reservoirs for water-power purposes is also a great advantage to the country in the valleys, as it reduces the water in flood periods and conserves it for use for irrigation

later in the season when it is greatly needed, and this point alone should serve to gain support for a water-power development. Unless each watershed is taken as a whole, and studied with a view to the most economical development, it is apparent that a great waste would ensue which would be almost permanent, as a properly installed plant represents a great deal of invested capital, and anyone would seriously hesitate before abandoning it, even for the purpose of effecting a more economical arrangement.

As to the position the existing water-power companies hold to-day, it would be gathered from the sensational utterances of the press and the skilfully written articles in some of the magazines (which however are not always based on facts) that even to-day a water-power is a monopoly, and that the people are suffering through them. It would be well, therefore, to look at a little of the work which has been accomplished by the development of some of our western water-powers, and the case I will cite, being a power company supplying power to a strictly agricultural community in the San Joaquin valley, is, I think, one of the best examples of the conservation of a natural resource that I have encountered. This company began operation about ten years ago, and its territory comprises a section of approximately one million and a quarter acres of land. At that time approximately 90,000 acres of the land was under irrigation, and all of the water in the natural streams was diverted and used, so that the remaining land which was most suitable for the raising of high-class products, when irrigated, was of necessity undeveloped owing to the lack of water, and was used only for the growing of grain and for grazing purposes. About this time it was discovered that approximately 150,000 acres of this land lying along the foot-hills was exceptionally well adapted for the raising of oranges and other citrus fruits, and as the profits on these products are extremely large, endeavors were made to irrigate these lands by pumping water from wells. Gasoline and steam engines were used for pumping, but owing to the unreliability and cost of operating these engines by unskilled men very little was done.

With the advent of electric power the country developed rapidly, and this rapid development continues. There are at present over 20,000 acres of land planted to these products, irrigated by water pumped from wells by electric motors, and the gas engine and steam engine have completely disappeared from the field. The value of the land which at the advent of electricity was from $2 to $10 per acre has increased to as high as $400 per acre, and the orange grower makes an average annual net profit of approximately $250 per acre. This $250 is interest on an investment of approximately $500 including the land at $100. per acre, the remainder being spent in planting and caring for his orchard for the first five years, when it comes into bearing. The irrigation which makes all this possible, costs on an average about $10 an acre per year for the power used, as the price charged is $50 per horse

power per year, and one horse power used in this way will irrigate on an average five acres. It may be farther noted that the cost of harvesting, raising and shipping the crop amounts to from \$300 to \$500 an acre, when the net is \$250, and it is therefore apparent that the power charge is an insignificant item of the expense, and that the consumer is not being very badly robbed by the power company, magazine articles to the contrary notwithstanding. The rate of \$50 per horse power per year is often compared with such prices as are charged at Niagara where thousands of horse power are sold to one consumer, whereas in this case the average size motor used is 7½ horse power. In a plant such as Niagara the installation is comparatively cheap owing to the large units, the short conduits, and the accessibility of the plant, but in the western plant mentioned, the output is comparatively small, and the plant is located many miles from a railroad, requiring the transportation of all machinery and material over rough mountain roads at great expense. The conduits are long, and constructed along steep mountain sides, thereby making the expense of hauling and hoisting one of the largest items of construction. The investment in distributing lines alone, due to the marketing of the power in small quantities over a wide area, amounts to approximately half as much as the cost of the rest of the plant. This illustrates also the necessity of treating each water-power separately when considering profit on prices charged, as all the water-powers which the writer has investigated operate under entirely different conditions, and it requires a very thorough knowledge of any plant to be able to discuss it, or the rate of charges, with any degree of exactness. This knowledge of the property can be gained only by an examination in the field. The plant described operates on rivers, the waters of which are used exclusively for irrigation. One miner's inch of water will irrigate about 5 acres in this section. The water-power company uses the water in the mountains and turns it back into the stream, whence it flows out to the valley and is used for irrigation, while the power-house transforms the energy of this water without diminishing it, and also sends to the valley in the form of an electric current over wires, energy which is used for the pumping of additional water from the underground reservoirs, and therefore is another irrigation project. In the above mentioned system approximately 100 second-feet are used and with this is generated approximately 7,000 h.p. which may be delivered to the pumping plants. Of this 100 second-feet a large proportion is of course lost in evaporation, seepage, etc., before it is applied to the land, but assuming that it were all delivered and placed on the land for irrigation it would be capable of irrigating about 20,000 acres, but in passing through the power-house it also generates enough current to irrigate approximately 35,000 acres by water pumped from wells. It can be seen that this is indeed conservation in its highest form.

A paper presented at the 236th meeting of the American Institute of Electrical Engineers, New York, April 9, 1909.

HIGH-VOLTAGE TRANSFORMERS AND PROTECTIVE AND CONTROLLING APPARATUS FOR OUTDOOR INSTALLATION

BY K. C. RANDALL

Outdoor or weatherproof transformers for ordinary distributing purposes, and for potentials up to about 2500 volts, have long been in general service. With the exception of arc lamps and series incandescent lamps, transformers have been about the only apparatus in high-tension service not protected by buildings. Distributing circuits operating at 6600 volts are now quite common. There are also a few 10,000-volt, and a small number of 15,000-volt, distributing systems using outdoor transformers.

Thus far, about 50 kilovolt-amperes has been the limiting capacity found in outdoor service, but there are a few exceptions. Outdoor transformers of 100 kilovolt-amperes capacity or greater, for any voltage, have been almost unknown. Recently, however, large transformers have been taken up; a few are in service, and most of them but for a short time.

The outdoor problem may be divided into two parts:

1. The production of satisfactory outdoor apparatus.
2. The application of outdoor apparatus.

The design and construction will be discussed, and the application will be considered.

Construction. Transformers for outdoor service may be built for any requirements that the ordinary indoor type of oil-insulated unit will satisfy. As to capacity, the limit of approximately 500 kilovolt-amperes will apply to the self-cooled type, depending somewhat on voltage and frequency. As with the

self-cooled indoor transformers, the case is the principal problem. It is difficult to obtain the radiating surface required for cooling very large transformers, and still retain a simple mechanical construction. Oil-insulated, water-cooled, outdoor transformers can be built for any capacity irrespective of voltage and frequency.

The first problem apparent in the development of out-

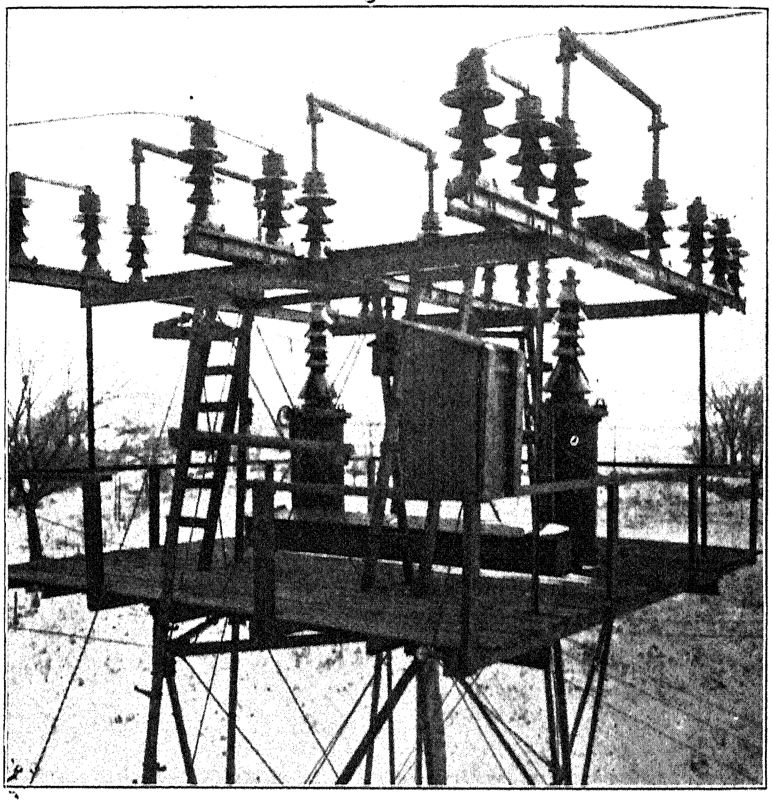


FIG. 1—Two 60,000-volt outdoor series transformers.

door transformers concerned the terminals: how to make them reliable in all kinds of weather and service. The next problem was to weatherproof the case satisfactorily. This much of the problem has been worked out, and now outdoor transformers up to 500 kilovolt-amperes capacity have been built, and units for potentials up to 60,000 volts are in service.

The downward projecting lead which issues from an over-

hanging pocket near the top of the transformer case is a quite satisfactory construction for moderate potentials. 10,000 volts or more can better be carried by upward projecting terminals, this arrangement being particularly attractive for convenience in wiring. About the same practice for placing outlet terminals serves for both indoor and outdoor transformers, so far as the general arrangement and convenience are concerned. The essential requirements of outdoor terminals are that they

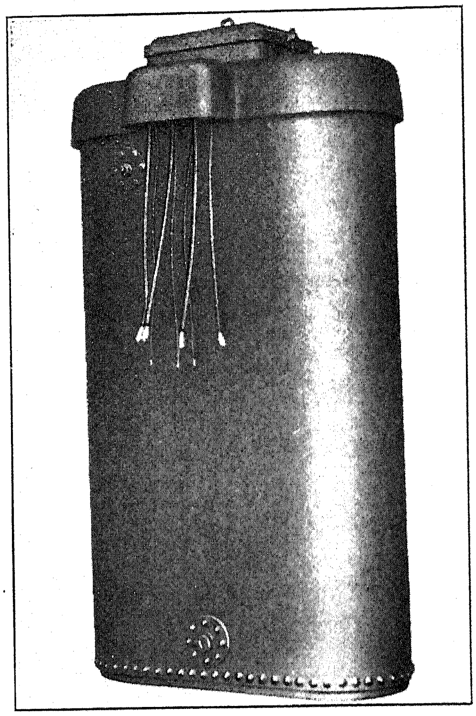


FIG. 2—This construction for outdoor service has been used for 6600-volt 25-cycle, three-phase transformers from 75 to 350 kilovolt-amperes.

retain their insulation characteristics and that they do not deteriorate by exposure to the elements. Outdoor terminals are larger and require much more room than those for indoor service.

Fig. 11 illustrates a self-cooled, 300-kilovolt-ampere, 33,000-volt, outdoor unit. The case is of corrugated sheet-iron with welded vertical seams. The bottom and top are cast on the corrugated shell. The resulting construction is strong, oil-tight, and is not subject to damage by the elements. The cover,

also of cast iron, has a considerable overhang or eave to guard the joint between it and the top of the case, and is fitted with the outlet terminals and a small inspection door. All joints are protected by an overhang and are designed for making tight with gaskets.

Outdoor water-cooled units employ the same general construction as the self-cooled type, except that the cases will usually be of boiler iron, and cooling coils with connections must be provided.

Weatherproofing outdoor transformer cases is doubtless best done by making the joints vacuum-tight. Eliminating leaks means that nothing can enter, and therefore a clean, dry unit would remain in that condition. Moisture is the only enemy to be feared, either in the form of rain, snow, or humidity. The "breathing" of wet air is the most important source of trouble because it is the hardest to eliminate. As a transformer heats, part of the air in the top of the case will escape if an outlet exists and new air will return as the initial temperature is resumed. Under certain conditions moisture which may have entered with the new air will condense. The amount of moisture which will accumulate in this way, in a short time is quite surprising. Well-made gasket joints, with deep, overhanging eaves and carefully sealed-in outlets, give good results.

Application. Whether outdoor apparatus is really desirable, involves a great many points even after it is proved satisfactory in the individual piece. Some of the more important considerations are: location and climate; cost of building and ground for indoor station; cost of corresponding ground for outdoor station; capacity of station; high-tension and low-tension voltages; number of high-tension and low-tension circuits; method of operation and control; method of cooling; attendance and supervision; instruments and their housing; and the cost of indoor versus outdoor apparatus.

A 20-kilovolt-ampere, 2200- to 220-volt transformer immediately suggests a pole installation. But if the figures are multiplied by ten, a 200-kilovolt-ampere, 22,000- to 2200-volt transformer suggests indoor service. If these figures are multiplied by three, a 600-kilovolt-ampere, 66,000- to 6600-volt transformer certainly has always demanded housing.

The large clearance required for exposed high-tension wiring and disconnecting switches; the expensive construction de-

manded for enclosed high-tension wiring and switch structures; the cells or compartments often used for transformers; the space for protective apparatus—all these operate to make the high-tension station costly as compared with the low-tension station. If all high-tension apparatus were placed outside, some kind of a structure would still be required in most cases for housing the instruments, the high-tension control apparatus, and the low-tension switchboard. If attendance were contemplated, some additional facilities might also be required. Under nearly all conditions where control of either or both high- and low-tension circuits is demanded, some housing will be required. Attendance, or at least frequent inspection, should usually be provided. When housing is needed for part of the apparatus, perhaps a large part, the expected advantage of placing the remaining apparatus outdoors may not become important, or may even not exist at all. In order to obtain the cost comparison, the cost of the station grounds and indoor apparatus must be balanced not only against the smaller building, but also against all the ground required for the outdoor apparatus, the outdoor apparatus itself, and the instruments and the indoor control apparatus.

If the high-tension circuits are many, and switching is contemplated, the outdoor arrangement appears attractive. If the low-tension plan calls for the control of the several circuits, the indoor arrangement looks desirable, as but little additional indoor space over that required for the instruments and high-tension control panels may be necessary for the low-tension switchboard. Furthermore, indoor low-tension switches and wiring, especially if remote control be not used, should be cheaper. Finally, when there has been provided a house that covers all but the high-tension pieces, it may be found that a small additional cost would have housed everything. Evidently no rule can be set down, as every case will require individual solution.

With all the large bodies of oil in such apparatus as the transformers and circuit-breakers outdoors and only instruments and remote-control boards indoors, the life and property hazard may be considered as less than in the equivalent but more congested indoor arrangement. The likelihood of careful inspection, however, is also much less, especially in bad weather. The difficulty of outdoor repairs except in the finest weather, is worthy of attention; though failures are very infrequent with well-built high-tension apparatus.

The successful outdoor transformer would itself be of less interest if outdoor switches and protective apparatus were not also available. Under such conditions, omitting the housing for the transformer might be of small advantage if protection for all other apparatus were still to be provided. The economy

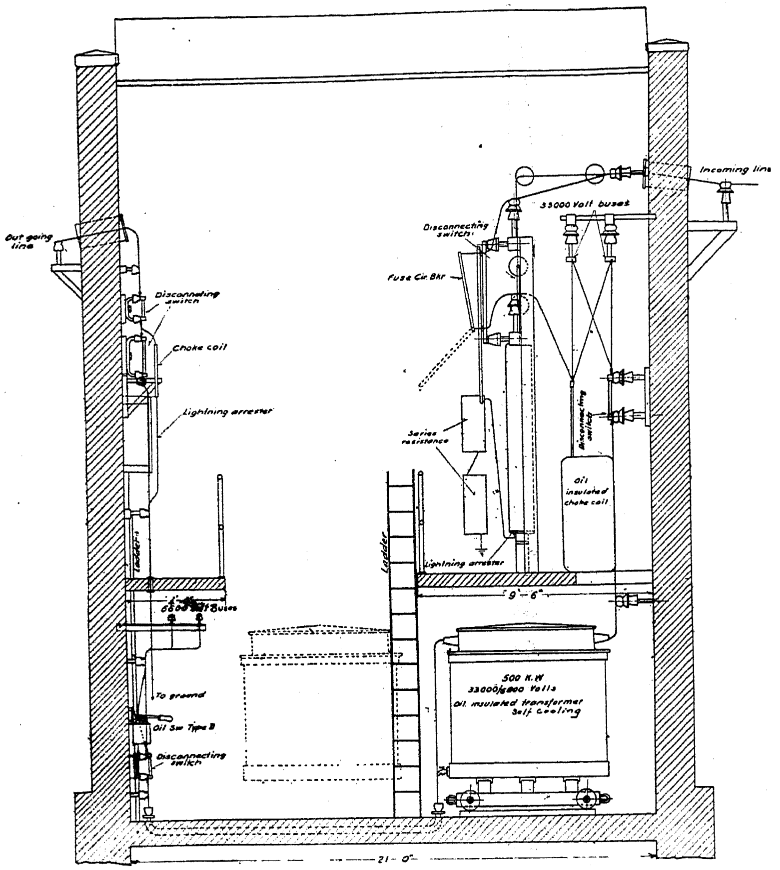


FIG. 3—Sectional view of 1500 kilovolt-ampere, 33,000-volt to 6600-volt, 25-cycle transformer station.

anticipated from employing outdoor transformers can generally be but partly realized without other outdoor apparatus.

Fig. 3 shows an indoor 33,000-volt station and Fig. 4 shows an equivalent outdoor station, No instruments are employed and no attendance is required. The station is entirely without a

building. Fig. 5 shows a 60,000-volt indoor station, and Fig. 6 shows an equivalent outdoor station of one-fourth the capacity. As an example of the growth toward outdoor stations, there may be cited the Lockport station of the Niagara, Lockport &

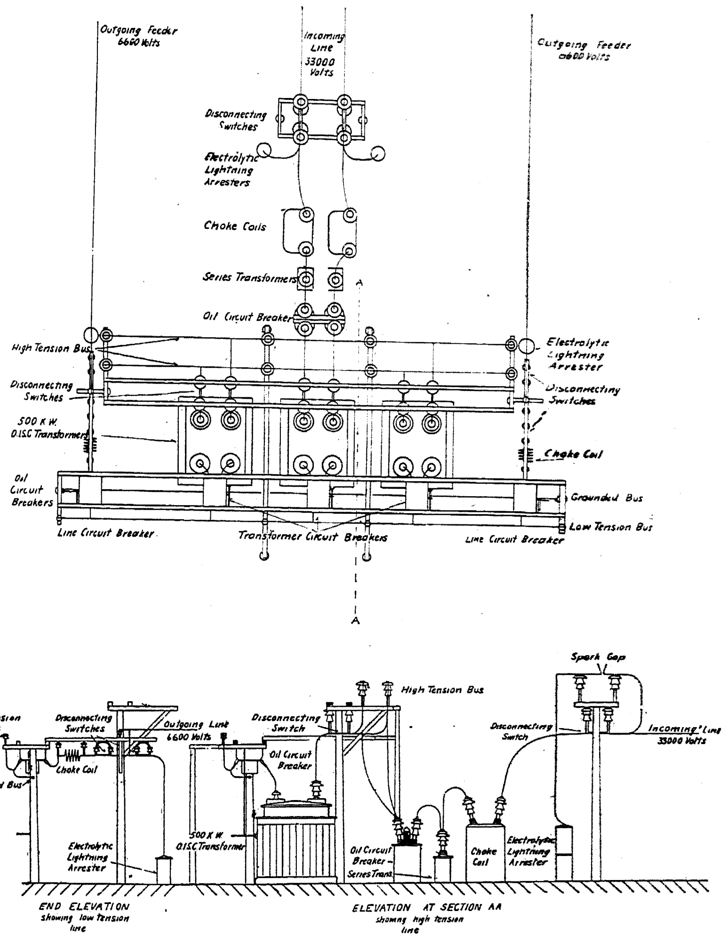


FIG. 4—Illustrates an outdoor transformer station equivalent to the one shown in Fig. 3.

Ontario Power Company. Some of the illustrations of this station showing outdoor lightning arresters and 60,000-volt busbars have already appeared in the Institute TRANSACTIONS.*

Considered purely as a circuit-interrupting device, whether

* Transactions A.I.E.E., 1907, Vol. XXVI, Part II, p. 1273.

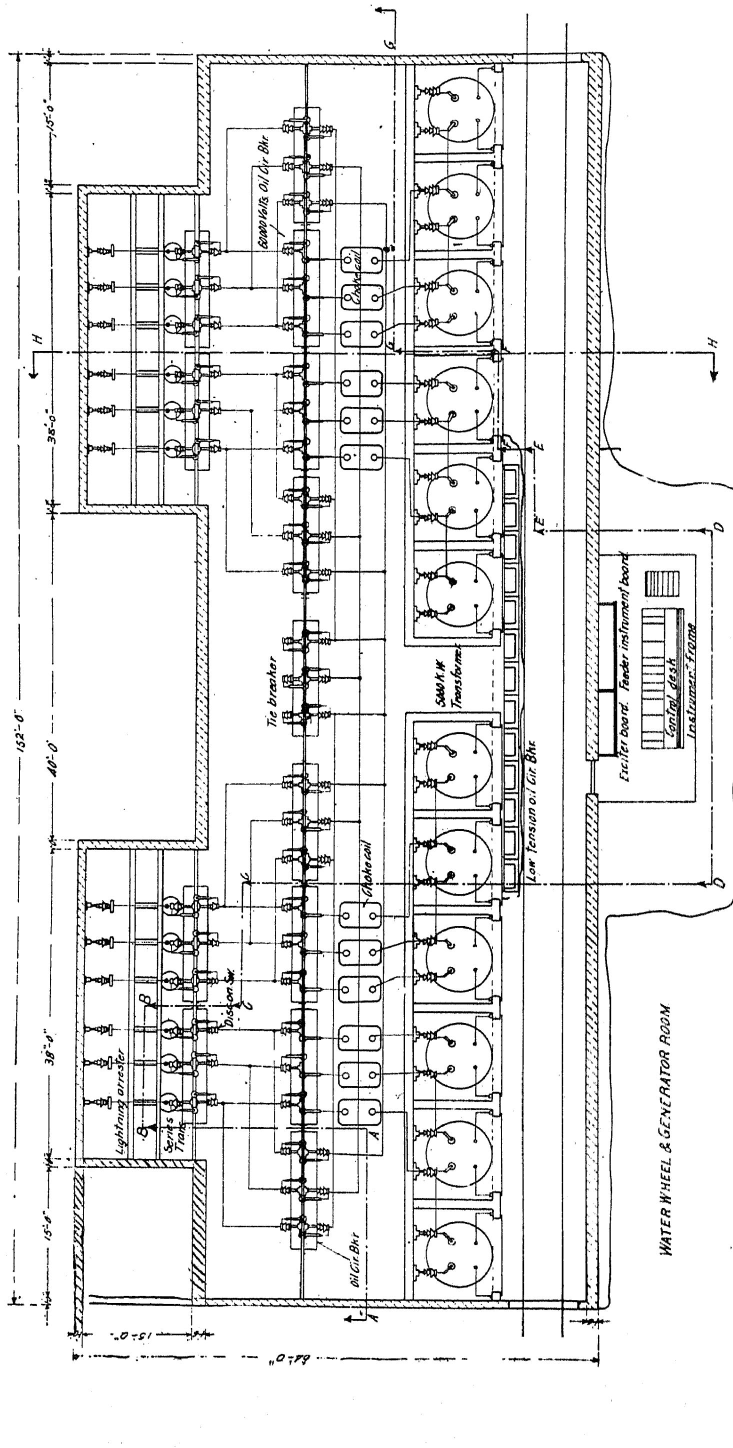


FIG. 5—Plan of 60,000-volt, 60,000-kilovolt-ampere, indoor transformer station.

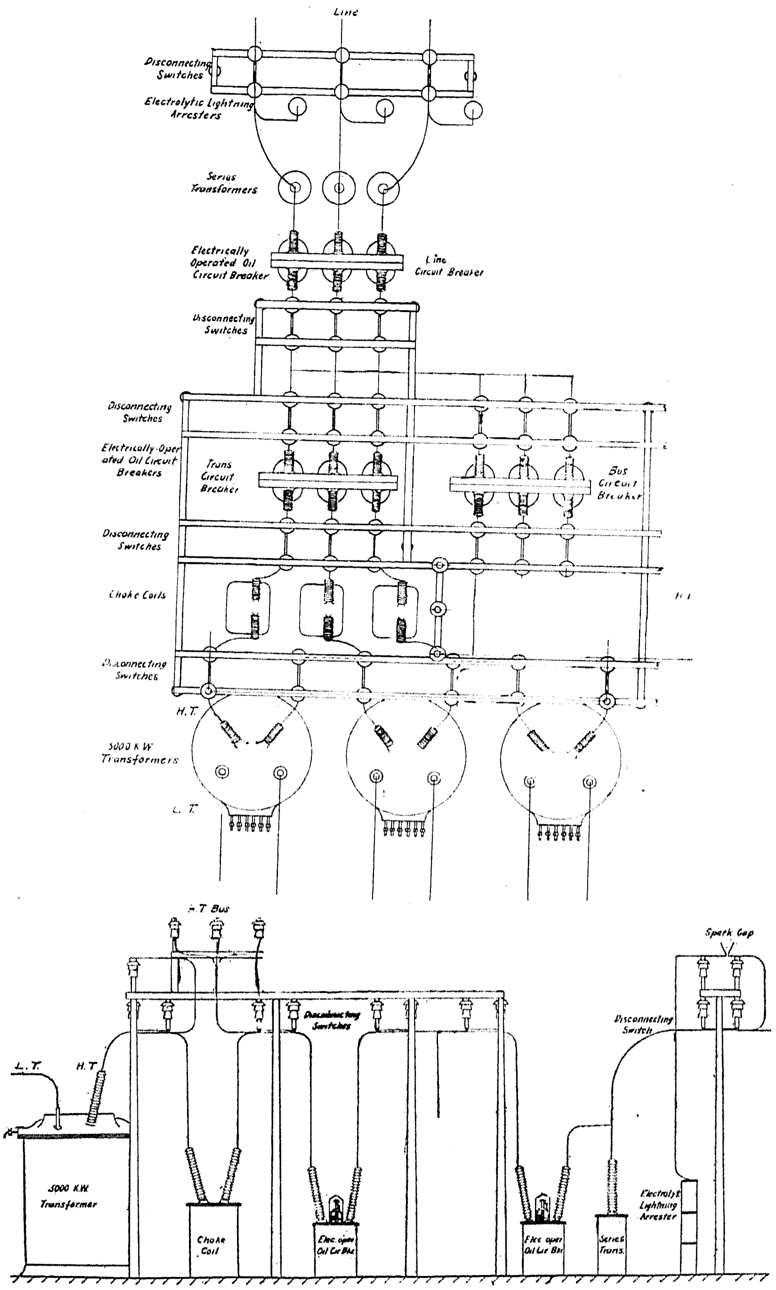


FIG. 6—Layout for outdoor transformer station, equivalent to one-fourth of indoor station, Fig. 5.

a circuit-breaker operates indoors or out does not affect the requirements. Outdoor transformer construction will take care of case and terminals—a suitable enclosing hood readily removable to protect the mechanism from the elements, snow and ice particularly, completes the changes necessary to make an outdoor unit. Non-freezing oil should be used in order that operation in severe weather may not be endangered. The ordinary methods of operation such as remote control, overload release, etc., are available just as in indoor practice. Outdoor disconnecting switches are in common use, and outdoor fuses have also found favor in some places.

The horn-gap arrester has been used considerably in outdoor service, both at terminals and along the transmission line, and the electrolytic type has been installed both indoors and outdoors. Arresters for all transmission voltages and designed for outdoor use are now available. The considerable amount of space indoors required for high-voltage lightning-arresters, and its cost, are serious; in fact, arrester houses are not unknown. Outdoor types may therefore be of particular and almost independent interest.

Outdoor choke-coils differ from indoor choke-coils only in respect to case and terminals; if of the oil-insulated type, the outdoor-transformer construction will apply. It is noticeable that about the same structural problems and solutions apply to outdoor transformer circuit-breakers and protective apparatus.

It has been suggested that by treating the high-tension transformer as part of the line, no high-tension circuit breakers would be required and no high-tension lines need enter a building. All switching would be done on the low-tension side, and when necessary fused disconnecting switches on the high-tension side would serve to cut off the transformer.

Outdoor stations should be paved and well drained around the apparatus. If this is considered too costly, individual foundations for each piece will serve. Transfer trackage with a truck, or a heavy hand truck will be found very convenient when moving apparatus.

When the stations are important it will usually be found desirable to arrange a room with repair facilities and connected with the outdoor station trackage, rather than resort to temporary weather protection or to remove the apparatus to a distance, should repairs be necessary. Such space would doubtless be convenient for many purposes such as cleaning and in-

specting. Fencing, to keep out intruders, is advised when more than the bus-bars are outdoors.

The general use of outdoor apparatus to the exclusion of indoor apparatus is, of course, not to be expected; but frequent

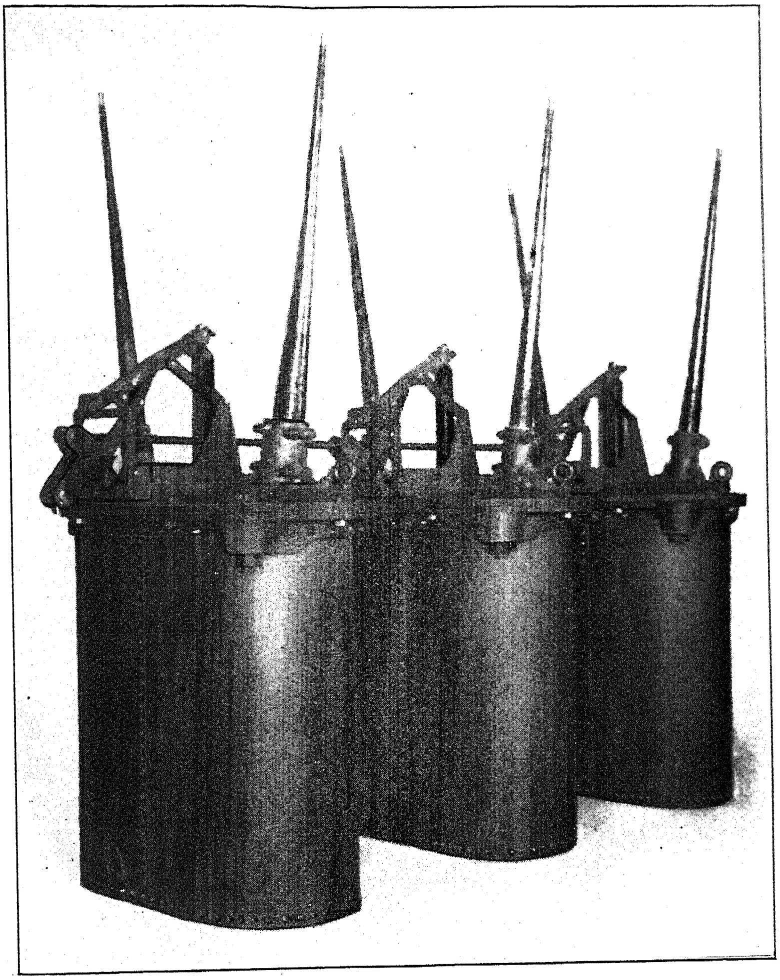


FIG. 7—Three-pole indoor, 60,000-volt, 300-ampere circuit-breaker.

applications will doubtless be made as a development of the general type of station just referred to. Transformer and switching stations present the greatest opportunities, as power houses and sub-stations employing synchronous converters or

motor-generator sets require so much housing that indoor transformers, circuit-breakers, etc., may be considered best.

If power is to be used within a short distance of a transmission line, a single transformation to the service voltage will be best. When an extended distribution is to be fed, a secondary transformation to the service voltage should be employed. The outdoor transformer will serve in either place, but with existing primary transformations the smaller secondary transformation affords a particularly attractive field.

Costs. Outdoor transformers, like the indoor type, increase in cost as the operating voltage is raised, and decrease in cost as the frequency of operation is increased. It is common to prefer a few large transformers rather than several small units. Also large transformers for the transformation from the high transmission voltage, with several smaller units on a secondary system, are preferred to a number of small units serving the consumers by one transformation from the transmission voltage.

The following cost estimates based on existing installations two equivalent indoor and outdoor 2000-kilovolt-ampere, 25,000-volt, 60-cycle stations—may be of interest.

Indoor station. High-tension apparatus inside of building. Transformers in pockets.

Approximate cost of steel building, frame erected	$3100
Inside of building, including steel work	2300
Total for building	$5400
Two 1000 kilovolt-ampere, three-phase, 22,000-volt to 440 volt, 60-cycle transformers delivered and erected	7200
Switchboard for above, including two incoming line panels step-down transformers' panel, and necessary oil switches	2500
Total for station	$15,100

Outdoor station.

Building, including outside bus-bar structure	$1020
Two 1000 kilovolt-ampere outdoor transformers as above, delivered and erected	7800
Switchboard as above (only low-tension apparatus and panels inside) all high-tension bus-bars and transformers outside, delivered and erected	2625
Total for station	$11,445

From these figures, the indoor station costs approximately 30 per cent more than the outdoor station.

Another example covers the approximate costs of a 3000-

kilovolt-ampere, 22,000-3000-volt, 25-cycle motor-generator sub-station.

Indoor station. Incoming lines 22,000 volts, two of which are installed. Building structural steel, all apparatus inside, transformers in brick compartments. High- and low-tension bus-bars and oil switches in compartments.

Length of building 110 ft., width 38 ft., equalling 4180 sq. ft.
Approximate cost of building ready for apparatus, steel work and compartments..$21,835
Three 1000-kilovolt-ampere, three-phase, 22,000/3000 volt, 25-cycle, step-down transformers, delivered and erected........ 15,000
Three 1000-kilovolt-ampere, motor-generator sets, motor synchronous type, 3000 volts operating at 80 per cent leading power-factor. Generator 550/600 volts, delivered and erected. 48,000
Three 45-kw. exciters, induction motor driven, delivered and erected... 4,500
Two incoming lines 22,000 volts, three-phase.
Three three-phase 1500-kilovolt-ampere, step-down transformers panels 22,000/3000 volts.
Two three-phase railway feeder panels 3000 volts, 3000 kilovolt-amperes.
Four three-phase lighting feeder panels, 3000 volts, 500 kilovolt-amperes.
Three panels for controlling motor-driven exciters.
Three synchronous motor panels.
One direct-current feeder and total output panel 600 volts, 3000-kilowatt switchboard delivered and erected.................. 20,000

Recapitulation.

Building	$21,835
Switchboard	20,000
Transformers	15,000
Motor-generator sets	48,000
Exciters	4,500
Total cost	$109,335

Outdoor station. High-tension apparatus outdoors on concrete foundations. Building to be 110 ft. by 19 ft. Station same as above as regards output. Ordinary type of building. No steel work.

Building	$7,480
Switchboard, same as above. Oil switches made weatherproof	20,200
Transformers made weatherproof	16,000
Motor-generator sets as above	48 000
Exciters, as above	4,500
Total	$96,180

In this example, as might be expected, the saving of approximately 13 per cent is not so large as for the simple transformer station.

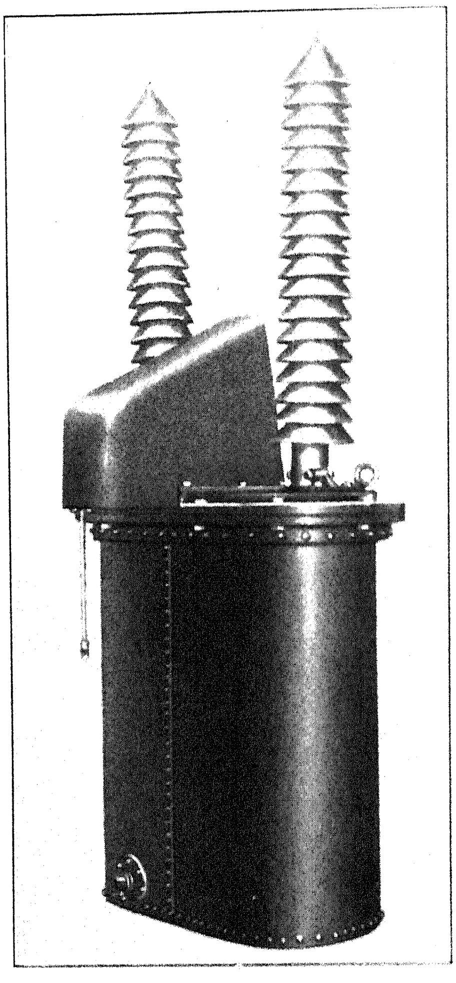


Fig. 8—Single-pole, 60,000-volt, 300-ampere, outdoor breaker. (This is one pole of three-pole indoor breaker, Fig. 7, arranged for outdoor service).

Service. For outdoor transformers the same care is required as for indoor transformers. An occasional test of the oil as obtained from the sampling pet-cock should be made. If attendance is at hand the usual load and temperature log should

be kept. This applies to self-cooled and water-cooled units just as in indoor practice. Thorough and detailed inspections should be made at such infrequent intervals as the operating conditions would indicate to be desirable.

In the summer, protection from the sun will probably be an advantage, and this may take the form of a simple fence high enough and so placed as to cast a shadow over the transformer

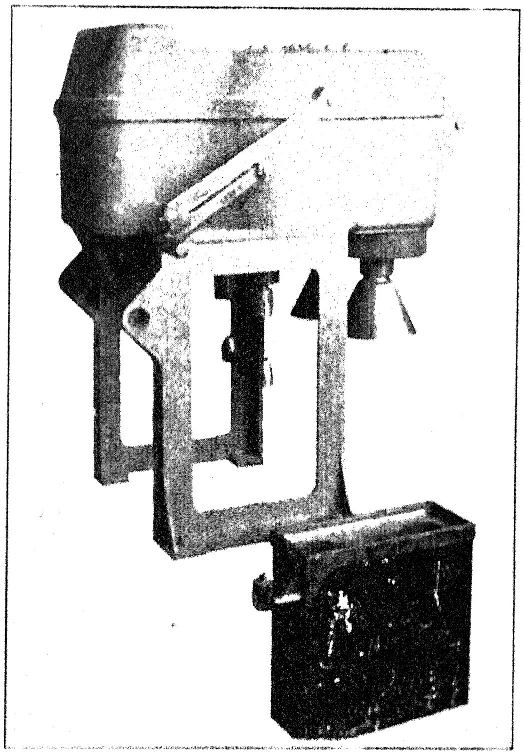


FIG. 9—11,000-volt, 300-ampere, remote-control, automatic, outdoor circuit breaker.

during the hottest hours of the day. At night the cooling conditions will be much more favorable, and the heat accumulated during the day will be discharged, so that each day is started with a cool transformer. The large amount of oil in self-cooled transformers represents a great thermal capacity; it acts as a sort of flywheel for the system, tending to smooth out temperature fluctuations.

Water-cooled units do not require shade, and if attendance is at hand their temperature can be held practically constant under varying load and weather conditions, if desired, even without automatic thermostatic control of the cooling water.

In extreme winter weather, if the transformer is likely to be shut down for a considerable time the cooling coils should be freed of water so that they cannot freeze and burst.

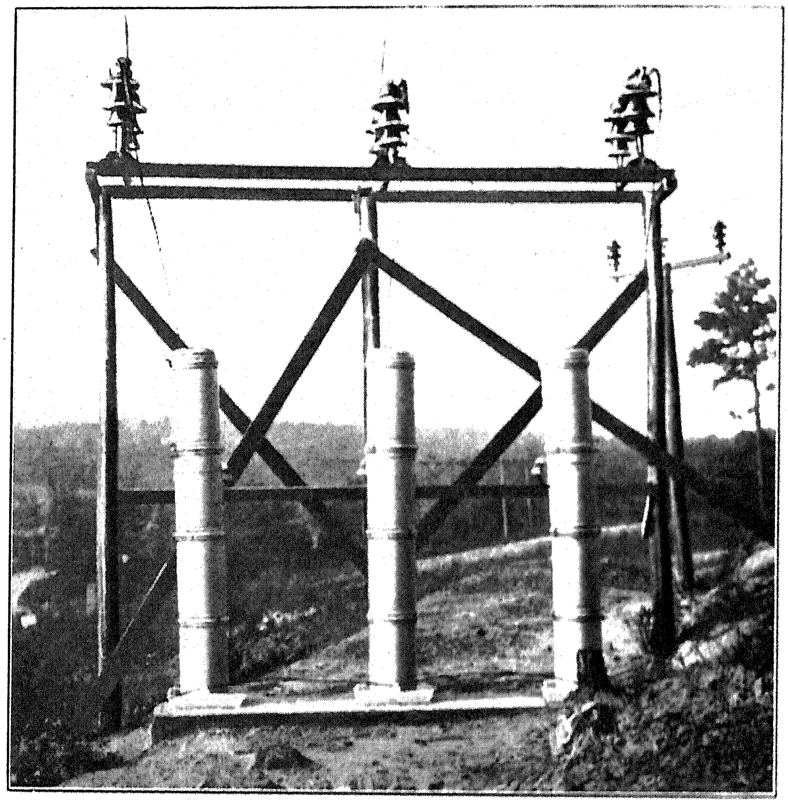


Fig. 10—Outdoor installation of 50,000-volt, electrolytic lightning arresters.

Transformer oil will thicken at about 0° cent., but does not harden so as to damage windings or case, as freezing water might do. When cold as this the insulating quality of the oil is reduced, but not so as to make it unfit for its purpose. In ordinary service such conditions need not arise, as the iron loss alone will usually suffice to keep the oil fluid. In extreme

weather conditions, self-cooled units may be kept warm by screening with a tarpaulin, thus reducing the effectiveness of the cooling surface. Similarly, the water-cooled unit may be operated without water, or with very little, to accomplish the same results. In some instances it may be found desirable to lag the water supply pipes during cold weather.

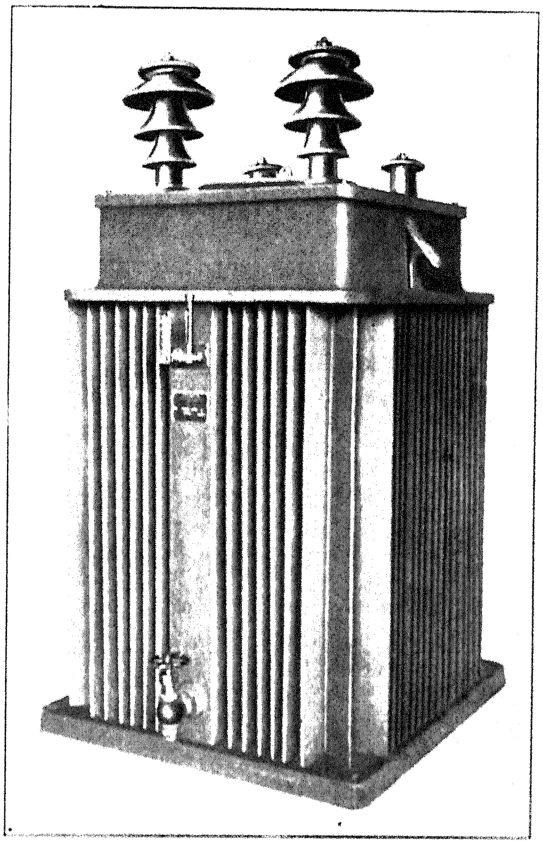


FIG. 11—300-kilovolt-ampere, 33,000/6600-volt, 60-cycle, oil-insulated, self cooled transformer. (Illustrates general construction for outdoor transformers of moderate capacity).

A self-contained sub-station made up of transformers, circuit-breakers, and choke-coils has been proposed. Such an arrangement, if three-phase, would require but three high-tension leads and would dispense with of twelve other leads otherwise required by switches and the choke-coils. If the choke-coils are omitted,

six leads are still saved. For capacities up to approximately 250 kilovolt-amperes this arrangement may be found attractive. Such units should be protected by fuses or other circuit-inter-

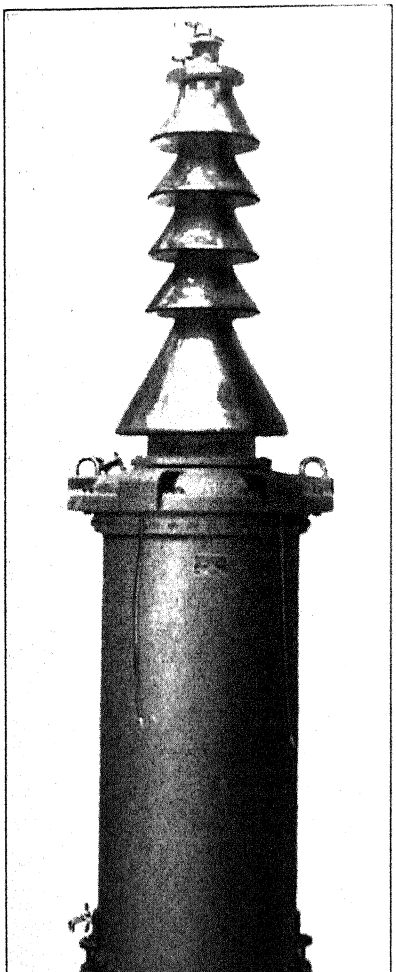
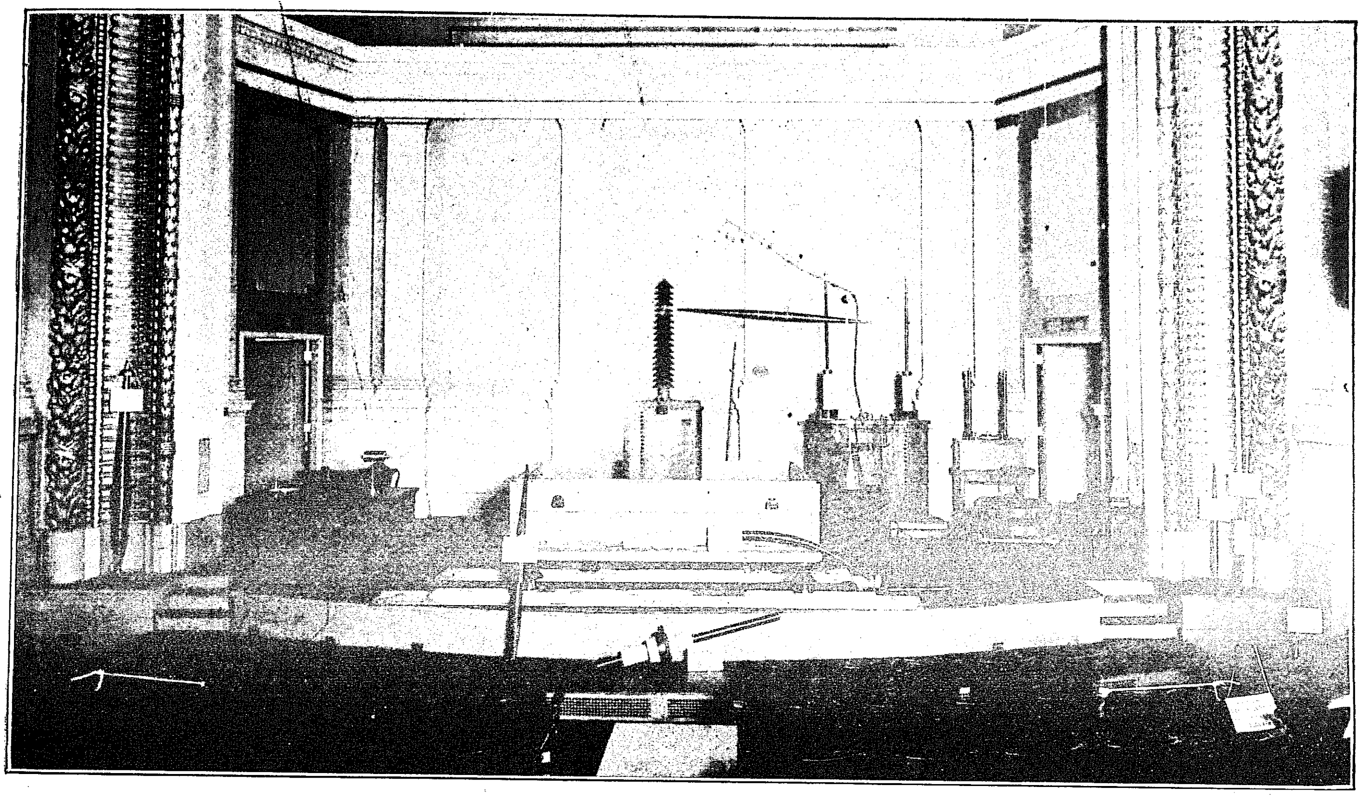


FIG. 12—60,000-volt, outdoor series transformer. (Fig. 1 shows two of these transformers installed).

rupting devices on the high-tension side, as a failure outside of the switch could not be relieved by it. Disconnecting switches should also be used.

A construction which permits of shipment in the case with oil is an improvement in transformer design which has been received with quite general favor, as the installation problem is much simplified. The usual unpacking, reassembling, and drying out, if necessary, is reduced simply to removing the blind flanges from the terminal openings and replacing the terminals in order to be ready for operation. Transformers arranged for shipment in their cases with oil, if made weatherproof, need never enter a building after leaving the factory, unless repairs or inspection should demand it.

Summary. The advantage of outdoor apparatus lies in cheapening the installation, due to a saving in building; there is also less life and property hazard. The disadvantages are absence of protection from weather when inspecting, overhauling or making repairs, and exposure to molestors. The problem, as a whole, was, first the transformer; second, the switch; and, third, the protective apparatus. All of these have been worked out and some experience obtained. The problem now is to decide when outdoor apparatus is warranted. This is a question of the station rather than the apparatus, and is subject to the individual conditions of each case.



Apparatus used for testing condenser type of insulation for high-tension terminals to 225,000 volts
Auditorium of the Engineers' Building, New York, April 9, 1909.

A paper presented at the 236th meeting of the American Institute of Electrical Engineers, New York, April 9, 1909

CONDENSER TYPE OF INSULATION FOR HIGH-TENSION TERMINALS

BY A. B. REYNDERS

The manufacture of apparatus for pressures of 88,000 volts or more resolves itself into the proper selection and arrangement of insulating materials. Of all the problems involved in this selection and arrangement, that of insulating the terminal wires where they pass through the case presents the greatest difficulty. When the terminal wires are carried through a metal cover the danger from breakdown is increased, yet the tendency of the latest designs is toward the use of metal tanks and covers. Even with present voltages the greatest difficulty lies in bringing out the terminal wires through the case. What may be termed "brute force" has characterized the proportioning of high-tension terminals up to the present time, as almost every class of insulating material in nearly every possible shape has been used to surround the conductor as it passes through the top of the case. The top itself has often been made of insulating material such as wood, simply to assist in insulating the lead.

One convenient way of analyzing the terminal problem is to consider it as a short piece of cable. The failure of such a terminal can occur in two ways; namely, by a puncture through the insulating material separating the inside conductor from the outside conductor or support, or by "creeping" over the surface from the end of the inside conductor to the end of the outside support.

Jona in an article on "Insulating Materials in High-Tension Cables" read before the International Congress at St. Louis in 1904, shows how the distribution of stress varies throughout

the dielectric, being inversely proportional to the radius. Furthermore, he shows that by grading the material according to its specific inductive capacity, it is possible to make the stresses nearly uniform throughout the dielectric.

Fig. 1 shows graphically the distribution of potential in an insulated cable for a homogeneous material. The ideal condition is obtained when the distribution lies in a straight line.

By the introduction of layers of different materials with

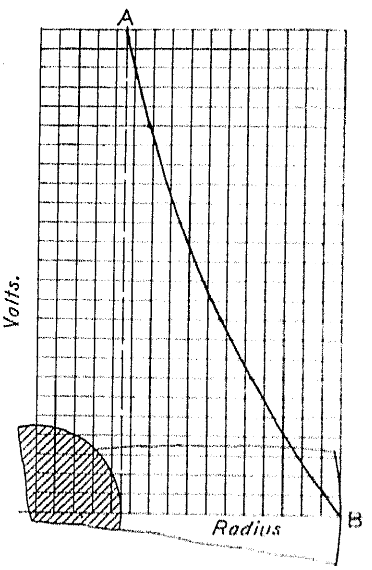


Fig. 1—Cable with homogeneous insulation: distribution of potential through dielectric.

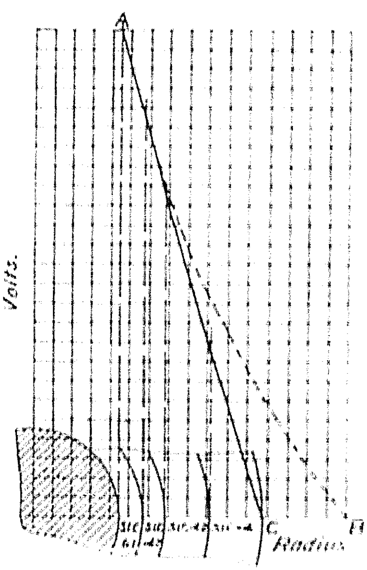


Fig. 2—Cable with graded insulation; distribution of potential through dielectric.

specific inductive capacities decreasing from the center outward, Jona obtains the result approximately as shown in Fig. 2, line A C. The curve A B is the same as in Fig. 1. By this method failure due to puncture in a properly designed cable is practically eliminated, but if the use of a piece of such a cable as the outlet lead from a transformer or a circuit-breaker is attempted, the greatest difficulty would be from breakdowns over the surface by "creepage" from the support to the ends.

A method that takes care of both puncture and creepage has been proposed by Ryan, Smith, Nagel, and others. This method

consists in dividing the dielectric by means of metal plates into a series of condensers of fixed capacities; for it is a known fact that if a difference of potential is impressed across a number of condensers connected in series, each will take its share of the stress in inverse proportion to its capacity. This principle has been utilized in constructing a terminal consisting of a small center rod or tube just large enough to carry the current, the rod being surrounded by alternative concentric cylinders of insulation and metal having the ends tapered in steps as shown in Fig. 3. The distribution of stress through the dielectric owing to the metal layers has been changed from a curve as shown in Fig. 1 to a straight line as shown in Fig. 2. Furthermore, the ends of the metal layers fix the distribution of voltage over the surface, which thus can be kept within safe limits. This design may be designated as a condenser type of terminal.

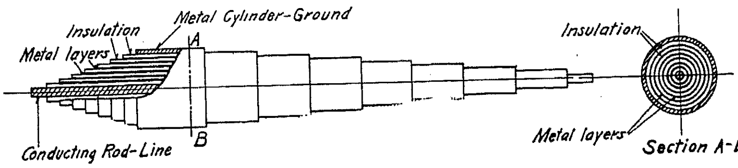


FIG. 3—Condenser type of terminal.

Before considering the mechanical design of this type of insulated terminal, it would be well to dwell on a few fundamental facts that will limit the design. It is obvious that in order to obtain the maximum efficiency from the insulating material every part of it should be subjected to a stress proportional to its strength. Further, if the dielectric material is homogeneous throughout, every particle should be strained the same as every other particle. This means that in a series of condensers employing a homogeneous dielectric throughout, each one should have the same capacity and the same thickness as every other one. In order to obtain this desirable result various difficulties must be overcome. It is the purpose of the following to show how this can be done.

The formula for the capacity of two concentric cylinders is:

$$C = \frac{k\,l}{2 \log \frac{r_2}{r_1}}$$

where

C = capacity in electrostatic units

k = specific inductive capacity of the dielectric.

l = length in centimeters of the conducting cylinders.

$r_1 - r_2$ = radius in centimeters of the inside and the outside conducting cylinders, respectively.

A glance at this formula shows that the capacity may be varied by changing the thickness between the conducting cylinders, the length of the conducting cylinders, or the specific inductive capacity of the dielectric.

It is now very generally known that the dielectric strength of ordinary solid insulating material does not increase proportionally as the thickness is increased. For this reason it is evident that varying the distance between the cylinders in order to obtain equal capacities will cause some layers to be thicker than others, and hence there is a waste of material.

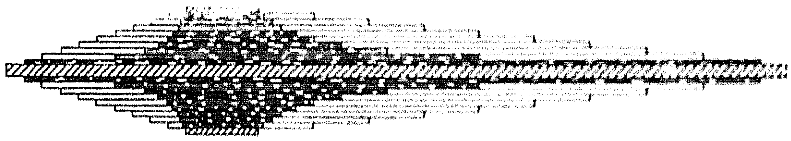


FIG. 4—Condenser type of terminal. Layers of insulation of equal thickness. Shaded portions show equal capacities throughout. Full lines show equal capacities on inside and outside layers and capacities, increasing towards center of insulation thickness.

The second method of varying the capacity—by changing the length of the conducting cylinder—is determined approximately, when equal capacities in all condensers are desired, by making the surface area, or the product of the length by the diameter, equal for all conducting cylinders. When the diameters of adjacent layers are nearly equal—a condition approached as the diameters increase and the thickness of insulation remains constant, the ends of the layers come very close together, and failure is liable to occur by creepage. In other words, the most economical design for creepage is to make equal steps with an equa difference of potential between the ends of each step. This cannot be accomplished by varying the areas alone, as shown by reference to Fig. 4. The shaded portion shows the shape which the ends must have for equal areas, while the full straight lines show the shape the ends must have if equal steps between layers are obtained. The best compromise that can

be obtained is to make the inside area and outside area equal and to allow all others to vary. The result is that the stresses are greatest at the center and on the outside, and decrease toward the middle.

The third means for obtaining equal capacity is limited by the variation in the specific inductive capacity of the available insulating materials. Unfortunately the available materials suitable for the manufacture of these terminals have very little variation in specific inductive capacity.

It is, however, by the combination of this third method with the best arrangement secured by the second method, that makes it possible to produce a terminal having equal distances

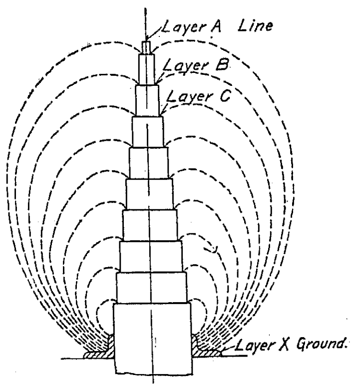


FIG. 5—Static field about condenser type of terminal.

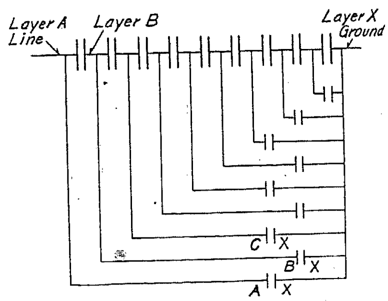


FIG. 6—Diagrammatic arrangement of condensers equivalent to Fig. 5.

and equal voltages between steps, and thus obtain the maximum economy from the minimum amount of material.

In all of the foregoing, only the capacity of the various metal plates with reference to each other has been considered, and it has been shown how it is possible to obtain equal capacities combined with other desirable features. If results on tests are to agree with previous calculations, there is one disturbing element which must be taken into account, especially at the higher voltages. This element is the capacity which each metal layer has to ground.

Fig. 5 shows in dotted lines what is possibly the static field surrounding a condenser type of terminal under service conditions. Fig. 6 shows an equivalent arrangement of con-

densers in which condensers *A-X*, *B-X*, etc., in parallel with portions of the main series, replace the leakages shown by dotted lines in Fig. 5.

Referring to these figures, layers *A* and *B*, with dielectric material between, form a condenser which will be called condenser *A-B*. In addition, layer *A*, and the outside grounded layer *X*, with the intermediate air or oil as a dielectric, form a condenser, *A X*, which is in multiple with the remaining system of condensers. The only effect of this condenser is to cause corona at the edges of the center rod; that is, if the stress to ground is great enough. To reduce this corona to a minimum, all corners should be rounded, as will be explained later.

In a like manner, layer *B* and the grounded layer *X*, with the intervening medium as a dielectric, form a condenser, *B-X*, which is in series with condenser *A-B*, and in parallel with the remaining arrangement of condensers. Again, in similar manner condensers *B-X*, *D-X*, etc., combine their capacities with the remaining arrangement. The net result of this leakage capacity is to decrease the apparent capacity of the condensers *A-B*, *B-C*, *C-D*, etc., at the line end and to increase the capacities of the condensers at the ground end. In order to correct for the effect of this leakage, the calculated stresses at the ground end should be made greater than at the line end by a percentage determined by experience.

Terminals designed in accordance with the above principles are constructed by rolling up on a metal tube, paper and mica, or paper alone, using as a bond some material like shellac. At regular increases by steps in diameter, for example $\frac{1}{16}$ in., a layer of tinfoil, making one complete turn, is inserted during the rolling process. Great care must be exercised to obtain accurate diameters, especially when the thickness of insulation is small. No air spaces or wrinkling of the paper are allowable, or poor results will inevitably follow. After being completely wound, the tube is placed in an ordinary engine lathe and the ends tapered in steps, the amount of taper depending on whether the end is immersed in oil or surrounded by air. During this turning process careful work is necessary to prevent the turning tool from " digging " into the insulation below the tinfoil, owing to concentric layers of insulation being irregular in shape instead of true circles. After the turning process is completed, the entire terminal should be treated with some insulating varnish in order to prevent the absorption of oil into that portion pro-

jecting into the oil, and the absorption of moisture in that portion which is in air.

With a terminal constructed by the above method, there yet remains one point of weakness; namely, the edges of the tinfoil are sharp, and the potential which exists between these edges and ground may be high enough to cause corona effects in the surrounding medium. If this medium is air the result of corona will be the formation of nitrous acid or ozone, which may cause a deterioration of the insulating material and an ultimate

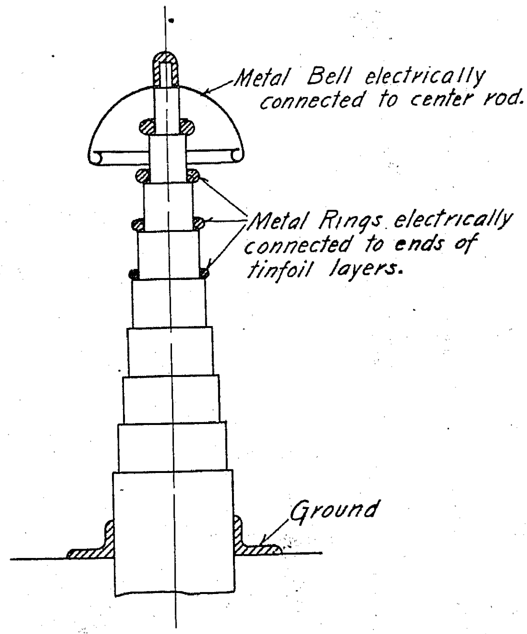


FIG. 7—Arrangement for preventing corona at edges of tinfoil layers.

breakdown. Up to the present time no harmful effects are known on insulation from corona in oil.

It is virtually impossible to roll up a tube of insulating paper and tinfoil and obtain all layers concentric and true circles. This will be strikingly shown if the ends are tapered in a straight line. Moreover, with such ends, the tinfoil, which is usually of a maximum thickness of 0.0045 in., is drawn out to a sharp edge and hence the formation of corona by the breaking down of the air in contact with these edges is vastly increased. Again it will be found that there is no fixed surface distance between

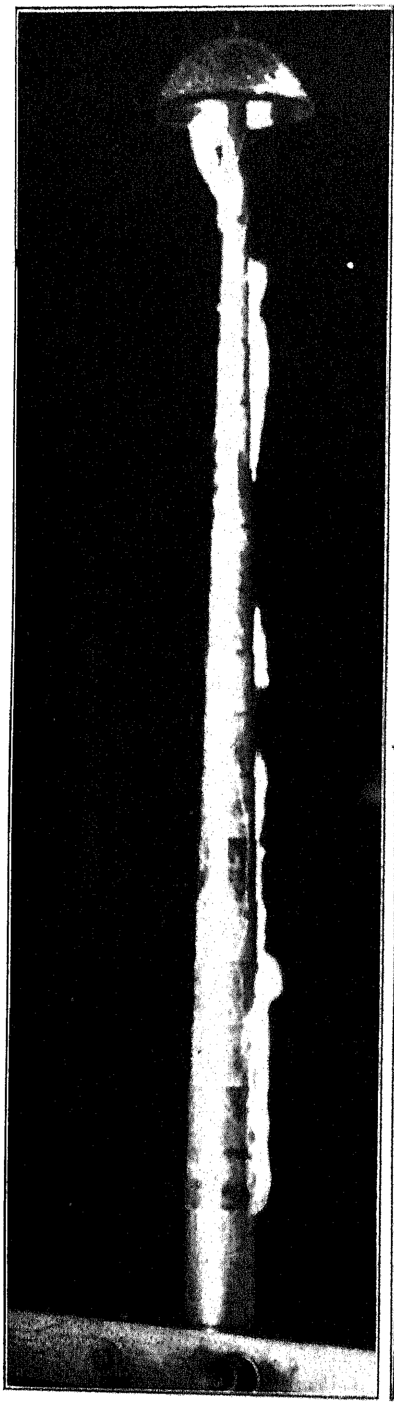


Fig. 8—Bulk type of terminal, breakdown 110,000 volts. First evidence of distress, 78,000 volts.

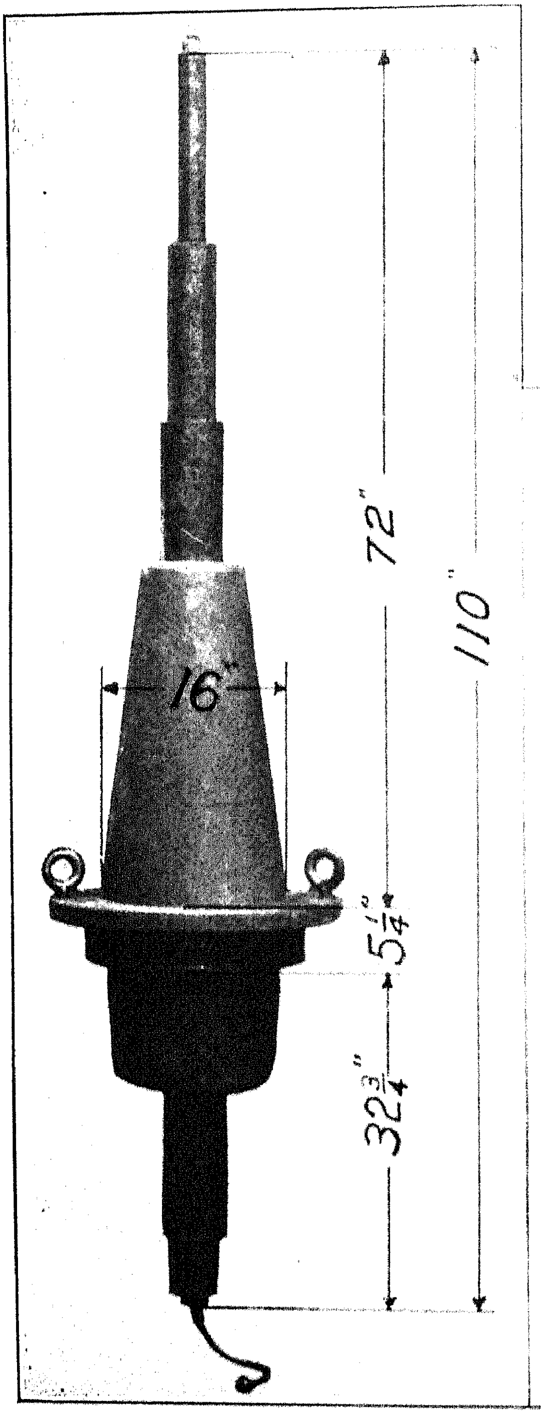
Fig. 9—Condenser type of terminal, breakdown 225,000 volts. First evidence of distress, 200,000 volts.

the ends of the tinfoil layers, due to variations in the insulation between them. For these reasons these ends are always stepped instead of being in one straight line. Even with stepped ends, it has been found that with extremely high voltages, 250,000 and more, the tin-foil edges are sharp enough to cause the formation of corona. As shown by Jona, Ryan, Berg, and others, corona occurs when the stress on the small layer of air at the surface (edge of tinfoil) exceeds the maximum stress which the air will stand. The remedy for such a condition is to make the radius of the end of the tinfoil layer large enough to reduce the surface stress to safe limits. By referring to Fig. 7 it will be noted that this result is accomplished by adding a metal ring of suitable diameter at the corners where the tinfoil ends. This ring is electrically connected to the tinfoil. Furthermore, the addition of a metal bell of large diameter on the central rod at the top not only reduces corona from this rod, but acts as a shield or guard ring for the first few layers, with a resulting appreciable gain in the creepage strength of these layers.

Figs. 8 and 9 show photographs of two terminals under dielectric stress near their breaking point. Both are of the same dimensions throughout. Fig. 8 is the ordinary solid homogeneous type, while Fig. 9 is the condenser type using the same dielectric material as Fig. 8. The stress on the terminal Fig. 8 is 105,000 volts, while that on the terminal Fig. 9 is 200,000 volts. An examination of Fig. 9 shows no flashes on the lower five layers. This is due to these five layers being short-circuited in order to cause the remaining part of the terminal to flash at 200,000 volts, this being the only testing voltage available at that time. If these layers had not been short-circuited, the terminal would not have flashed at less than 225,000 volts.

Figs. 10 and 11 show two terminals designed for the same test of 200,000 volts for 1 minute. The dimensions will show the saving effected in using the condenser type of insulation.

The question naturally arises: If this style of insulation is effective for terminals, why could it not be used for the insulation throughout the transformer? The terminal proposition is one of concentric cylinders, and, if the particular insulation in the transformer is of a similar shape, there is no reason whatever why the condenser type should not be used; its effectiveness, however, is less and less as the diameters of the layers become greater, and the difference between diameters becomes less.



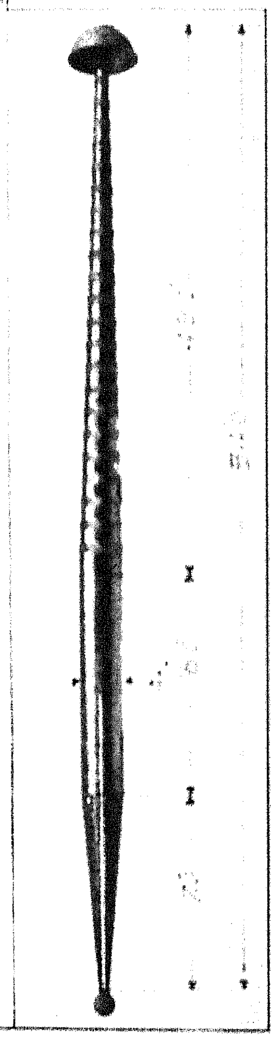


Fig. 10.—Bulk type of terminal. Test, 200,000 volts for one minute.

Fig. 11.—Condenser type of terminal. Test, 200,000 volts for one minute.

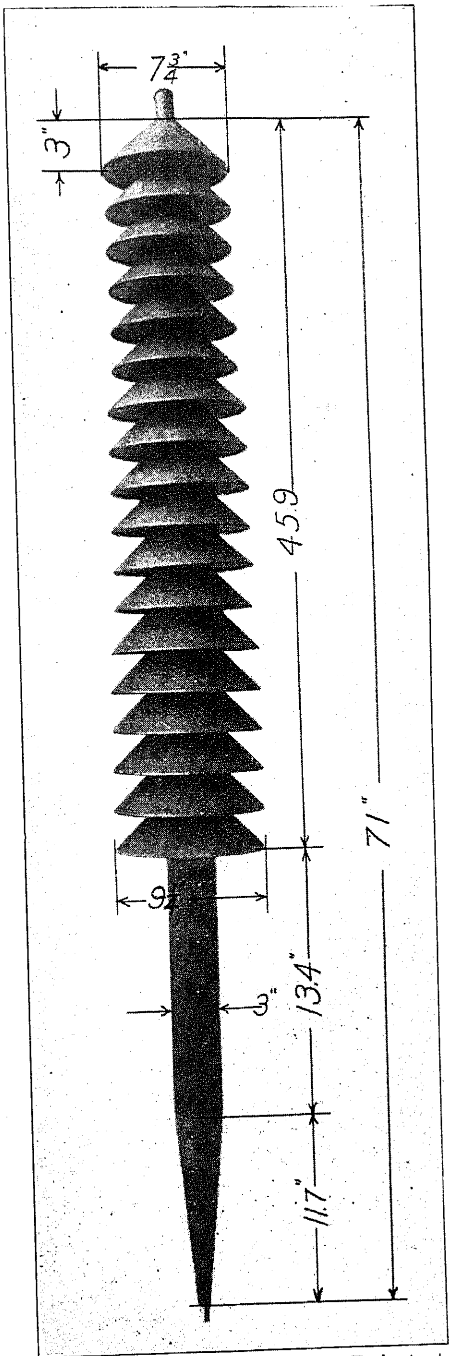


FIG. 12—Outdoor type of condenser terminal. Rain test: first spark 125,000 volts; intact at 150,000 volts, 1 minute; breakdown 200,000 volts.

In other words, as the layers approach a flat surface the potential gradient approaches nearer and nearer to a straight line.

If the condenser type of insulation in the form of a tube is used between primary and secondary or primary and ground in a core-type transformer and the metal layers in the tube are connected electrically to points of equal potential in the windings, then not only is the distribution of stress in the dielectric fixed and hence the total dielectric strength increased, but the action of the condensers assists in the suppression of surges due to outside disturbances.

It is probable that by the use of the above method of connecting the winding and the metal layers together, and the use of the condenser type of terminal, transformers of 300,000, 400,000, or 500,000 volts may be made on a commercial basis.

There is yet another possibility of the condenser type of terminal which may be mentioned; namely its use for outdoor service. Such a terminal is constructed as mentioned above except that at the end of each metal layer is placed a bell-shaped metal cap in electrical contact with the metal layer. The space on the surface of the terminal between the caps is covered by a cylinder of porcelain or similar material. The completed terminal has the appearance shown in Fig. 12. This terminal without the bell-shaped metal caps has the same dimensions and materials as the terminal shown in Fig. 9. The metal caps vary in size, having a maximum diameter of 9.25 in. and a depth of 3 in. Upon test in a rain, falling at a rate of between 0.2 and 0.3 of an inch per minute, and driving horizontally at times, this terminal showed a faint spark at 125,000 volts, stood 150,000 volts for one minute with an occasional flash between caps, and broke down at 200,000 volts at the end of a few seconds.

Such a terminal will stand rough handling and will solve to a great extent the difficult problem of broken insulators. If the metal discs are replaced by porcelain, a gain will be made in the insulation strength, although first cost and maintenance will be greatly increased.

Discussion on "High-Voltage Transformers and Protective and Controlling Apparatus for Outdoor Installation," and "Condenser Type of Insulation for High-Tension Terminals." New York, April 9, 1909

W. S. Moody: Some years ago I attempted to build leads such as described by Mr. Reynders. I found that the difficulties encountered, without the special facilities which have evidently been developed for building the lead before us, made it virtually impossible to gain anything by using this construction.

There is one feature of such a lead that requires more or better insulating material when the condenser cylinders are introduced than when they are not. With the condenser plates in place, every weak spot in the insulation is in series with every other weak spot, whereas a lead covered entirely with insulating material is not weakened greatly by any one of its weak spots, because this weak spot will not ordinarily, by the mere law of probabilities, be in series with any other weak spot. It is possible then that the use of condenser plates in such an insulation, and notwithstanding the better distribution of internal strain, will make a lead no more and perhaps less capable of withstanding a given strain than if the condenser plates were not used. But this may be no serious objection, for, as Mr. Reynders says, the problem of obtaining sufficient insulation to withstand the puncture is not difficult.

It would seem, therefore, that by far the greatest value to be obtained from this construction is in the equalization of the strain along the exterior surface of the leads. There would seem to be no question that the practically equal distribution of strain that can be and has been obtained in the lead shown us does require the minimum length of lead to withstand a given voltage, provided one confines himself to a plain cylindrical or conical surface for the exposed part of the terminal.

My rather limited trial of this form of insulation was sufficient to convince me that such a lead would be a difficult one to produce under ordinary manufacturing conditions, and that it would be, when completed, rather a delicate affair mechanically. I therefore turned my attention to other forms, which have given very satisfactory service on operating pressures all the way from 100,000 to 500,000 volts.

Nearly 8 years ago, when only the most courageous were beginning to think of 60,000 volts as a commercial possibility, I gave to my assistant, J. J. Frank, the problem of building a 500,000-volt transformer. In the preliminary tests leading up to the design of this transformer, it was found when an air or oil space was under electrostatic stress that the introduction into this space of several sheets of metal lowered the voltage required to puncture; whereas the introduction of several sheets of insulating material—consisting of something that was inferior to oil or even air as insulation—markedly increased the volt-

age required to puncture. It was therefore decided to subdivide the oil—which was chosen as the only material to provide satisfactorily against puncture for a pressure such as 500,000 volts—by insulation cylinders rather than by metal cylinders.

In attacking the problem of providing surface insulation for 500,000 volts, it was clearly apparent that the 15 to 18 ft. of such surface that would be required could hardly be attained in a straight line, but that some form of irregular surface, such as that used in line insulators, must be resorted to. These ideas, combined, resulted in the production of a lead about 16 in. in diameter where it passed through its support, and about 80 in. long each side of this support. This lead has been in most frequent use for six years in connection with all sorts of special testing, and has been subjected to from 300,000 to 650,000 volts.

This lead consists of a tube of low grade insulating material, tapering in diameter both ways from the point at which the lead is to be supported, the tube being filled with a high-grade insulating liquid or semi-liquid compound, such as oil or asphaltum. The tube is made up of sections telescoping into each other slightly, and between the sections are rings of similar material projecting beyond the sections to give very great leakage distance on the surface. In the later forms of these sections the projecting part is made integral with the section, so that the section has the general appearance of the petticoat of a line insulator. After a suitable cement has been placed on them, all the sections are drawn tightly together by the conductor itself, which is a heavy rod or tube. The space between the walls of the tube and the conductor is divided into several sections by thin cylinders of insulating material. The entire air space in the lead is then filled with the insulating compound.

A lead used for 100,000-volt transformers is regularly tested at 200,000 volts, some of the leads having withstood 225,000 volts for 15 or 20 min. The dimensions of one of these leads are, diameter, 7.5 in., length, 39 in.

It would seem, therefore, that the very long leakage surface, which is so easily attained in this construction, enables one to reduce materially the length of the lead to stand a given strain as compared with a lead without the irregular surface but with a most perfect distribution of the strain, and that no excessive diameter has to be used at the point where the lead passes through the cover. My experience is that there is little difficulty in providing space for leads of quite large diameters in such cases of transformers as are ordinarily built for 100,000 volts and more. It is desirable though to keep the length of the lead above the cover as small as possible, as the transformer is usually the tallest piece of apparatus that goes into the station, and its over-all height has, therefore, often a direct effect on the cost of the station.

It is evident that one or more of the internal insulating cylinders in this construction could be metallic; and if they were metallic

the lead could be made to have a somewhat better distribution of the external strains. But as the lead is now little longer than is necessary to provide a safe factor in the air distance from terminal to cover, only a slight reduction of length could result.

It seems to me that if the principle of distributing the strain on a terminal by a condenser is to be applied extensively, a lead of robust mechanical construction with one, or, at the most, two or three, condenser cylinders is more likely of ultimate adoption than one requiring the refinements of manufacturing conditions and delicate handling required by the lead described in the paper. Moreover, for indoor use at least, it will usually be desirable to use a form of lead whose length from terminal to cover is not greatly in excess of the distance which the test voltage will jump in air.

The problem of having a transformer outdoors is not a new one, of course. All the earlier transformers were placed outdoors as are the present small transformers. The only new points to consider in building large units, as Mr. Randall has said, are in taking extra precautions that the transformer be weather-proof, and providing easy means of repairs.

With reference to their being weather-proof, I believe that in the large units, involving so much money, we should never depend on the joints being absolutely tight, because the best made joint is liable to loosen up. I think it would be very much better if the transformer were kept under a slight internal pressure all the time, so that when leaks developed they would be outward rather than inward. A little gas generator could maintain a pressure on top of the transformer, or an auxiliary tank of oil could be used to keep a slight head of oil on the transformer all the time.

There is one difficulty with transformers installed in this way; that is, the insulating qualities of oil at low temperatures. Oil, when frozen, is ordinarily about as good an insulator as when liquid, but just about the freezing point, especially after it has been frozen, its insulating qualities are greatly reduced. It may be necessary to use some new oil, or some new method of refining oil must be worked out before we can be sure that the transformer that is left outdoors in a cold climate may not have its insulating qualities greatly reduced at low temperatures.

Percy H. Thomas: As Mr. Randall has stated, the crux of the matter of the outdoor station is the advantage in cost of plant; in most other respects the arrangement is, perhaps, disadvantageous. Perhaps it is easier to summarize the difficulties than the advantages. Mr. Randall has well pointed out the chief causes and annoyances of failure; the extra operating expense; the difficulty of inspection, which is really a very important one; and the matter of being able to open the transformers and switches at any time. If the climate is at all bad, I think it would preclude the outdoor apparatus. In any such climate as that of New York where there is much sleet and snow and where

one would not always open the apparatus and look at it. I think there would be much difficulty experienced.

The question of repairs is also important. Mr. Randall suggested a special repair house. Would not the cost of a repair house justify the additional cost of putting the whole apparatus in a building? The necessity of working in a cold or exposed place makes any repairs much more difficult, and much slower in their execution. These facts must be clearly borne in mind in weighing any saving in first cost that can be made by omitting the building.

I will call attention to the danger of electrolytic arresters freezing. They seem necessary on high voltage work, and if there is a long cold season the question comes up whether it is safe to have them remain frozen.

Often in high-tension work the transformers indoors are protected from fire by fire-proof cells. If put out-doors without cells, unless they are much more widely separated, the transformers presumably lose most of the advantage of that isolation; or if cells are used the expense will approximate that incurred in the indoor construction. The matter of lighting an open space is a small item which is sometimes lost sight of in outdoor installations.

In the examples Mr. Randall pointed out, the saving for outdoor stations is entirely incommensurate with the amount of risk and difficulty incurred, but I judge that this would not be true in all cases. In the first case the interest at 10 per cent on the excess cost of the housed plant would be $[illegible] a year, which might be exceeded several times by the cost entailed through the increased difficulty in handling one or two cases of trouble. In the other case, with a little larger plant, 10 per cent interest on the increased cost of the housed plant would be about $1,[illegible].

In the terminal described by Mr. Reynders we have an ingenious use of theory to increase the effectiveness of the terminal material. I think there is no doubt about the correctness of the theory, as is shown pretty clearly by the tests we have seen; but there are some difficulties to be borne in mind. Broadly speaking, the thing which is accomplished by this terminal is the favorable determination of the fall of potential along the surface of the bushing. Puncture through the material is relatively easily cared for; but to prevent a concentration of potential locally on the surface of the terminal, when the conductor has a very small diameter, it is necessary to take some precaution. As has been clearly explained, by making use of the fall of potential within the solid material and balancing the area of the small condensers, equal steps on the surface of the terminal are obtained. As the result of this follows a condition which, while theoretically dangerous, very probably will never arise; that is, if the terminal is designed for ordinary commercial frequencies, the larger current of the small condensers formed by the tin-foil, over and above

the true surface leakage current, is the important factor in determining the distribution of potential. If, however, the leakage current over the surface or through the terminal itself should be larger than the charging current of the tin-foil condensers, then the fall of potential over the surface would determine the potential between these tin-foil layers, and the potential drop would be relatively much higher on the first few sections.

As far as the surface leakage is able, it will disturb the condition of equal distribution of potential, putting a very high strain on the inner layers. This condition is, fortunately, not likely to be serious. If, however, we should make a direct-current high-potential test, the charging current to these internal condensers would become practically zero, and the potentials would be determined by the leakage current over the surface, in which case the distribution potential would be much disturbed. Fortunately, in the frequencies higher than normal which we are much more likely to have, the effect is the other way and only increases the safety of the terminal itself.

Mr. Moody called attention to the lining up of the weak spots. This is a matter which should have careful consideration. I believe, however, that the uniformity of material is satisfactory in apparatus of this sort, and unless actual trial shows a weakness in this direction, there is not much to be feared. There is another danger, however, in that if a leak, even a small one, does occur between two layers of tin-foil, the current passing through the leak will be very much increased by the presence of the conducting plates, because the full charging current of all the plate will pass through the weak point; whereas if the conducting material were absent, only a very small current would pass through any given weak local path, not passing through the insulation from terminal to terminal.

I am inclined to believe it the part of wisdom not to attempt to pass the high-tension terminals through a transformer top with an opening of small diameter. It would be better to provide a 20-in. opening than to have a terminal passing through an 8-in. opening in the metal cover. The strains and tendency to corona discharge are certainly greatly increased by the close proximity of the ground to the high-tension parts. It would seem that much effort should be made to have the mechanical construction of a larger insulating diaphragm more suitable for supporting the terminal rather than to attempt to find some way of carrying the high potential through a small opening.

If there were time it would be interesting to discuss a number of alternative forms of high-potential terminal. I do not believe the last word has been said on the subject yet, and in closing I call attention to the condition shown in Mr. Reynders' paper—of a very small diameter of conductor in proportion to the outside diameter of the insulation. That ratio emphasizes the tendency to concentration of potential drop in one por-

tion of the insulation. It is perhaps a favorable condition for the shaping of the tin-foil condensers of this particular type of terminal—the condenser terminal—but some other form of terminal having a larger diameter of conductor would, in that very fact, have an advantage, so to speak, over the condenser form, by relieving automatically in quite large measure the tendency to local concentration.

D. B. Rushmore: The reason for the demand for outdoor sub-stations is the desire to keep the original cost low. That, to my mind, is the only advantage that outdoor sub-stations have, so that instead of asking the question "Where can we use these?" it should be "Where must we use them?", for unless financial considerations make it necessary, there seems to be no reason for their use. There certainly is not apparent any engineering advantage in using them.

In certain climates, however, where either of the extremes of heat or cold predominate, it is a question whether or not to put them outdoors. For instance, in the great heat of Arizona there would be a decided question concerning the desirability of installing large transformers outdoors. There not only the temperature rise, but the final temperature reached, is going to determine the life of the insulation. In that climate, lightning-arresters or any other installation must be protected in all possible ways from the heat, not that absorbed by conduction and measured by temperature rise, but the great heat that comes from the direct radiation of the sun.

In the extreme climates in the north the cold to a certain extent will prevent the operation of mechanical devices through the thickening of the oil. As long as everything runs without interruption a great many conditions are possible which are not advisable in emergencies. It is for the emergency condition that a large part of the apparatus and installations have to be designed. Under severe climatic conditions, great difficulty would be met with in taking care of a large installation where most of it is exposed.

The freezing of the electrolyte of lightning arresters has been a serious problem, and the development of an electrolyte which will not be affected by freezing is the result of several years of experience. An electrolyte has been found which is not affected by freezing, but there is no indication that it is at all helped by that condition. It is, however, undesirable to have the apparatus more exposed than is necessary; for any transmission line now in operation there are a sufficient number of disturbances to make us consider seriously the question of any further exposure of the apparatus. In most of the long high-tension transmissions, and where on a 100,000-volt transmission line a few horse power are taken off, we will be confronted with a serious condition regarding the cost of installation. It is here that perhaps we shall be most strongly forced to consider the outdoor sub-station.

As I watched the experiments to-night, I was reminded of an installation which was worked under good conditions for a few weeks, and was then followed by a breakdown of the insulators. This is simply an example which goes to show that the breakdown of insulation is not an instantaneous one. Wherever we have a brush discharge, there we have the formation of ozone and, under certain conditions, of nitric acid. This is apt to bring about a slow deposit over the insulating material, and in that way, at the end of a certain time, to cause a breakdown of the insulation. While this is an interesting device, which has been considered in other forms before, in the particular cases that I have in mind it does not seem desirable for mechanical reasons.

Paul M. Lincoln: I believe I am not exaggerating in saying that the lead described by Mr Reynders marks the greatest advance in the high-tension art that has been made during recent years. One of the advantages of this lead and one of the most striking exhibits is shown in comparing Figs. 10 and 11. These transformer bushings are made for the same operating voltage. The smaller one—of the condenser type—has a test voltage of 2.5 times normal, while the larger one has a test voltage which is only twice normal.

When this device was first suggested, the lining up of the weak points of the insulation was thought to be a fatal defect; not until after the thing had been tried out thoroughly was it found that this defect was not fatal.

The method of construction is important. The terminals and bushings are made of paper wound up into form under a heavy pressure and a high temperature. The different layers are put together with a bond of shellac which goes into the joint of the rolling paper. The temperature is high enough to melt the shellac as it goes in, insuring a perfect bond between the adjacent layers. By that means a construction is obtained in which weak points are practically eliminated. When finished the tube is virtually a solid mass.

I would take exception to Mr. Moody's intimation that the lead lacks robustness of structure. I think there can be no bushing made which is more robust in its construction than the one shown.

There was one point that was brought out in the rain test which strikingly confirms one of the points brought out by Mr. Reynders. He has indicated that in designing the terminal it was necessary to sacrifice a perfectly even distribution of potential along the terminal, and that the potential was somewhat higher in the middle than at the ends. It was noticeable that when the terminal was flashing over it was mostly the middle condensers which were discharging, with very little discharge at the ends. This fact illustrates one of the points of construction which Mr. Reynders mentioned.

The spark-gap, which is a shunt to the terminal, has been seen under test. The gap is set at 27 in., so that the voltage which

was applied to the terminal was fully 250,000 at the time of breakdown, as the spark-gap was in continual discharge.

E. M. Hewlett: I have been interesting myself for some time in the development of Mr. Frank's oil- or asphaltum-filled insulator which is now being used with high voltage transformers. I hardly see the necessity of a complicated design when the same result can be accomplished in a simple way.

The bushing of this insulator consists of a built-up shell of insulating material which is filled with a high-grade insulating liquid or compound, such as oil or asphaltum, with concentric cylinders placed inside of the bushing to distribute the potential. This arrangement results in an insulator having the same arc-over value as the equivalent spark-gap.

Considering storms, accumulation of moisture, general climatic conditions, and the trouble of making repairs, it seems as though the outdoor installation could never be a serious competitor of the protected station. In cold climates it would be necessary to use a special oil for the switches or to heat the oil in order to keep it of the proper viscosity for the satisfactory operation of the oil switches.

S. Piek: Doubt has been expressed as to the reliability of outdoor terminals for high-potential transformers. The Niagara, Lockport & Ontario Power Company has had in permanent operation since October, 1908, six outdoor 60,000-volt series transformers, which have operated most successfully during the severe rain, snow, and sleet conditions of the last winter. On the strength of this performance there is no hesitancy to adopt this type of terminal for outdoor power transformers.

The saving in investment made possible by the use of outdoor transformers would be considerably more than the comparative figure in Mr. Randall's estimate, had the indoor station been provided with the facilities which are considered as arguments in favor of the outdoor installation. For instance, a steel building costing $3100 for housing transformers of 2000-kw capacity would hardly have enough floor room and head room to lift a core out of the transformer bank. A building designed to afford such facilities will cost considerably more, possibly double the figure used in the estimate. Furthermore, no allowance has been made for heating facilities which are essential to permit of repair work in any season of the year.

The outdoor transformer arrangement follows closely the traditional indoor station layout. If the money now spent for choke-coils and lightning-arresters were partly applied to the improvement of the transformer installation, the result might be an almost indestructible transformer, which could be placed at any point on the line without requiring much attention for its maintenance.

The Niagara, Lockport & Ontario Power Company has felt the need of outdoor transformers for a long time, and could probably increase its load immediately by 3000 or 4000 kw. if outdoor 60,000-volt transformers were to be obtained.

Guido Semenza (by letter): It is natural that reaction from any exaggerated condition should itself be exaggerated. The indoor high-tension sub-station has in some instances reached such a high degree of complication with walls, partitions, cells, and conduits, that the general feeling is that things have been pushed beyond the necessary limit. Then we are brought by the reaction to the opposite extreme—no walls and no roof. This new form of sub-station requires very careful investigation, and Mr. Randall is to be thanked for having brought the matter before the Institute in such a clear and impartial manner.

In comparing Fig. 3 with Fig. 4 in Mr. Randall's paper, we are impressed by the simplicity of the outdoor arrangement; but if we consider it well we find that some of the objections pointed out by the author and others cannot be neglected. First of all, I quite agree with Mr. Randall that if a part of the station requires housing, the extra cost for housing the whole will not be of great importance. Measuring instruments and controlling panels will not be improved by rain and summer sun. Nowadays a marked tendency is shown toward being able to read instruments in every part of the plant, so that in many cases the outdoor arrangement would not be feasible.

As Mr. Randall states, outdoor apparatus should not only be water-tight, but also air-tight. I am very doubtful if a perfect condition of air-tightness is possible at the inlets, where substances of a very different coefficient of expansion are used, such as porcelain, mica, iron, copper, and compounds. Apparatus repaired under conditions of rain, mist, or snow are liable to deteriorate, and we do not always find customers inclined to wait for a sunny day before having their current put on again. Outdoor sub-stations are not easily protected from direct lightning strokes.

With reference to insulation, I venture to point out the fact, often observed, that in moist air, when porcelain and iron are in contact, the iron rust gradually penetrates the glazing of the insulator, causing a breakdown after a time.

Mr. Randall figures out the cost of first installation only. This is not sufficient. If we take into account the cost of operation, I fear that the larger liability of breakdown of open-air insulators and terminals, the necessity of frequent repainting, of cleaning off dust and insects, and the more accurate inspection required, will more than offset the lower cost of first installation. Thus I feel inclined to conclude that a fair solution would be to simplify sub-stations by making them more similar to the arrangement shown in Fig. 4, but to cover them with a roof, and wall them in.

Exaggeration in partition walls and cells was justified in the early years of high-tension work when switches were considered a dangerous device; nowadays we have more confidence in them and can do away with excessive protection.

A. E. Kennelly (by letter): The introduction of metallic layers into the substance of a dielectric for improving the distribution of electric stress, has been known and used to some extent in the

construction of high-tension induction apparatus. It promises to be of considerable assistance in the construction of high-tension terminals in the manner described in Mr. Reynders' paper. This construction not only establishes equipotential surfaces within the dielectric, with a geometrical distribution which is under the control of the designer, but it also limits the range of influence of an accidental flaw in the dielectric to the distance between adjacent conducting layers in the vicinity. This capability of injecting artificial equipotential surfaces is a fortunate property of alternating-current tensions not possessed by steady, or direct-current tensions.

If a condenser type of high-tension terminal were used on direct-current pressure, and any slight leakage to ground occurred from the layer of tin-foil nearest the central conductor, it would only be a question of time for that layer to sink to ground potential through the leak, and thus bring the entire difference of potential stress on the innermost wall of the dielectric; whereas, with alternating current no such cumulative action can occur, and the influence of external leakage current in adding to internal stresses is only in the proportion of the leakage in current to the total condenser current through the dielectric mass.

One of the problems of aerial transmission insulator design may be regarded as analogous to the problem presented in the design of a condenser type of terminal. That is, the glazed surfaces of the insulator should be disposed in such a manner that when they are temporarily converted by rain water into equipotential surfaces, the stresses on the intervening dielectric layers in the insulator shall not be materially altered, and, in particular, shall not be locally increased to a dangerous intensity.

It is doubtful whether the introduction of concentric cylindrical tin-foil layers into the uniform dielectric of a cable can be of utility beyond limiting the radial influence of flaws in the dielectric; because the equipotential surfaces in such a cable are already concentric cylinders, so that in the absence of flaws the introduction of thin tin-foil changes nothing. When, however, as in transformer terminals, the construction is in three dimensions and the equipotential surfaces are no longer simple concentric cylinders, the insertion of the tin-foil alters and controls the potential distribution.

J. S. Peck (by letter): In reading Mr. Randall's paper, two questions suggest themselves to an engineer somewhat familiar with the conditions governing the use of high-voltage apparatus in Great Britain: 1. Will the Board of Trade and the Home Office approve of an outdoor installation? 2. If they do approve, will not their approval be qualified by the imposition of such measures for the protection of the public and operatives, as would make the cost of the outdoor installation as high or higher than the indoor one? Assuming, however, that such severe restrictions are not imposed, and that, as in America, each engineer has practically

a free hand to lay out his system as he sees fit, the question appears to resolve itself into a discussion of the conditions under which an outdoor station may be desirable.

The only real argument in favor of the outdoor high-tension station is its low cost. The argument of reduced fire-risk is of secondary importance, while regarding risk to life there is considerable doubt if the outdoor station is as safe as the indoor one, though there is something to be said on both sides. So far as cost is concerned, it would appear certainly that the outdoor has a considerable advantage over the indoor station, but when one notes the way in which the appearance of the handsome building of the Lockport station of the Niagara, Lockport & Ontario Power Company referred to by the author is disfigured by the forest of poles outside it, the possibility of a compromise is suggested. It may be found that in the long run it will pay to protect transformers, switches, and arresters from the direct action of rain, wind, and snow, by some cheap form of building or shed which will be no more offensive to the aesthetic taste than an outdoor station consisting of a mass of poles, insulators, wires, metal tanks, earthenware jars, and fences for keeping off the sun.

The strongest argument against the outdoor station is the possibility of damage to the apparatus through the absorption of moisture. This applies particularly to the transformers, which, being alternately heated and cooled, tend to breathe out hot air and breathe in cool air, the latter possibly being saturated with moisture. As the author suggests, while it is possible to make the case vacuum tight, this is a difficult and expensive matter; and even if vacuum tight when installed, it is an open question whether the case will remain so. Apparently the transformer case shown in Fig. 11 is not vacuum tight, and it would be interesting to know whether the costs of the outdoor stations given would be increased appreciably by making the transformer cases vacuum tight.

In Europe it is almost common practice to protect all transformers from the weather, even those of small capacity and low voltage. The very low mortality rate of transformers in Europe as compared with those in America, even though built to the same specification, leads to the belief that protection from the weather is an item of considerable importance, which certainly will not grow less as the voltage is increased.

Another danger with the outdoor station where water-cooled transformers are used, is the freezing of the water in the cooling coils in the event of a transformer being cut out of circuit for several hours. It is extremely difficult to remove all water from the coils, and a frozen coil usually means a burst one. When the transformers are placed inside a station, it is usually possible to keep the temperature of the station above freezing point.

Mr. Randall apparently assumes that 500 kw. is about the limiting size for self-cooling transformers, but in Great Britain there are a number of 1000-kw, 20,000-volt, 40-cycle three-phase

transformers in operation. There is no reason why 1500 to 2000-kw., 50-cycle units of the self-cooling type should not be made, even without auxiliary cooling tanks.

On account of the relatively heavy expense of terminals for very high voltages, and of the difficulty in making them weatherproof, the advantage of using only one terminal on each transformer becomes more apparent whenever the star connection is used with grounded neutral. It should be noted also that with this system the series transformers may be grouped about the neutral point, so that practically no insulation is required for the terminals or between the high tension and ground. Where it is desired to measure the total output of two or more groups of transformers, the secondaries of the series transformers on the different groups may be paralleled.

As stated previously, it is doubtful whether the outdoor station is ever likely to come into favor in Great Britain on account of the possible objections of the Board of Trade and the Home Office and of the aesthetic feelings of the people, but in America, where the use of outdoor transformers of small size is already universal, the outdoor station seems bound to follow. The only question, then, is to determine the limits of size, voltage, etc., beyond which it is better to house the apparatus which may be damaged by the weather and which it may be desirable to inspect or to cut in and out of service from time to time. What these limits are must be determined by experience.

Ralph D. Mershon: Both the papers presented this evening represent a distinct advance in the art of high-voltage transmission. Mr. Randall's paper especially deals with a subject in which I have been interested for a long while. I believe that eventually outdoor high-voltage apparatus will be the rule rather than the exception. I can well remember the indulgent smile with which I was met on making suggestions along these lines several years ago. Even Mr. Randall himself, who is most ready to consider any suggestion which may appear to contain elements of progress, seemed to have grave doubts as to the possibility of successful outdoor high-voltage apparatus.

There are two suggestions made by Mr. Randall which especially interest me. One of these is that of considering the high-voltage apparatus a part of the transmission line, and switching the apparatus and line as a unit. I think in time this course of procedure will prove desirable in many cases where large amounts of power are to be handled, and the transmission is over long distances. The other suggestion is that of putting all the high-voltage apparatus in one tank, so as to require the minimum number of terminals. This is a matter to which I hope the designers of high-voltage apparatus will give careful attention.

It would be interesting to know from Mr. Randall the minimum temperature at which the oil switch could be operated, in view of the fact that the oil will thicken as the temperature is

lowered. It would also be interesting to know as to the possibility of putting the complete oil switch, including the mechanism, inside the transformer tank and, possibly, underneath the oil.

I agree with Mr. Piek that Mr. Randall has not allowed enough for the cost of his buildings. In designing a transformer station it is surprising how the cost runs up, and the various details that have to be taken care of are numerous. The point Mr. Pick made is a very good one; that is, if we have room to handle the transformer in the station we will require more room than Mr. Randall provided.

Mr. Peck raises a point which has always interested me; that is, the fact that in using single-phase transformers with the high-voltage side connected in Y, we need only one high-voltage terminal for each transformer, and that by such a method of connection we may locate our series transformers near the neutral point, thus avoiding the necessity of insulating them for high voltages. If such course be pursued, it seems to me that it might possibly pay to go further and bring a lead out from the high-voltage winding near the neutral connection, from which could be actuated the voltage-measuring instruments and the reverse relays where such are required.

Mr. Peck's suggestion brings to mind another point which has always interested me, and that is the desirability of being able to connect both sides of the transforming apparatus in Y, resulting in the same simplicity on the low-voltage side as that obtained on the high-voltage side. As is well known, we hesitate to follow this course now because of the difficulty met with when the neutrals of generating apparatus are connected. This difficulty is generally assumed to be due to the effects of the third harmonic. It is very desirable that this question be given closer investigation, experimental and theoretical, than it has heretofore received, with a view to overcoming the difficulty if possible.

If separate single-phase transformers are used in three-phase transformation, with both high-voltage and low-voltage windings connected in Y, each transformer and the line cable connected to it may be treated as a single-phase unit; that is, the three-phase transmission circuit may be considered as made up of three single-phase circuits 120 degrees apart, and handled accordingly, so that where the return current through the ground would not be especially objectionable, we might, in cases of emergency, switch out a part or all of the transforming apparatus connected to one line conductor and run for a while on the other two phases. Such a possibility as this would greatly increase the flexibility of a transmission system. In this case our switching apparatus would preferably consist of single-phase switches arranged so that they might be operated as such, or, when desirable, operated simultaneously, thus constituting a three-phase switch. Under such conditions the suggestion made by Mr. Randall, of putting

the switch in the same tank with the transformer, becomes very attractive.

Of the many written contributions received, one of them, for the reading of which there is not sufficient time, deals with the idea of locating the high-voltage bus-bars and some of the remainder of the high-voltage apparatus on the roof of such small building as may be required for sheltering the sub-station operator and the switchboard measuring and control apparatus. This is a suggestion which has been made a number of times, but, so far as I know, has never been fully worked out. It opens up a subject worthy of further study, as possibly leading to desirable results in economy, reliability, and safety.

Mr. Reynders' paper is a beautiful exemplification of the application of theory to practice, and the advance which it marks should do much in forwarding high-voltage transmission work through the cheapening of reliable apparatus.*

W. S. Franklin: It has occurred to me that I might briefly consider the accumulation of charge upon a high-voltage transmission line, an accumulation which continuously discharges to ground and which is called the "static discharge" by transmission engineers. The maximum voltage that can be created between a pair of single-phase transmission lines and the ground is equal to $\frac{2\sqrt{2}\,C\,E}{C'}$ where E is the effective alternating voltage between the transmission lines, C is the capacity per unit length of the pair of transmission wires with respect to each other, and C' is the capacity of the pair of transmission wires with respect to earth. Thus, a maximum pressure of 560,000 volts may be built up between a pair of transmission wires and earth when the voltage between wires is 40,000 volts effective, and when C is five times as great as C'.

The theory of this effect, as I have worked it out, is based upon the great mobility of the negative ions in the atmosphere as compared with the positive ions, and indeed the above formula is based upon the assumption that the mobility of the positive ions is negligible in comparison with the mobility of the negative ions.

The building up of an excessively high voltage between a pair of line wires and earth does not depend upon a potential gradient in the atmosphere but, so far as the above formula is concerned, depends solely upon the existence of free ions.

N. J. Neall: There is one curious arrangement shown for the outdoor stations that indicates the persistence of established practice, and that is the lightning-arrester equipment which is placed at one particular end of this group of apparatus. I see no reason whatever why it should be used on only one side of the group, so to say. In fact, let us suppose that these sets represent future

* Before the close of the meeting, Mr. Mershon said that the thanks of the Institute were due to the Electrical Testing Laboratories for the loan of the testing apparatus used for experimental purposes at the meeting, and to the New York Edison Company for installing it.

practice with very high voltages where the transmission lines will be provided with overhead grounded wires. I observe that we have here no overhead grounded wires for the station. I think it only fitting, however, that we should supply the equipment with a static shield also, but the preparation of that static shield will not be so simple when all the wiring now shown is taken into account. A modification of this—but to some extent begging the question—would be to scatter lightning-arresters throughout the group, just as many as the station would afford.

G. Faccioli: The so-called condenser type of transformer terminal has attracted the attention of manufacturers and engineers for years. In May, 1906, the *Elektrische Bahnen und Betriebe* gave a full description of such leads and the complete theory thereof. In that paper, as in the present paper, the terminal is compared to a short piece of cable; but a transformer terminal is under very different conditions from a short cable, and the distribution of the stresses is not the same in both cases. In a transformer lead the inside conductor is much longer than the outside grounded plate, and, therefore, all formulas which apply to cylindrical condensers where the inside and the outside plates are of the same length, do not apply to the case of leads.

A simple experiment shows this clearly. Let us take a rod 45 in. long and 0.25 in. in diameter, and let us put this rod in the center of a metallic cylinder 10 in. in diameter and 2 in. long. We ground the outside cylinder and apply to the rod high potential from the terminal of a transformer whose other terminal is grounded. We find that as soon as we reach 40,000 volts, part of the rod covered by the grounded cylinder glows. This means that at the surface of that portion of the rod the electric stress is higher than the disruptive strength of air.

If we raise the voltage, corona will gradually spread on each side of the center of the rod. If we increase the width of the grounded cylinder, we find that the first part of the rod to glow is always in the center and very short in length, showing that the distribution of the electric field is not uniform, but is the strongest in the center and decreases gradually on either side.

Now let us put a thin copper cylinder 5 in. in diameter and 10 in. long concentric with our rod and the grounded disk. If we test the apparatus, we find that corona appears in the center of the rod at 43,250 volts, which is very little above the 40,000 volts found before. This shows that the introduction of the intermediate concentric cylinder in the electrostatic field has changed its distribution very little. We can conclude from this test that the condenser type of lead must have comparatively small diameters and short steps between adjacent tin-foil layers. A small diameter necessitates an abnormal length of lead, for we know that in order to reduce corona and creepage effects, an increase in diameter is far more efficient than an increase in length.

For instance, the condenser lead illustrated in the paper has a length above ground of 49 in. Successful leads for the same voltage have been built with only 39 in. above ground. Their diameter is, however, 7.75 in. in the center. Such leads do not arc over at 250,000 volts, although their length is 10 in. less than the condenser lead.

The condenser type of lead, however, gives a better distribution of potential on the surface of the lead; in fact the equipotential surfaces are straightened by tin-foil cylinders and tend to relieve the crowding of the voltage near the grounded plate. On the other hand, the mechanical construction is very delicate: the lead has to be built cylindrical and then machined to smaller dimensions, and the metallic cylinders connect in series all air bubbles and other faults which are very apt to occur in this kind of construction.

Furthermore, Mr. Jona, in his article referred to in this paper, brings out the point that the introduction of metallic cylinders in the mass of the insulation decreases the resistance to puncture of the apparatus. He gives the example of a copper wire 4 mm. in diameter, insulated to 8 mm. in diameter, then covered with a thin metallic cylinder, next with 3 mm. more of rubber, and finally with a lead sheath. At 8000 volts the insulation between the small wire and the metallic cylinder is punctured; but if the metallic cylinder is removed the insulation is not punctured up to 15,000 volts. The phenomenon is not clearly understood.

We consider in our calculation the different layers of insulation as independent and subjected to a certain difference of potential from one surface to the other. This view, according to Jona, is probably too "static." The phenomenon of "puncturing" is dynamic, and consists in the flow of a certain amount of energy which cannot reach one layer of insulation without going through the other layers. It seems that the various layers of insulation give a considerable mutual assisting action, which is evidently lost if they are separated by metallic surfaces.

In conclusion, as far as puncturing is concerned, this new lead has no advantage over the bulk type of lead, but the fact that the distribution of potential on its surface is much more favorable makes this lead a very desirable apparatus.

C. L. de Muralt: One point which I should like to emphasize is that in the design of our transformer leads—which are the first part of the power-house equipment in connection with the line—we are working against a constantly decreasing margin of safety. It is comparatively easy to take care of the line working-pressure. Even the excess pressures produced by static charges and high-frequency discharges on the line can be handled. But the very apparatus which annihilates the effect of these outside excess pressures may, in connection with the transformers, produce resonance effects which bring on local excess pressures of magnitudes more and more dangerous the higher the line pressure goes.

Transformer terminals and transformer windings have a certain capacity against ground and a certain self-induction. Lightning-arresters usually have a certain ohmic resistance and a spark-gap of some kind. These, with the ground, form a resonance circuit.

So long as I $R^2 > 4$ L, we are on the safe side and there will be no resonance. If I $R^2 < 4$ L, we will have excess pressures which may easily reach multiples of the working pressure. It is therefore necessary to make the ohmic resistance of the lightning-arrester very high. At the same time this resistance should be as low as possible to handle readily the very large currents produced by the outside excess pressures. We have here two absolutely opposing conditions.

I have worked considerably along this line and I offer a solution which I have found safe for very high pressures. Metallic rods connected to the ground are first insulated and then surrounded by a series of metallic discs. The lowest disc is connected to the ground; there is a gap between every two discs, and the last disc is connected to the line by means of a high resistance and a spark-gap. We thus have a series of spark-gaps and also of capacities. Several such devices are then placed in parallel. Each one has a resistance high enough to prevent resonance. The resistance of the group in multiple is low enough to allow any current—thousands of amperes—to pass to the ground without trouble.

Of course the spark gaps will not operate all together, as they cannot be thus closely regulated. One starts, and as the excess pressure increases others are brought into service until relief is obtained. By these means we have been able to keep the excess pressures within 10 per cent of the working pressure, thus giving a very large margin of safety to transformer terminals even with very high line pressures.

V. D. Moody: As covered by the summary of Mr. Randall's paper, the problem from the manufacturer's view involves three distinct features; the transformer, the switching apparatus, and the protective apparatus. To these items there should be added another and most important factor which is involved; that is, the saving in cost of the plant in locating the apparatus outdoors instead of indoors. As concerns the high tension bus-bars, there is no good reason why these should not be located outside in every case, as they are practically a termination of the transmission line.

The apparatus directly involved in an outdoor installation comprises lightning arrester equipment, choke-coils, oil switches, and transformers. With the adoption of the aluminum-cell lightning-arrester, which is built to-day for outdoor installations, there remain three factors for the designer to deal with. At the present time there are in outdoor operation numerous transformers wound for 10,000 volts; and there should be no reason why outdoor operation would not be possible with higher voltage. As for the oil switches, these can be made water-proof as readily as transformers.

In transmissions of 10,000 volts, where self-cooled transformers can be applied with the downward projecting insulated lead which issues from an overhanging pocket near the top of the transformer case, an outdoor application is perfectly safe where the transformer is located on an elevated platform; but for higher voltages, where the oil-insulated, water-cooled type of transformer is necessary, with the leads projecting out of the top of the case, several factors are to be considered. Owing to the weight, this transformer would necessarily have to be located on a foundation. In climates where severe snowstorms are encountered, the transformers and oil switches should be protected by some kind of shed, otherwise they are likely to become submerged, grounding the system. This shed is also a necessity in the summer months, to protect the oil-insulated apparatus from the intense heat of the sun's rays, as well as acting as a shield to the insulators of the apparatus from brick-throwers and sharpshooters. In addition to the shed covering the apparatus, the latter should also be surrounded by a wall or fence, in order to keep the inquisitive out of the danger zone.

In high-voltage transmissions the transformers should be protected on the high-tension side by disconnecting and automatic oil switches. On this point I cannot agree with the suggestion:

> * * * that by treating the high-tension side of the transformers as a part of the line, no high-tension oil switches would be required, but fused disconnecting switches placed on the high-tension side would serve to cut out the transformer.

This might apply on a 6600-volt line, but even in this case I would recommend oil switches. The fuse proposition on a 6600-volt line is a very expensive one, as good protective fuses are costly, especially owing to the number likely to blow. At the present stage of the art, in speaking of high-tension apparatus, we have in mind voltages from 40,000 up; fuses for such voltages would be absolutely prohibitive as well as poor practice.

In describing an outside station, Mr. Randall says that as no instruments are employed, no attendants are necessary. I do not agree with this statement unless the station is located in close proximity to another station containing the low-tension apparatus, so that the switching can be readily looked after, or the switches are electrically operated from a distance. If a station were connected on the high-tension side by only fused disconnecting switches, any possible trouble that might occur—blowing a fuse—would cause an inconvenience that might not only be annoying but costly to the consumer.

If it is desirable to locate the high-tension apparatus outside, the ideal arrangement for a switching or transformer station where no secondary machinery—as motor-generators—is necessary, would be to locate the apparatus—including the lightning arresters, choke-coils, oil switches and transformers—under a

shed, the station being surrounded by a fence in populated districts, the low-tension apparatus, consisting of switchboard, etc., being housed. The transformers should be protected by electrically operated oil switches, operated from the switchboard by the station attendant, and by disconnecting switches on the high-tension side. If the low-tension voltage for distribution is 2300 volts, hand-operated automatic oil switches, mounted on the back of the switchboard, and disconnecting switches should be used between the secondary of the transformers and the low-tension bus-bar. If the low-tension voltage is 440 volts or thereabouts a saving could be made in the switching apparatus on the secondary by using only disconnecting switches between the transformer and the bus-bar, the outgoing feeders being protected by triple-pole, carbon-break circuit-breakers with overload release.

In using water-cooled transformers, it is essential in such a layout that attention be given to the cooling-water piping to prevent bursting from freezing, which necessitates the flow of water continuously through the coils. It is also necessary that the attendant know at all times that this water is flowing. This can be taken care of by inserting a quick-flow indicator in the main water line, which should run through the station, and one on the incoming water piping at each transformer. This device shows the operator at a glance whether or not the water is flowing, and can be seen 25 ft. distant. To my knowledge it is the only safe device on the market for such an indication. It is now made electrically operated so that when the water stops flowing a lamp will be lighted.

I had the pleasure of assisting Mr. Randall in the estimates of costs of indoor and outdoor stations. In making these calculations it was apparent that in stations containing secondary machinery, as motor-generator sets, or synchronous converters, the percentage of saving in placing the high-tension apparatus outdoors, instead of in compartments indoors, is very small, as there is involved only the cost of the building against the total cost of the installation plus the additional cost of weather-proofing the outside apparatus. It was evident that this percentage of saving decreases as the size of the station increases for a larger amount of secondary apparatus. In such cases the saving would not warrant locating the apparatus outdoors, due to the fact that with the transformers and switching apparatus in compartments they are absolutely protected from the weather, are more accessible for inspection or overhauling, and out of the way of molesters. The life hazard is less also. In simple transformer or switching stations, such as tapping from the transmission line to feed an outlying district for lighting and motor work, there is no doubt that by locating the high-tension apparatus outside, protected by a fence and shed covering, with the low-tension switch-board and switches housed, a saving in cost is effected well worth considering.

As regards locating oil switches outside of a station, under a lean-to, as suggested, I have at present in hand such a problem. The plant in question is now generating 36,000 kw. at 60,000 volts, delta. The output of the station is to be increased to 56,000 kw. by the addition of two vertical-type water-wheel-driven 10,000-kw. generators, and the transmission pressure raised to 85,000 volts, Y, with grounded neutral. The switches and bus-bars are now installed in masonry compartments, and it would be impossible, owing to conditions existing, to use a 100,000-volt tubular switch which was desired, erected in the station, without tearing out the present bus-bar structure, involving a large expense.

The problem reverted to whether the insulating qualities and rupturing capacity of the present switches could not be increased to take care of the new conditions. The manufacturers found after a careful study of the situation that the pots of these switches could be made 6 in. deeper and 2 in. larger in diameter, increasing the oil space approximately 50 per cent and doubling the expansion chamber or air space above the oil, thus greatly increasing the insulating and rupturing capacity of the switches. From tests it was found that the insulation was satisfactory, but as there were no facilities in the factory for testing the rupturing capacity, this is not known; but there is no question that the switches are perfectly safe for the high-tension side of any bank of transformers, the maximum normal capacity of the largest bank being 18,000 kw. It is deemed advisable to install an outdoor 100,000-volt switch with a rupturing capacity of 200,000-kw. and this proposition is now before the engineers of the manufacturing companies. There is no doubt in my mind that the design of such a switch is not only possible, but will be called for frequently in the future.

These outdoor switches will be used only on the outgoing lines from the power house and incoming lines to the sub-stations, operating at 85,000 volts, Y, with grounded neutral. I am quite sure that these switches can be built, as the manufacturers are anxious to submit prices to take care of such a proposition.

M. W. Franklin: Fig. 1 in Mr. Reynders' paper shows the potential curve in the dielectric between the surface of the conductor and the grounded external circumference of the dielectric. The statement to the effect that it shows the distribution of stress is an error. Stress, which in this case corresponds with intensity of electrification, is the space-rate of change of potential and is therefore represented by the graph, not of the potential, but of the first derivative of the potential with respect to space.

The equations for the potential at any point between two coaxial cylinders must satisfy the condition.

$$\Delta^2 V = 0 \tag{1}$$

The equipotential surfaces are coaxial cylinders and therefore (1) may be written

$$\frac{d^2 V}{d r^2} + \frac{1}{r} \frac{d V}{d r} = 0 \tag{2}$$

which on integration gives:

$$V = c \log r + C_1 \tag{3}$$

when $r = R_1$, $V = V_1$ and when $r = R_2$, $V = V_2$, therefore

$$V = V_1 \frac{\log \frac{R_2}{r}}{\log \frac{R_2}{R_1}} + V_2 \frac{\log \frac{r}{R_1}}{\log \frac{R_2}{R_1}} \tag{4}$$

where V_1 R_1 and V_2 R_2 are respectively the potentials and radii of the inner and outer cylinders, and r is the distance of any point at potential V from the common axis.

The intensity of electrification at any point R at a distance r from the axis is

$$\mathfrak{F} = -\frac{d V}{d r} = \frac{-(V_2 - V_1)}{r \log \frac{R_2}{R_1}} \tag{5}$$

This represents the force.

In the case of a homogenous, continuous dielectric, V_2 is at the outer circumference and is equal to zero. The equation (4) may then be written:

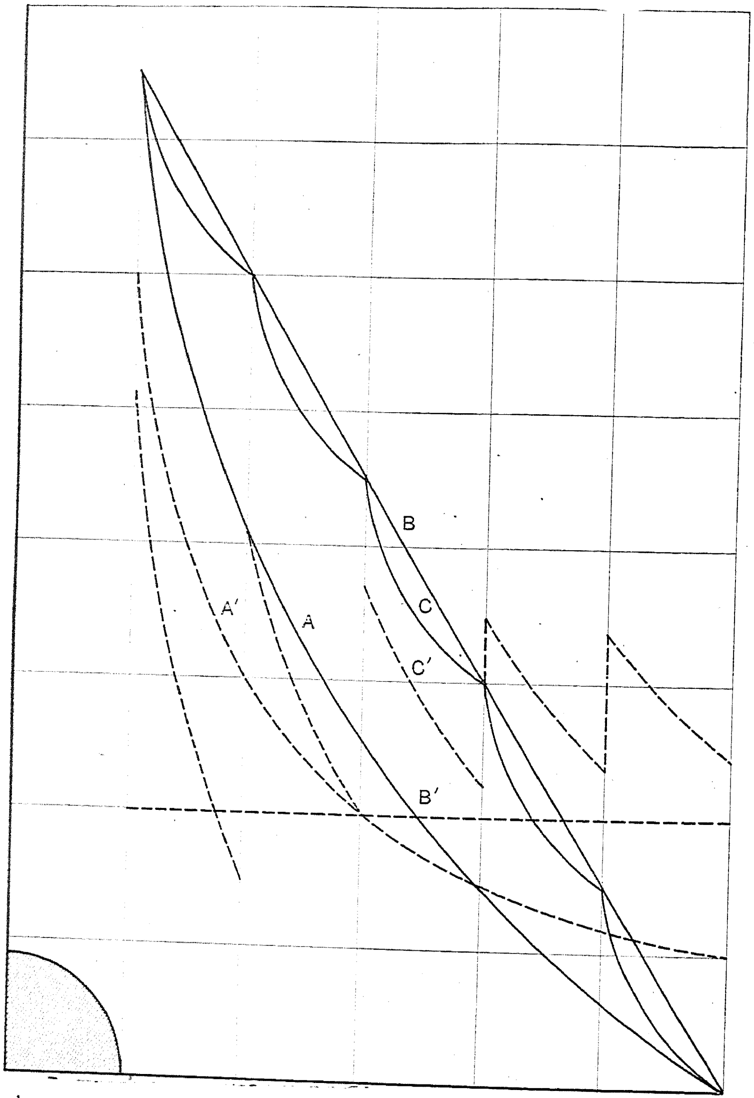
$$V = V_1 \frac{\log \frac{R_2}{r}}{\log \frac{R_2}{R_1}} = c \log \frac{C_1}{r} = k \log \frac{1}{r} \tag{6}$$

The graph of this function (curve A in the annexed illustration) is the potential gradient. Equation (5) reduces to:

$$\mathfrak{F} = \frac{V_1}{\log \left(\frac{R_2}{R_1}\right) r} = \frac{C_2}{rg} \frac{1}{r} \tag{7}$$

and the graph of this function is the stress diagram and is shown in curve A'.

The condenser bushing here presented has for its object the attainment of a uniform potential gradient, as shown in curve B. The reason for this is that the derivative of such a function is a constant, as graphed at B', which shows that the force at any point in the dielectric is constant.



A, potential in a uniform dielectric; A′, stress diagram corresponding to A; B, ideal uniform potential curve; B′, stress diagram corresponding to B; C, potential for a series of concentric condensers; C′, stress diagram corresponding to C.

To this end, the dielectric is divided into a series of concentric cylinders, each forming the dielectric of a condenser whose plates consist of sheets of tin-foil interposed between them. The difference in potential between each plate and the next succeeding is a constant quantity, so that a line through the positions of the plates would indicate a potential curve as at B.

Were the number of plates infinite this would be strictly true throughout the dielectric, but in the case shown the stress gradient in the individual layers of dielectric must exert upon the results an influence which is not negligible.

The equation (4) for potential shows that though the potential differences between pairs of adjacent plates be uniform, the potential in the contained dielectric is not a straight line, but a logarithmic curve determined by equation (4). This is shown in the curves C.

The stresses in each section of the dielectric are shown in the curves C' which are the first derivatives of the curves C.

Referring to equation (4), it will be seen that the shapes of the logarithmic curves C differ as the position of the shell changes with respect to the distance from the center. The differences could not be shown in the small diagram.

Reference to equation (5) shows that the potential difference $f(v)$ being a constant for all shells, the constant $\log \frac{R_2}{R_1}$ varies with varying distance from center. The ratio $\frac{R_2}{R_1}$ becomes smaller while the value r becomes greater.

$\text{Log } \frac{R_2}{R_1}$ is a discontinuous function, and does not therefore lend itself readily to pure analysis, but correctly formed stress diagrams are plotted for each of several values of the function. These curves show that the stress diagram, far from being a straight line or even a smooth curve, is a series of discontinuous curves which introduce serious stresses in the dielectric. They also show that the stress distribution is very irregular, being greatest near the conductor, as with a uniform dielectric.

Geometrical considerations as mentioned in the paper necessitate the employment of various dielectric media to obtain equal capacities. This is unfortunate, as the different layers will expand and contract unequally, and mechanical strains of a serious nature will probably result.

The impossibility of avoiding sharp ridges and prominences on the surface of the foil will result in localized high potentials which will probably cause puncture.

Coaxial cylinders when charged are in a state of unstable equilibrium. When they are displaced even a small amount the

capacity rapidly rises, the rise being proportional to the square and higher powers of the displacement of the centers. In the imperfectly rolled cylinders the irregularities correspond to equivalent displacements of center, and the stresses across certain limited areas of dielectric will be greatly increased.

K. C. Randall: The joints which were described as "vacuum tight" were so called as a measure of good joints. Mr. Moody's suggestion to maintain a weak pressure, so that there would be a slight outward pressure in order that any leakage would be outward instead of inward, is one that has frequently been looked upon with favor. It is true that the oil may freeze and become temporarily less effective through losing its insulating qualities, but the usual insulation margin in the quality of oil is sufficient to care for such a condition. Oil that will withstand 50,000 volts may drop down to 10,000 or 15,000 volts, and yet this very considerable depreciation in the insulating qualities of the oil may occur and the oil still be serviceable. The ordinary substations are not carefully heated. Furthermore, the heat due to the loss in most transformers will prevent the oil freezing solid. It is obvious that should the water in the cooling pipes freeze, the pipes would split and the next time water was turned on the transformer would probably be ruined. The obvious remedy is to adopt some means to keep the water from freezing, or arrange to drain the pipes.

Mr. Thomas pointed out the troubles of inspection and investigation, and particularly the daily routine of looking after the apparatus. That is more difficult, it is true, but the underlying fact is that nowadays our apparatus is getting better and better. A few years ago an occasional failure, or even a frequent failure, was not very much thought of. To-day a good deal of trouble is raised at a single suggestion of failure. How many moderately large transformers are taken apart and cleaned, or even looked over thoroughly? Once a year probably some of them are examined, but a goodly share are not. It would be difficult to repair an outdoor unit in a snowstorm, no doubt. The reference made to the heliostat seems to have been taken seriously by some one.

To summarize what I have said in the paper. Outdoor apparatus is not a panacea. There are places where it can be and will be used; there are some places where it is used to-day.

Mr. Mershon asked at what temperature cold oil would stop switches from operating. I cannot say positively, but I believe there is a non-freezing oil which at a temperature 35° fahr. below zero will still permit switches to operate.

In an outdoor installation the isolation of the units will be cheaper than for indoor stations, and consequently the security resulting from wide separation of units is more cheaply obtained.

Tight joints for transformer cases are equally desirable for indoor and outdoor units, so far as "breathing" is concerned, but

the outdoor units require protection against rain, storm, dust, etc., not needed by indoor apparatus.

The standard 150-mil gap will give a test of from 6000 to 10,000 volts in oil taken from a tank with water under the oil. There are, doubtless, many transformers for 10,000 volts and over which are operating with water standing in the bottom of the case. It is also likely that an average test value of many oil samples taken from transformers in moderately high-tension service would not exceed 20,000 volts. The writer recalls a 60,000-volt unit which continues in perfect service, though the oil at one time tested only 15,000 volts.

These are some of the reasons for feeling that the limited depreciation of the oil dielectric due to freezing—which is quite improbable—will not endanger modern large outdoor transformers.

The field of the outdoor transformers is limited, but there are some notably economical installations. The Ontario Power Company is reported to have saved $2000 by an installation of 60,000-volt outdoor series transformers alone.

There are a few large outdoor 6600-volt transformer installations which are just as simple and satisfactory as the small house-to-house transformer installation.

Except in the very remote case of failure, it will not be necessary to inspect or work around outdoor apparatus, save in favorable weather chosen for the purpose.

A. B. Reynders: In Mr. Moody's remarks the lining up of the weak spots in the insulation was mentioned, and several subsequent speakers have brought out the same point. It is true that the weak spots tend to line up, especially if the insulation be thin. We have made terminals with three layers of 0.005 in. paper between tin-foil layers, and the terminal on test broke down at a relatively low value. The thickness now employed in the design practically eliminates this weakness. Mr. Moody also stated that the design employing oil, separated into layers by barriers of insulating materials, gave a very high dielectric strength. This is true, as the barriers of insulation act as an interference to the lining up of the particles of oil when under stress. This arrangement is not in any sense a condenser terminal.

The length of the condenser terminal above the top of the tank has been criticized as excessive. I wish to say that these terminals were purposely designed long. Observe the 66,000-volt terminal. This is used on an oil circuit-breaker. The operating mechanism is placed between the poles. As this mechanism extends some distance above the cover, and as it is necessary to obtain somewhat more than sparking distance between the top of this mechanism and the tip of the terminals, it required long terminals. Compare the terminal having a test of 180,000 volts with the terminal having a test of 250,000 volts. The latter is used on a transformer. It will be noted that the surface insulation distance in air is about the same as on the 66,000-volt circuit-

breaker terminal. It will be further noted that the outside grounded band is extremely long. This is required by the design of the transformer tank and cover. It is necessary to have the lower end of the outside grounded band under oil and also to have the upper end of the same band on a level with or a little above the top of the cover. Owing to the "dome" required by the cover to withstand pressure, this long distance is necessary. Observe the short distance required under oil—in this terminal having 250,000-volts test, it is only 12.5 in., making 20,000 volts per inch.

Mr. Thomas spoke in favor of terminals of large diameter. This meets with no objection in the design of transformers, but for circuit-breakers it is practically impossible. For instance, a 100,000-volt circuit-breaker would require a terminal of the "bulk" type 20 in. in diameter, while the tank required would not permit the use of such large terminals.

Mr. Rushmore stated that brush discharge is detrimental, that it is doubtful if the condenser terminal will stand the test of time. We have made these terminals for 2 years and some of them are operating to-day at 200,000 volts to ground.

Mr. Neall thinks that manufacturing limits are reached in the terminals shown in this exhibit. I can state that we are now building a terminal for a test of 400,000 volts for 1 min., having approximately a diameter of 10 in. and a length of 117 in. We have a design under way for a terminal for a 500,000-volt transformer—one end grounded—having approximately a diameter between 15 in. and 20 in. and a length of 15 ft.

Mr. de Muralt stated that the factor of safety in terminals is too small for ordinary conditions of working. At the present time we are designing terminals for a test voltage which in commercial service is 2.5 times the normal operating voltage. The stress chosen for this test voltage is never more than one-half of the breakdown value of the insulating material, hence the terminals have at least a factor of safety of five.

Two of the speakers touched upon the theory of this terminal and attempted to show that either the addition of tin-foil did not improve the puncture value or else it was liable to produce harm in time. I will not take the time to answer these arguments, but wish to refer them to Jona's paper previously mentioned and particularly to a paper by Rudolph Nagel, published in the *Elektrische Bahnen U. Betriebe* for May 23, 1906.

Ralph W. Pope: I have here an insulator presented to the Institute by one of our charter members, E. P. Roberts of Cleveland. The inscription on the insulator is as follows:

Insulator taken from old telegraph pole in Northern Wyoming about 1887 or 1888, by Wilbur C. Knight, mining engineer, and presented to E. P. Roberts, member of the A. I. E. E. The telegraph line was the first transcontinental and was built by Ezra Cornell, founder of Cornell University.

This type of insulator is peculiar. The early insulators were simply of glass; this insulator is of glass inside, and covered with a wooden shield. The object of the wooden shield is interesting. It was found that the glass insulators formed an excellent target for the marksmen of the plains, and the shattering of the glass was a never-ending source of amusement when the marksmen hit the insulator. Hence, this form of wooden shield was devised, so that the glass would not shatter. Consequently this type of insulator was generally used all through the plains. It was commonly known as the "nigger head" insulator.

F. G. Baum (by letter): In the early work of the transmission companies on the Pacific coast problems had to be solved within

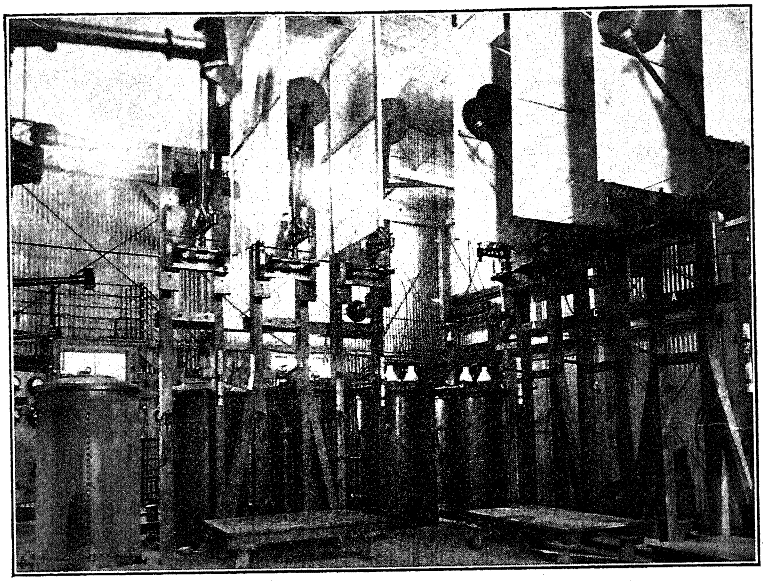


FIG. 1.—Interior of sub-station, showing early type of switches.

certain prescribed expenditures. What we were trying to obtain was a satisfactory delivery of power to small consumers without great expense or danger to persons or property. In some cases the transformers were set outdoors on ties and a shed built over them to keep out moisture, all switches, fuses, etc., being also outdoors. Many of the stations had no operators and often were visited only monthly, and the simpler the layout the better.

Fig. 1 shows one of the earliest interior sub-stations, with single-pole switches mounted on a wood frame, making the fire hazard very great. It was this sort of thing that kept the men on the alert day and night. Now nearly all the air-break switches

and fuses are installed outdoors. Fig. 2 shows a modern type of switch. Fig. 3, 4, and 5 show some of the earlier types which may be interesting from a historical point of view.

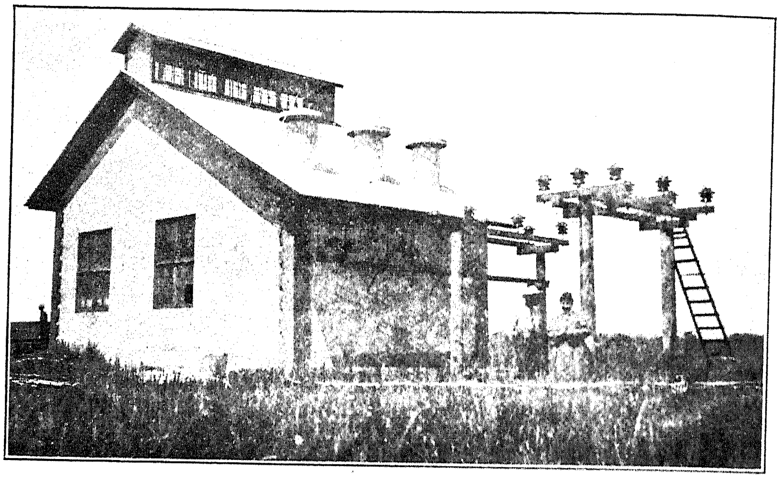


FIG. 2.—Modern outdoor switch with fuse.

As voltages increase and power consumption becomes more general, there will arise a demand for transformer apparatus and

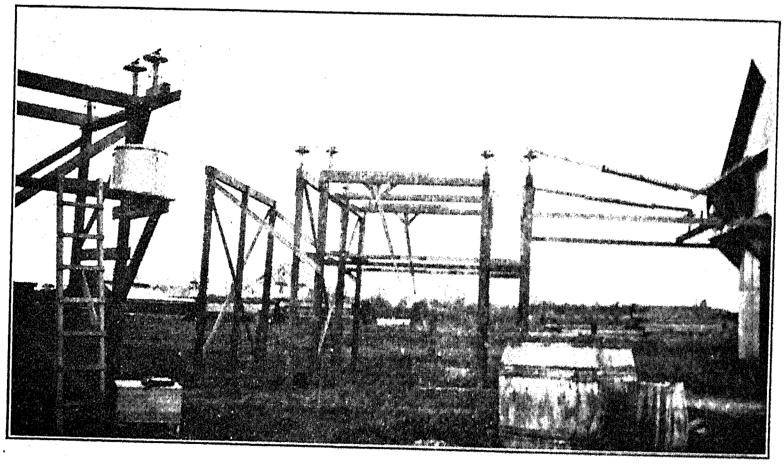


FIG. 3.—Early outdoor switches. Compare the choke-coil lightning-arresters with Fig. 10 of Mr. Randall's paper.

switches to be mounted outdoors, thus at once reducing the investment, facilitating inspection, reducing fire risks, and decreasing operation costs.

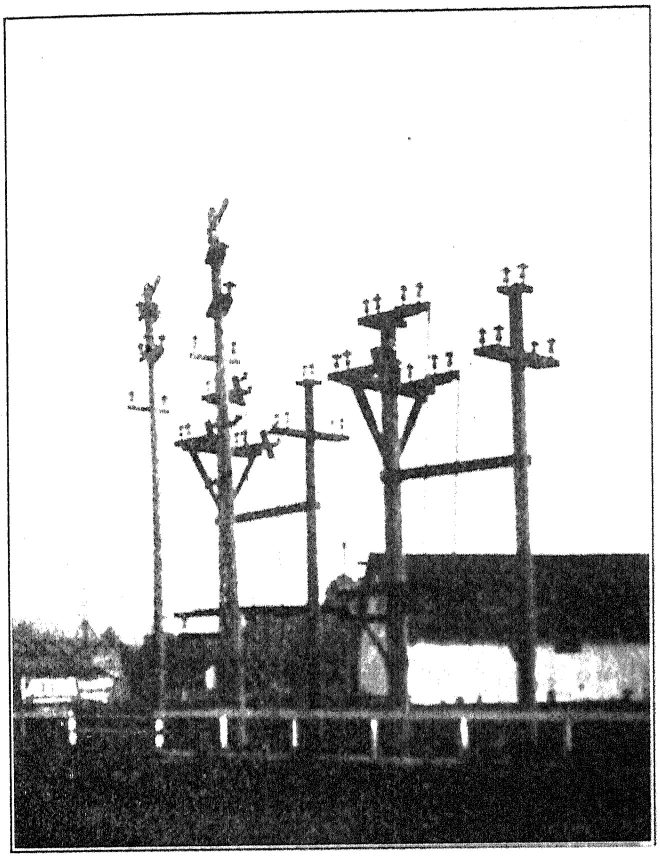


FIG. 4.—Early outdoor switches.

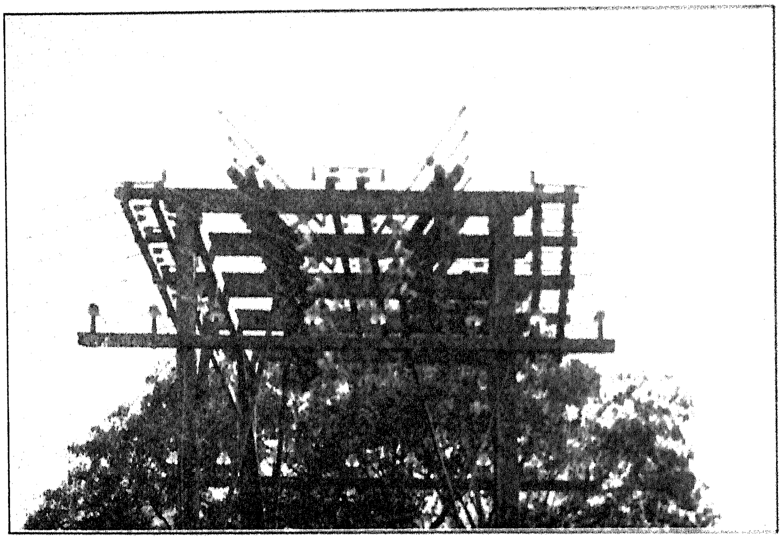


FIG. 5.—Early outdoor switches.

O. S. Lyford, Jr. (by letter): Mr. Randall's paper relates to one of the most important steps recently taken in the field of high-tension transmission. The indoor sub-station, with apparatus operated at 40,000 volts or higher, is an absurdity. Not only is the cost of housing excessive, but because of the obvious necessity of economizing space as much as possible an excessive amount of engineering expense is incurred in developing the most compact safe arrangement.

The merit of the outdoor station will be most apparent where the amount of power to be transformed is small. Transmission companies are continually facing the problem of taking off blocks of power of 500-kw., or less at isolated points. At 40,000 volts, or higher, the cost of a sub-station, with the usual controlling and protecting devices suitably housed, is not very different for 100 or 1,000-kw.

The cost for the small sub-station, even of the outdoor type, will always be disproportionately high, but much can be done toward minimizing the cost by concentrating the transformer, circuit-breaker, and choke-coil in one tank. This plan was proposed at the Niagara Falls convention in 1907 and is referred to in Mr. Randall's paper. The merit in saving terminals is apparent from an inspection of Figs. 4 and 6.

Mr. Randall also refers to the recent development of a transformer construction which permits of shipment in the case with the necessary oil. If we can have a self-contained sub-station unit, ready for setting up in the field, and requiring with it only a small shelter for low-tension switching equipment and for repair purposes, the problem will be greatly simplified and the cost brought within reason.

Referring to the difficulty with moisture, the extent of breathing of a transformer is hard to realize without actual experience. The variation of oil level with change of temperature is much greater than would be represented by the respective coefficients of expansion of the oil, transformer core, and windings. This is apparently due to the porous and somewhat loosely built up insulation which absorbs and expels the oil as the temperature changes. This tendency to breathe can be provided for without admitting moist air to the interior of the transformer tank, if an adaptation of the U tube, with an oil seal, be used.

Referring to the terminals for outdoor transformers, the illustrations indicate that porcelain and composition types are both being experimented with. The writer has yet to see any composition material that has proved adequate for use in outdoor high-tension work. The vitreous quality and high glaze of porcelain seem to be essential.

Carl Schwartz (by letter): Two advantages are claimed in favor of the outdoor station: first, the cheapening of the installation due to a saving in building; and secondly, less life and property hazard.

Which type of installation, outdoor or indoor, is the most

economical is not determined by low first cost alone. Any difference between the two schemes in cost of operation and maintenance should be capitalized and balanced against a difference in first cost. Also the value of safety of operation will have to be most carefully considered, even if this feature cannot be estimated in dollars and cents.

Reliability of service in generation and distribution of electric power has become of greater importance from year to year, and is recognized to be in almost all cases the most important feature. It will be admitted that delicate apparatus with complicated connections is apt to operate more reliably, can be better cared for and maintained for less money, if installed in suitably constructed buildings instead of outdoors. Inspection of oil switches placed outdoors is frequently difficult; dust, rain, and snow may cause serious disturbances at any time, if not during inspection.

Mr. Randall says:

> In order to obtain the cost comparison, the cost of the station grounds and indoor apparatus must be balanced, not only against the smaller building, but also against all the ground required for the outdoor apparatus, the outdoor apparatus itself, and the instruments and the indoor control apparatus.

In the estimate, however, the larger area occupied by the outdoor arrangement is not considered, and the saving in first cost shown in favor of the latter is to be reduced accordingly.

The two estimates for an indoor and semi-outdoor motor-generator sub-station show a saving of about $13,000 in favor of the outdoor scheme. This saving will be reduced to about $4,000 if the size of building is made more proportionate to the apparatus to be accomodated, and if the same class of building be assumed for both cases. Also, the same class of building will probably be erected whether some of the apparatus is installed indoors or outdoors, the general requirements being entirely similar. There is hardly reason for constructing a building of a cheaper class if only the smallest part of the apparatus be taken outdoors.

Hence it appears that the outdoor type of station may be somewhat more economical in case of a very simple transformer or switching station of high voltage with few or no instruments.

For a motor-generator station or a station requiring numerous instruments and attendance, the first cost of an outdoor station is not appreciably, if any, lower than of an indoor station, greater economy in operation and maintenance and greater reliability always being in favor of the latter.

As far as less life hazard is concerned, the outdoor station is distinctly more dangerous unless it is protected against intruders in a most thorough manner. The property hazard, if this is meant to apply to the station building proper, even taking into account large quantities of oil stored in switches or transformers, is, as experience shows, extremely small if the station is sufficiently sectionalized and otherwise well designed, and is far overbalanced by the better protection afforded to the apparatus.

Concluding, I wish to say that we are fortunate in having available apparently well designed apparatus such as that described in the paper for use outdoors even under adverse climatic conditions. We know of circumstances where apparatus of such type can be used to advantage. I believe, however, that it would not be wise to promote the building of outdoor stations and to develop apparatus for them, to save in first cost by sacrificing reliability, while increasing costs of operation and maintenance. It would rather appear advisable to endeavor to limit the use of outdoor-type transformers, switches, etc., to cases where complicated connections and other undesirable features result, should such apparatus not be located close to the line and should it become necessary to bring high-potential wires with perhaps numerous crossings to and into buildings. This applies, for instance, to sectionalizing switches of high-voltage trolley line systems where the location of such switches away from the line would cause serious complication, though in this case the use of high-tension switches of the outdoor type is to be considered as one of the disadvantages of the system.

J. B. Whitehead (by letter): The most general expression for the capacity of two concentric cylinders is

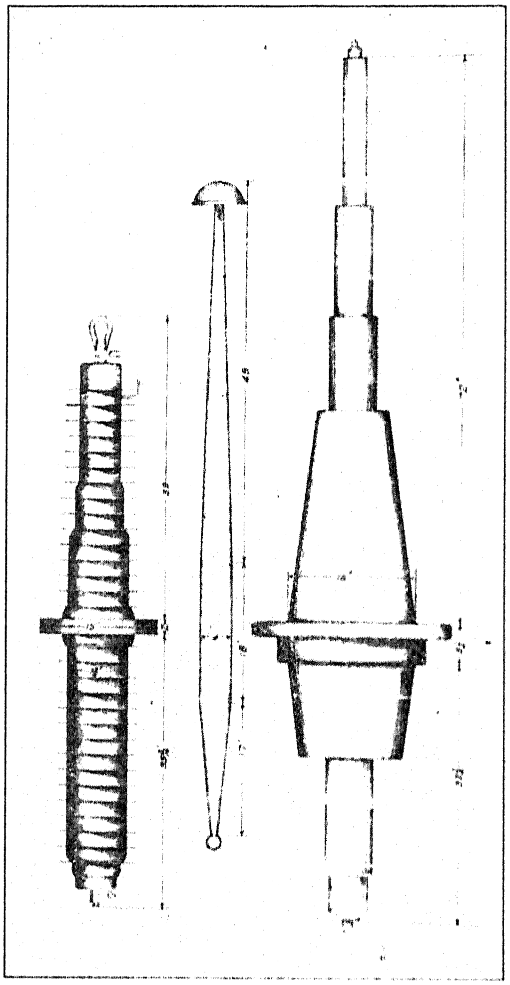
$$C = \frac{k\,l + \Delta\,l}{2 \log \frac{r_2}{r_1}}$$

Δl is the correction term which depends on the length of the cylinders, their diameters, and the shape of the ends. The exact value of Δl leads to complicated expressions if it is attempted to express it for all cases. It is not probable that the requirements of terminals such as are here treated would necessitate taking account of this corrective value. The value of Δl for two cylinders 60 cm. in length and of radii 12.5 and 14.5 cm., respectively, with square ends, is between 2 and 3 per cent. Consequently, for the long narrow tubes as used in the terminals, even the difference of length of successive tubes would probably not introduce any error which could not be allowed for under the conditions of construction. It is well to note, however, the suggestion of this type of insulation for use in transformers between primary and secondary and primary and ground. In such cases the dimensions would frequently lead to short cylinders of large diameters. If the exposed surfaces of the ends go to large values in order to prevent creepage, it is not impossible that it will be necessary to take account of the end correction in the simple expression for the capacity concentric cylinders.

Referring to Mr. Randall's paper, a preventive for the trouble arising from the breathing of moist air in transformers might be found in the use of a double valve in which the ingoing air is carried through a drying agent and a free-one-way outlet is provided for expansion. In this way no pressure due to increased

temperature would be applied to gaskets and other joints which it is aimed to make absolutely air-tight. It would be interesting to know also whether any investigation has been made as to the value of drying agents located in bulk within the transformer case.

John J. Frank (by letter): Supplementing the remarks by Mr. Moody relative to the oil-filled lead which we have developed, I



would like to call attention to the illustration herewith showing a comparison of dimensions of such a lead with similar leads shown in Figs. 10 and 11. This lead has actually been tested and shows a failure by arc from terminal to ground of 250,000 volts. Attention is called to the reduced height of the lead as compared with the condenser type, Fig. 11, and the solid type as

shown in Fig. 10, which reduction in height was referred to by Mr. Moody as a desirable feature in the design of any lead.

W. L. Waters (by letter): The grading of insulation in high-voltage cables was first suggested by O'Gorman about 10 years ago, as a means of reducing the excessive amount of insulating material required in a high-tension cable with homogeneous dielectric. It was found, however, that the variation in the specific inductive capacity and dielectric strength of available insulating material was too small to obtain any very great commercial advantage from this arrangement. The condenser-type of terminal was next suggested about 4 years ago as a means of obtaining uniform stress on a homogeneous dielectric. As Mr. Reynders has pointed out, this type is limited by the necessity of obtaining sufficient leakage distance between the various metallic cylinders on the outer surface of the terminal. The terminal which Mr. Reynders has developed by combining the above two principles is the first really practical commercial type of high-tension terminal.

When the condenser type of insulation was first proposed, it was suggested that it might also prove advantageous for use in alternators and transformers. Its use for slot insulation on high-tension alternators was suggested as a means of avoiding the brush discharge from sharp corners on the laminations of conductors, and of avoiding the weakening effect of the air spaces usually present in a dielectric of considerable thickness built up in layers. Up to the present, however, it has been considered that in this case the difficulty of making a satisfactory mechanical construction more than counterbalances any possible advantage which might result. The advantages of this type of insulation for transformer work, as Mr. Reynders has pointed out, are very doubtful except in special cases.

It has long been recognized that the terminals were a weak point in a high-tension transformer, and that the solid type of terminal with uniform dielectric was of poor design on account of its high cost and great bulk, and on account of the difficulty of obtaining the dielectric perfectly homogeneous and free from defects. Mr. Reynders apparently decided that it was not worth while attempting to improve the solid type of terminal further, and took up the condenser type as being more promising.

L. L. Perry (by letter): Any engineer who has calculated the cost of a building to house switching apparatus for four or five high-tension circuits and a simple bus-bar, will appreciate the work done along the lines covered by Mr. Randall's paper. In planning several important transmissions, we have estimated on intermediate switching stations for the purpose of cross-connecting the good half of a defective circuit with the good half of a second defective circuit; and we have always concluded that it was not wise to spend the money until actual operation had demonstrated that the cost was warranted, especially since, at the beginning, the load is light and more spare circuits are available. It appears

to me that for switching stations there is bound to be considerable development in the field treated of in this paper.

When we postponed the building of switching stations we hoped that the ground wire and the horn arrester would give more ample protection from lightning than the facts now show. Notwithstanding the great progress made in the last few years in arresters, it appears to-day that the greatest enemy to long-distance transmission is lightning. Our own experience has been confirmed by that of other engineers. A reasonable degree of protection has been obtained for the stations, but we still have before us the problem of the protection of the line insulators miles away from the stations. The overhead ground wire, the horn arrester, and the electrolytic arrester do not, so far as our experience goes, afford full protection. Since it seems improbable that full protection will be ever obtained, there are good reasons for putting in, on important transmissions, a certain number of switching stations, depending on the value of reliability. Such devices as have been used in the past have been crude, and there now seems to be a demand for more elaborate apparatus. Probably we shall soon use outdoor oil-switches on many of our lines of highest pressure, and their use on lines of lower pressure will increase.

Most transmission engineers have been confronted with the problem of tapping an important transmission for the supply of a few horse power; as a rule not only is there no money in it for the power company, but such tapping is a menace to the main transmission. If inexpensive devices are available it will be commercially possible to do things that the high cost of the older types of station prohibited.

But it would seem to many of us that high-tension outdoor transformers could have but a very limited application, for where reliability of service is of much value it will be found cheaper to house the transformers. As electricity is more and more applied to electrochemical and electrometallurgical work, where often low power cost is of paramount importance, the minimum investment will rule and will tend to develop the outdoor station.

Nevertheless it would not be surprising if the application of such outdoor apparatus extended to transmissions for which its advocates do not now urge its use. We have had occasion to build some 6000-volt street-transformer vaults for feeding general light and power mains. Some of these have been built to stand flooding, and in them water-proof switches, transformers, and fuses have been employed. We have built others where there is no danger of flooding and the ordinary type of switch and fuse apparatus has been used. Contrary to our expectation the water-proof type seems preferable for many good reasons. It will require long experience to determine which is more reliable in service. The compactness, safety, and ease of repair of the water-proof type are great advantages. These vaults are not, of course, comparable to high-tension stations, for we used lead-

covered cable for bus-bars and leads; but they have been cited to show that sometimes quite unexpected results may be obtained where such radical methods are adopted.

It is to be noted that the author does not go into the question of reliability. Most engineers would probably assert that an outdoor station such as that described by Mr. Randall could be of advantage only where a very low price is charged for power.

Assuming 20 per cent on the investment to cover all annual costs such as interest, operation, maintenance, repairs, depreciation and profit, the yearly cost will be:

Indoor station, 20 per cent of $15,100 or	$3020.
Outdoor station, 20 per cent of $11,445 or	2289.
Yearly saving	$ 731.

The yearly saving on the basis of 2000 kw. is only $0.37 per kilowatt capacity. If power be sold for as low as $16.00 per kilowatt-year, 37 cents seems no large sum to pay for extra reliability. But if power be sold for $32.00 or $64.00 per kilowatt-year, the 37 cents seem inconsiderable. These assumptions were that in each case 20 per cent covers all annual charges; but it would seem that the repairs, maintenance, and depreciation on the outdoor station must be relatively higher.

J. N. Kelman (by letter): The switch shown in the accompanying illustrations was designed for outdoor service on 30,000-volt systems. It consists of a weather-proof, three-pole, oil switch and two sets of disconnecting switches installed on a galvanized-steel tower. The oil switch proper consists of three separate iron tanks, each containing mechanism that gives a double horizontal break per pole, this break being so located as to leave two-thirds of the total volume of oil above it. Over each tank is placed a two-part insulating cover which serves to hold the leads in place. The leads are wrapped with varnished cambric for their entire length and brought out of the tanks and down to the disconnecting switches through heavy porcelain tubes, into which they are sealed with a special compound, then passing through the covers of the tanks and the plates in the eaves, both of these being of a high-voltage insulating material mechanically strong and moisture proof. The leads are well extended below the eaves to give ample creepage distance to ground under any weather conditions, the porcelain tubes serving to protect the cambric from the weather, porcelain being used for this purpose as the best and most reliable material known at this time. The lower ends of the leads in the tanks are sealed into the tubes with an oil-proof cement which prevents the cambric insulation from "wicking" out the oil.

The switch is made weather-proof by closing the spaces between the tanks on the sides and the bottom with heavy galvanized iron securely held in place by bolts, and by placing over the top a two-part roof of the same material sufficiently large to give ample clearance. For convenience of inspection the roof sec-

tions are hinged at the ridge pole, and either side may be raised as desired after loosening a few thumb screws. The weather-proofing was given most careful thought, and it is claimed to be

FIG. 1.—Outdoor switch and tower for 30,000 volt system.

impossible for rain or snow to enter the switch housing, either when falling quietly or when driven by high wind. The housing is designed to permit of circulation of air, thus keeping the tem-

perature of the air within it about the same as that of the atmosphere. It is expected that the oil will absorb more or less moisture from the atmosphere. The insulation is designed to have a high factor of safety even with oil of very low insulating value.

As an insulation test this switch was erected and connected to a 50,000-volt line as follows: while the switch was being erected rain commenced falling, but as it was desired to get as complete a test as possible the work was continued in the rain; the tanks were filled with oil and the switch was connected to the line ready for the test. The pressure was then raised to 60,000 volts and

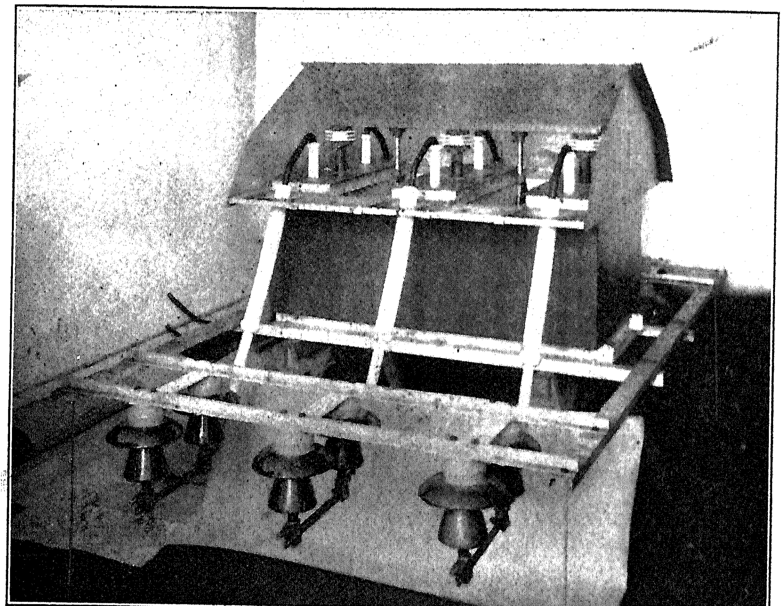


FIG. 2.—Weather-proof oil switch, leads encased in porcelain tubes.

held at that point for 10 min., a separate generator being used for this purpose. The line was then cut back into regular service at 50,000 volts, which was maintained on the switch continuously for 20 days. Rain was falling steadily during the installation and continued to fall steadily while the 60,000-volt test was being made and for over 24 hours thereafter. Following this rain and during the 20-day test there were a number of showers accompanied by high winds, and one storm which lasted over 12 hours During all these rains 50,000 volts was maintained on this 30,000-volt switch, with a generating capacity of over 25,000 kw. to hold up the voltage; but notwithstanding the fact that this was nearly double the voltage for which the switch was designed

there was absolutely no indication of distress or weakness at any point. The switch was tested on the lines of the Kern River Co., at Los Angeles, Cal.

August H Kruesi (by letter): The desirability of placing high-tension bus-bars, wiring, disconnecting switches, horn gaps

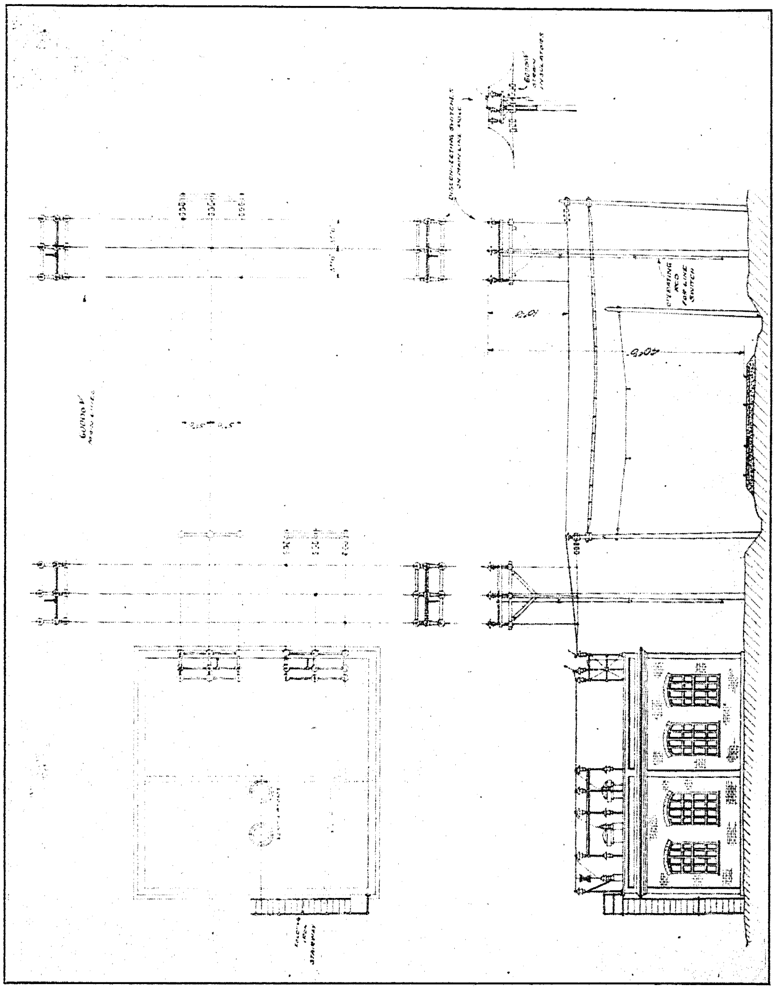


FIG. 1.—General arrangement of 60,000-volt converter sub-station and lines.

for arresters, etc., out of doors is forcibly called to the attention of the designer in dealing with voltages above 33,000 on account of the very large clearances required for the higher potentials. A year and a half ago this led to a design in which practically all high-tension wiring is placed on the roof of sub-station buildings, as in Fig. 1, herewith, showing the development of special

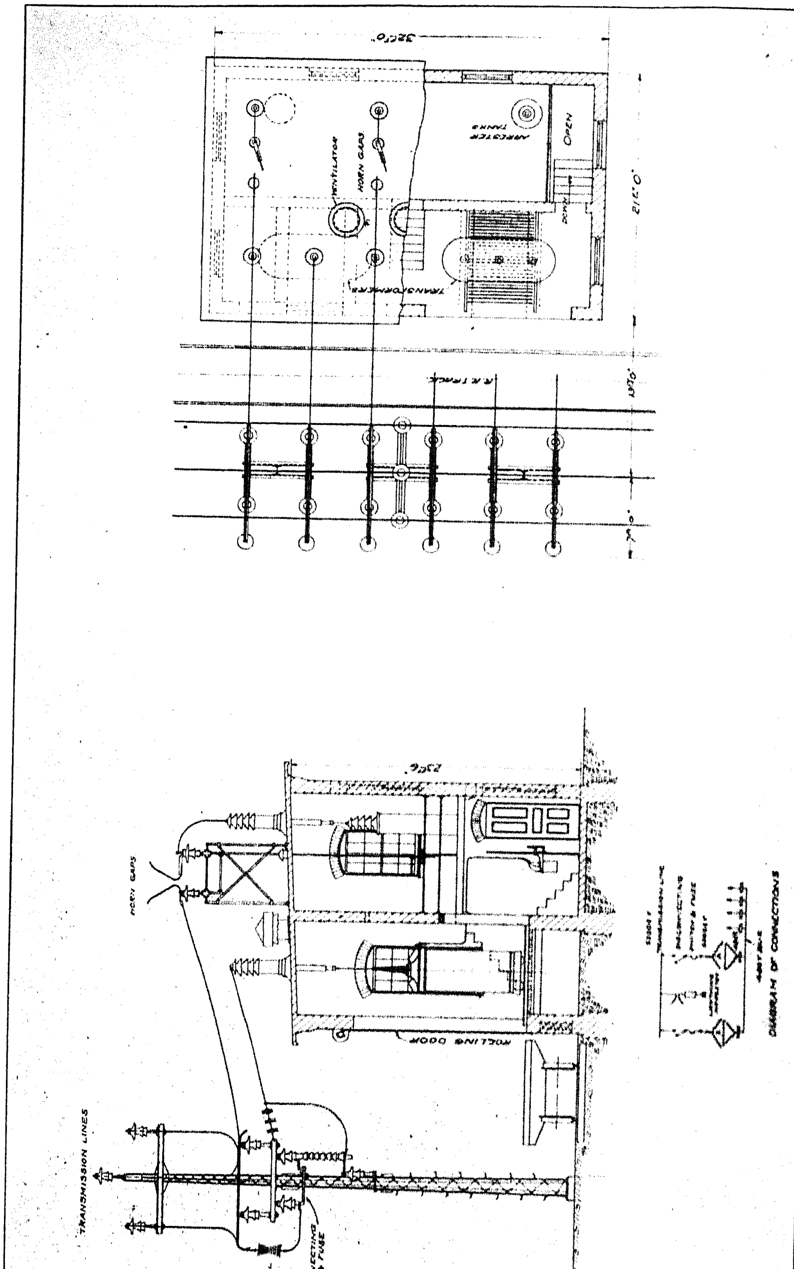


FIG. 2.—General arrangement of 200-kw., 60,000-volt transformer sub-station.

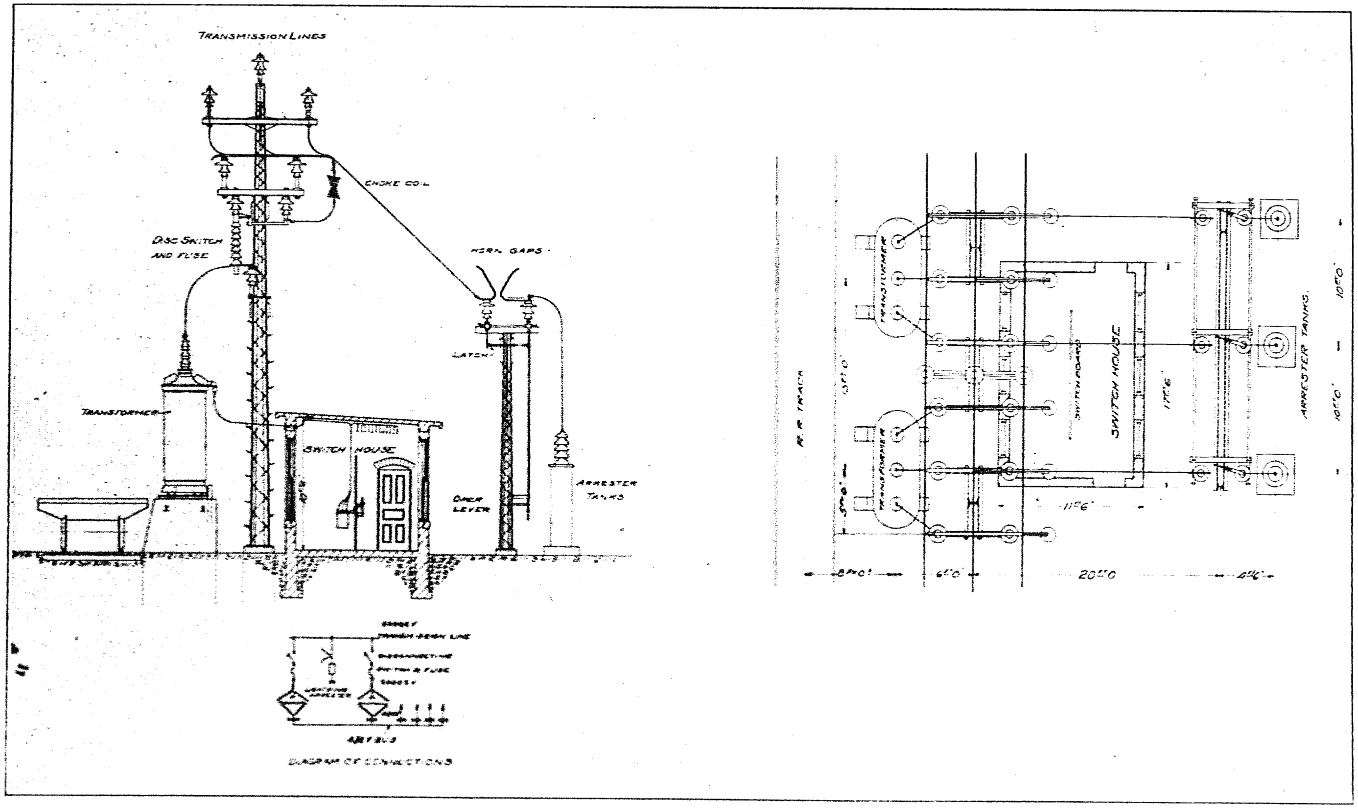


Fig. 8—General arrangement of 200-kw., 60,000-volt transformer sub-station.

weather-proof disconnecting switches for operation from the floor, roof-entrance bushings, etc. Wherever a building is used for the apparatus this roof construction may be considered superior to mounting horn gaps, bus-bars, etc., on a number of poles. Ground space is economized, appearance is improved by reducing the number of poles, and the object of the poles; namely, to raise the high-tension wiring and switches to a safe height, is accomplished while the apparatus is not rendered so inaccessible as when mounted on poles. In fact it is more accessible than if placed inside in the upper part of a much higher building. The horn gaps and disconnecting switches may be operated from the inside of the building. The entire construction is suitable to all climates. Where converters, or other moving apparatus requiring attendance, is to be used, I consider this construction of the outdoor type to the extent indicated, very desirable.

The advantage of making the transformers weather-proof and placing all apparatus except the low-tension distributing switchboard outdoors becomes more apparent in the case of small high-tension transformer stations, which need only occasional attendance and may be protected by fuses on the high-tension side. The fuses require considerable clearance and may well be placed outside. In general, such stations will require some low-tension switchboard equipment, so that a small building will usually be necessary. A relatively large increase in the size of building is required to house the transformers. The use of fuses and the placing of recording watt-hour meters in the secondary circuit eliminate current and potential transformers and make a simple installation possible. Figs. 2 and 3 show comparative designs for a 200-kw. transformer sub-station. It is estimated that steel poles, a fire-proof brick building, and the best construction in all other respects, would show the following approximate costs:

Comparative estimates, 200-kw. transformer sub-station.
One transmission line, no high-tension oil switches.

	Indoor	Outdoor.
Two 100-kw., 60000/480-volt three-phase oil-cooled transformers	$ 5200	$ 5600
Switching equipment, insulators, one lightning-arrester Two 480-volt transformer panels Three 480-volt power feeder panels One 480-volt lightning feeder panel	3307	2847
Freight, installation, wiring of apparatus	1330	1250
Steel poles, cross-arms, and concrete bases	252	386
Building, piers for transformers, excavation, grading, etc.	2740	1022
	$12,829	$11,105

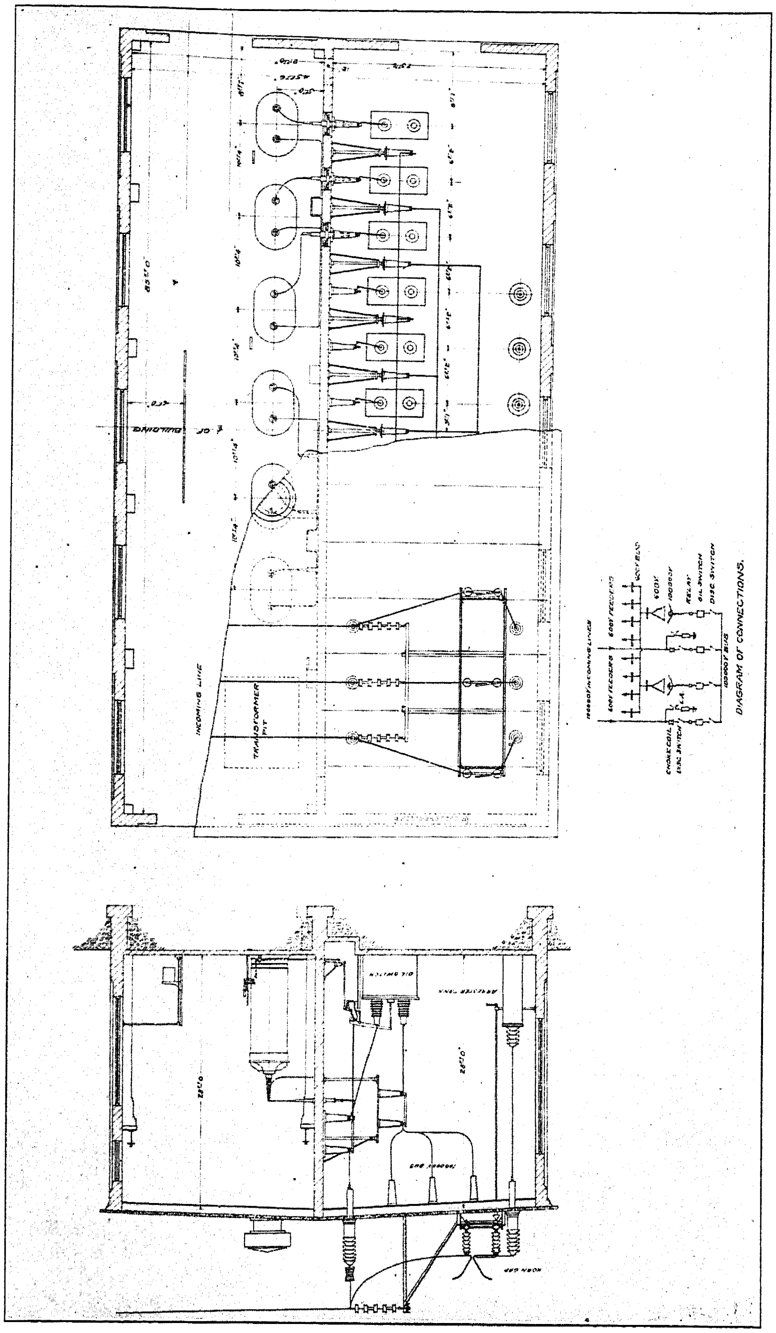


Fig. 4.—General arrangement of 2400-kw., 100,000-volt transformer sub-station.

The outoor construction shows a saving of $1724, or 15 per cent. On the other hand, the ground space required is about 20 per cent greater and the actual amount saved is so small as not to be a determining factor.

It would appear that the higher the line voltage to be dealt with the more advantageous outdoor construction should prove. By way of comparison, and to extend the comparison to much higher voltages than those covered in Mr. Randall's estimates, Figs. 4 and 5, showing designs of both types for a 2400-kw. sub-station at 100,000 volts, are added, together with the following approximate estimates:

Comparative estimates, 2400-kw. transformer sub-stations.
Two transmission lines with high-tension oil switches.

	Indoor	Outdoor
Six 400-kw. 100000/600-volt three-phase, oil-cooled transformers	$25,800	$27,750
Switching equipment, insulators,		
Two lightning-arresters, as per sketches		
Two incoming line panels with high-tension oil switches		
Two low-tension transformer panels complete		
Six power feeder panels complete		
Two lighting feeder panels complete	12,029	13,739
Freight, installation, wiring of apparatus	5700	5625
Steel poles and cross-arms, including concrete bases, steel framing on roof	104	433
Building, piers for transformers, excavating, grading, etc.	12,041	5870
Total Cost	$55,674	$53,417

The indoor estimate includes a 15-ton hand-operated crane and runways.

The small difference in cost—$2257, or 4 per cent—would not be decisive in any case, but the real difference in cost is masked in this design by the use of oil switches, not only for the transformers, but for the two transmission lines as well, and a building of considerable size in either case for these switches. The use of outdoor oil switches or fuses would make the building much smaller and eliminate 24 line entrance bushings. The outdoor type of installation is, therefore, dependent for a satisfactory showing on the placing of all high-tension apparatus outdoors. Merely locating the transformers outside does not accomplish very much in a large station, and, on the other hand, the oil switches may be considerably increased in cost and still show a substantial saving for the outdoor construction. Here again, as in the small station, the use of series relays for the oil switches, air-insulated choke-coils, and instruments on the low-tension side very materially simplify the arrangement.

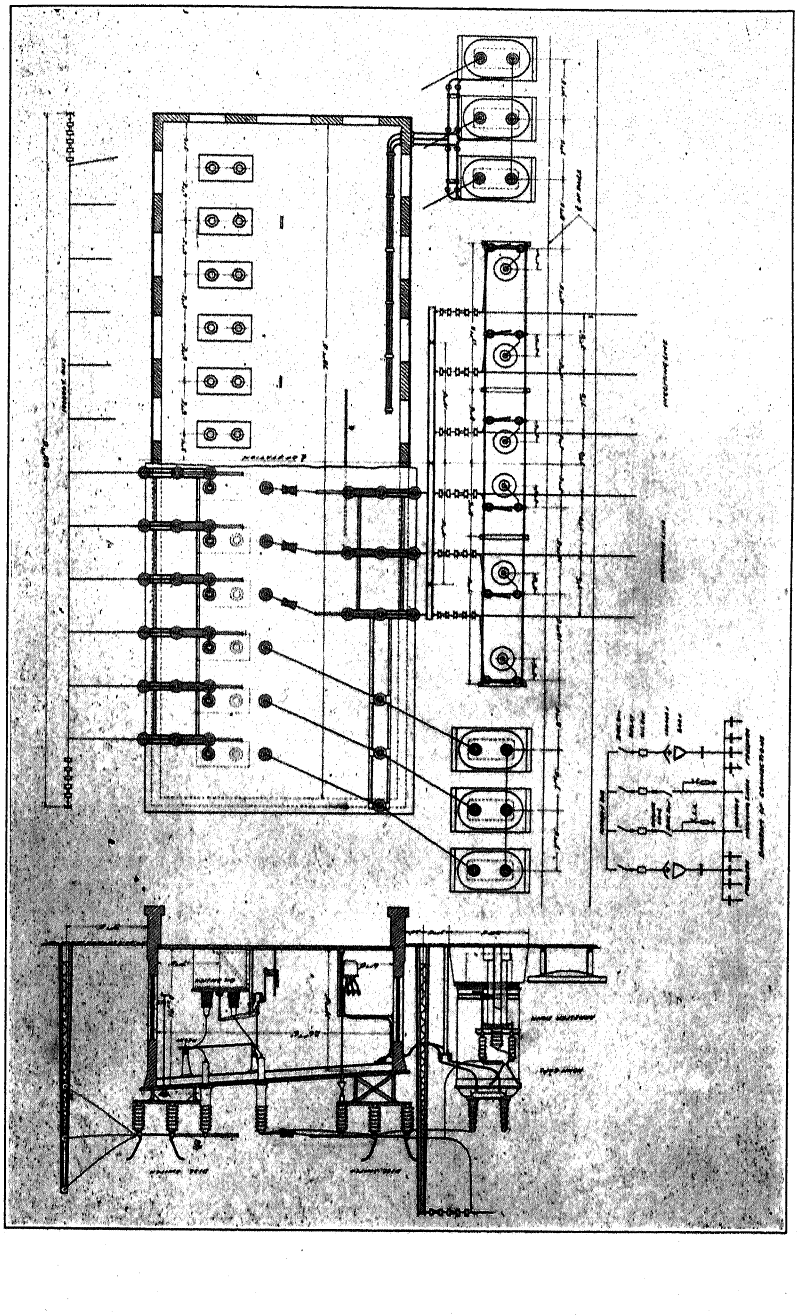


FIG. 5.—General arrangement of 2400-kw., 100,000-volt transformer sub-station.

The necessity for making repairs on the spot may often determine the type of construction. A building covering all apparatus would generally be equipped with a crane, and the facility for repairing is a distinct advantage of the indoor construction. Exposing disconnecting switches to the weather will generally require a more expensive and complicated type of switch to avoid trouble from sleet and to insure safety to the operator. To work an ordinary knife-blade disconnecting switch indoors at 100,000 volts, a dry pole of about 16 ft. in length is considered necessary. To work a switch outdoors at the higher voltages in all kinds of weather, it seems imperative that a grounded metallic mechanism be used.

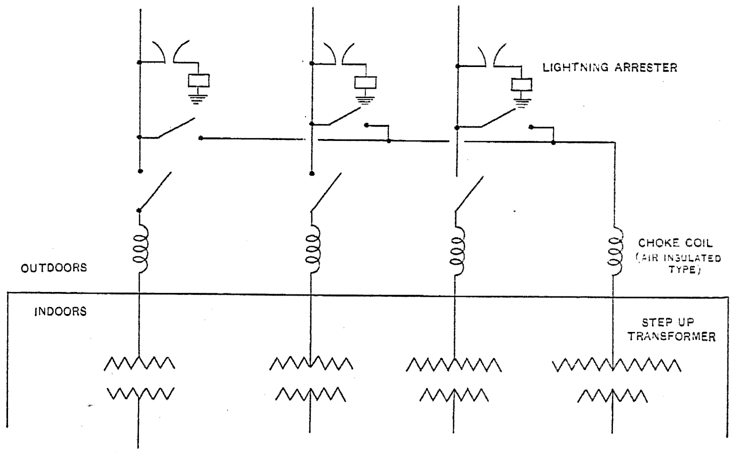
D. Kos (by letter): The interesting subject which has been treated in Mr. Randall's paper apparently thus far has been receiving attention only in America, as to my knowledge no outdoor high-tension switching equipments have been installed by European firms. Though at first sight the proposition looks very advantageous, there are a few points on which I would like to make some comments. When comparing the estimated costs for an indoor station and an outdoor station, it appears that only $200 is set aside for weather-proof coverings of oil switches, while it is estimated that the transformers can be made weather-proof for $1,000, or 6⅔ per cent more. It seems to me that these estimates, especially the first one, are somewhat low.

In addition, for outdoor service a heavier type of insulator must be used, and also an increase of cost must be allowed for series transformers and choke-coils. The outdoor structure is made of wood, which is rather a short-lived material at best. For reliability after a certain number of years from the time of construction, this certainly cannot be compared with inside construction work where no use has been made of wood as a building material. Here we have again the same comparison that was discussed rather often a few years ago when the advisability of using steel transmission towers or wooden poles was considered. Nowadays the general engineering opinion seems to be unanimous in favor of steel towers for very high voltages. The use of a wooden structure for an outdoor switching equipment for very high voltages cannot therefore be considered for a first-class job. These considerations largely alter the aspect, and when the outdoor station will be again estimated in accordance with the above the real difference of cost will probably be rather small.

But even if this difference of cost might amount to, say, more than 10 per cent, it remains to be considered, at least for the generating station, whether an indoor station should be chosen under any circumstances, as the advantage of repairing and overhauling during all conditions of weather outweighs a rather considerable extra expenditure. For sub-stations of small importance, however, Mr. Randall's suggestions should be carefully considered, as here the cost of switch gear per kilowatt is considerably higher than in the case of large sub-stations, so that the question of sav-

ing money versus decreasing facility of repairs assumes quite a different aspect in the two cases. For the generating station, the money saved per kilowatt is a minimum and the importance of reliability and facility of repairs is a maximum, so that at least here an outdoor station seems to be out of place.

It should be added, however, that though it be less desirable to install outdoors the transformers and other oil-filled apparatus, this objection does not hold good for bus-bars and plain disconnecting switches. For lightning-arresters it may even be an advantage. Outdoor stations, to the extent that they are used by Mr. Mershon in the Niagara, Lockport and Ontario Power Company, do not seem to have any disadvantages; but whether it is good engineering to go further along this line, as suggested by Mr. Randall, may reasonably be doubted, except for small sub-stations. For extremely high voltage the auxiliary apparatus, as oil switches, series transformers, oil-choke-coils, etc., become so



cumbersome, voluminous, and costly and introduce so many new danger points in the system, that it is time to consider whether it is not better to do away with all of these as much as possible, and place all switches and instruments on the low-tension side only. For choke-coils ordinary air-insulated coils could be used outside and the end windings of the transformers should be reinforced an extra amount. The high-tension part of the switching equipment becomes therefore as shown herewith.

This switch shows three outgoing lines without oil-filled apparatus on the high-tension side and one spare transformer which can take the place of any of the other three. To simplify the sketch, the scheme has been shown with one line instead of three lines per feeder.

The arrangement is simple and without unnecessary vulnerable points on the high-tension side. Another set of lightning-arresters

can be added on the transformer side of the choke-coils, if necessary. The only objection that can be raised is that there are no series transformers on the high-tension side, so that no separate protection of transformers can be applied according to the Merz and Price method. If desired, the transmission line as a whole, including step-up and step-down transformers, can be protected according to the Merz and Price system, provided that both sets of transformers are connected in the same way; for instance, Y on the high-tension and delta on the low-tension side.

A paper presented at a special meeting of the American Institute of Electrical Engineers New York, April 16, 1909.

FUNDAMENTAL PRINCIPLES OF INDUSTRIAL EDUCATION*

BY HERMAN SCHNEIDER

It is intended to take the size of the problem, the field that the problem covers, and the end to be attained, and then to build synthetically a solution.

The problem. The problem of education, viewed from the points of industrial progress and the stability of the State, is the training of the mass for efficiency in industrial and civic service. The problem is as big as the United States, because of the interdependence of industrial effort, and because ultimately all education must be framed to strengthen the stability of the nation. In other words, the chief end of education is so to instruct the individual that all individuals as civic units will combine to make the best possible State.

In the United States there are over 18,000,000 children in the public schools; over 17,000,000 of them drop out when the law permits them to, and they go into agricultural, commercial, and

*The aim of the Educational Committee is to lay before the Institute members the problems of industrial and technical education. Last year the committee devoted its attention to the problems of technical education. It has been deemed wise this year to present some phases of more elementary industrial training. To this end, Dean Schneider was invited to give the members of the Institute the results of his study and experience along this line. This he has done in a brief paper which is designed to provoke a discussion of the practical aspects of the training of men as mechanics, draftsmen, etc. A plan for applying the coöperative idea in electrical engineering education was outlined in a paper by Mr. M. W. Alexander, at the annual convention last year. Dean Schneider believes that the coöperative principle applies to industrial as well as to technical training and that it can be carried out in a practical manner.

industrial life. In the industrial centers they go into the factories and stores, but up to the time of leaving school they have not received one iota of industrial training that is worthy of the name. The size of the problem may perhaps be understood more clearly if it is stated in this way: if these 18,000,000 children were stretched across the country in a straight line, giving one linear foot of space to each child, the line would reach from the upper end of Maine to the lower end of California. That portion which goes through the high schools would reach across the state of California. The rest of the line from Maine to the eastern border of California is drawn into commercial, industrial, and agricultural life at about the age of 15 years, with no industrial training prior to going to work and practically no schooling thereafter. This proportion holds in any industrial town or city.

Scope of the problem. The writer has heard many discussions on industrial education and has read many papers by very able writers. In almost every case the discussion has been contributed by the members of a society that represents a particular branch of industry, or the paper has been written by a man who feels that the trade with which he is most familiar is the most important of all. Members of the American Institute of Electrical Engineers, for instance, in discussing this problem, would usually not consider the educational situation from the point of view of the master tailor, the telephone company superintendent, the head of a department store, the pottery owner, or the building contractor. Investigation on this subject, covering a period of 9 years, discloses the curious fact that a man in almost any business considers the art of his trade as more difficult than the art of any other trade, and he inevitably insists that there must be trade schools to teach the children who go into his particular field of work. He is insisting at present that the public schools shall include specific training for his trade; and there is a general clamor from individuals and societies representing certain industries for the introduction of training in their particular trades into the public school system. Consider for a moment what this means.

If you take the industrial directory at the end of any city telephone book, and attempt to make a list of the trades therein shown, you will find, in a town like Cincinnati, that when you get to the letter E, you have almost one hundred distinct trades. The proposition of putting equipment into the public school

system of Cincinnati to train the children for all the different trades in Cincinnati would make so top-heavy an organization, and necessitate so many buildings, that the imagination is staggered. Further, in many lines of industrial work the machinery becomes obsolete about every 7 years; if, then, the trades are to be properly taught in the schools, the equipment for the whole system must be changed at least once in 10 years. The advocates of trade instruction in the public school system evade these very essential facts by saying that the more important trades only should be taught. There are two phases to this latter proposition:

1. Who shall decide which are the more important trades? And how shall public support be obtained from all sources for these few trades?

2. Are the children to be taught a few trades, leaving all the other trades neglected, and leaving the predilections of the children out of question entirely? Or will only a few be trained in the more important trades, and the rest be allowed to shift for themselves as heretofore?

Should this be attempted, it would simply be a partial solution, and a very small one at that, of the whole problem of industrial education. If, for instance, we teach in the public schools the plumbing, machinist, wood-working, and moulding trades, what would be the solution for all the children entering the numerous other trades and commercial occupations?

There is, further, the taxpayer to take into consideration. Let us assume that you and I are residents of the same town, in moderate circumstances, and paying about the same amount of taxes. You have a boy and a girl and I have a boy and a girl. Your boy desires to be a machinist, and the public schools will take care of him; your girl desires to be a stenographer, and the public schools will probably take care of her. My boy wants to be a printer. Have I not the same right to demand that the public schools shall teach my son to be a printer, as you have that they teach your son to be a machinist? Shall the public school system say to my boy: you have got to be a machinist, plumber, moulder, or wood-worker, or go without a trade training? My girl wants to be a telephone exchange operator. Will you tell her that she has got to learn her work without any training? Is it fair for the owner of the machine shop to tell the telephone company that it must train its own help, when he demands that the public schools train his help?

In view of the foregoing, it should be obvious that any attempt to put trade schools; that is, schools that teach trades, into the public school system, would result in a tremendous expenditure of money for skilled teachers and for equipment which is not commercially productive. Any attempt to put in a few of the trades would afford only a slight solution of the question. The big problem would still be before us.

The situation. The size of the problem, and its scope, cause us to be confronted by a peculiar situation. In the first place the problem is so comprehensive that there is probably only one organized institution capable of carrying it; namely, the public school system. It ought not to need any argument to prove that private enterprise can not furnish the solution when an average city has 10,000 to 20,000 children to be educated every year, if they are to have industrial education. Figures are available, if they are necessary, to show the futility of private endeavor in industrial education. In the second place, to install a series of trade schools in the public schools is impossible, because of the expense and the inevitable complications.

Let us take account of stock. Every industrial center has a group of school buildings with its quotas of trained teachers. It also has many factories and commercial houses. Under present conditions most of the children who leave the public schools go at once into the industries or stores, and there is no connection whatever between school and shop. The children go to work not because they want to, but because they have to become little bread-winners. Here, then, are many children working in some capacity, and a series of public schools carried on only for those who are fortunate enough to be able to continue in their school work. Since the public school system is the only organized institution capable of dealing with all these children at work, and since the children are also learning a trade and earning money sufficient for their simple wants, it seems that the only complete solution of the problem is a system of coöperation between the schools and the factories for efficiency training and civic training of the young people after they have found their work.

How, then, can this coöperation be built up within the limits of reasonable economy? Fortunately, there are three or four experiments going on which indicate that a coöperative system is feasible in the trades, as it has been shown to be in engineering training. Several of these will be briefly described to indicate the various methods of solution.

The coöperative system. The fundamental principle of the coöperative system is very simple. It is this: the technique or the practical side of the work is taught only in a shop or store which is working under actual commercial conditions, and the science underlying the technique can be taught properly only by skilled teachers. All questions as to who shall supply the school teachers (the shop or the public); the hours the student works, and the hours he is taught; the periods of alternation of shop and school work, if alternating periods be used; and the curricula of the schools—all these are matters of detail to be considered for each particular case.

The economy of the system is at once apparent. In an engineering school, for instance, twice the number of students can be taught at about two-thirds the expense as compared with the four-year theoretical system. The same is probably true in industrial education, for under the coöperative plan the schools will not require any physical equipment; all their money can be used for brains and for buildings for teaching purposes only. There is a further economy to the student. In this case he is earning while he is learning, while under the trade school system he does not earn until he has completed his trade education. Parenthetically it should be stated that no displacement of the present high school courses is contemplated.

At Fitchburg, Mass., the coöperative system is a part of the public school system. The students are divided into two groups which alternate every week. That is to say, this week one-half of the students are in the day school and one-half are in the factories. Next Monday morning these groups will change, and those who are in the school this week will go to the shops, and those who are in the shops this week will go to the school. Since the public school becomes a part of the apprenticeship system, it has a voice in the organization of the apprentice course in the shop, and is *in loco parentis* to the boy, so far as his shop work is concerned. It ought to be obvious that the boy will receive a fair training in the shop because the school is in a measure watching over this phase of his work.

It is not intended, of course, that this plan should apply only to factories. It will apply to a boy learning the tailor trade, the butcher trade, the baker trade, or any other trade. It is necessary, however, to obtain two boys to alternate in the shops.

The course in Fitchburg is of 5 years' duration. At the end of that time the student has been taught the simple science

underlying his trade; he has been taught shop mathematics; he has been given a certain amount of cultural work; he has become a fair mechanic. In brief, he is at the beginning of specialization.

It has been found further that the money earned every alternate week, and for full time during the summer, is sufficient to pay for the simple wants of these children. It is also true that there is seldom an instance when a child is hindered by financial conditions from taking such a course.

A number of changes in the system of administration have been found necessary, and these details will be considered to show certain phases of the working of the system. In the first place, the grade school principal is responsible to the school authorities for every boy in his district. When the boy leaves school at the legal age, the grade school principal is required to follow him and ascertain what he is doing. He then consults the parents of the boy and the boy's employer, and the question of the future of the boy is immediately shifted to them. The employer is urged to enter into a combination with the school in the effort to make the boy more efficient at his work, and to give him a little more general education than he would otherwise obtain. The fact is impressed upon the young man and his parents that he ought to make a choice of a trade for life, if possible. If it is found after he has begun work that he has no aptitude for that which he has undertaken, an attempt is made on the part of the school principal, the parents, and the employer of the boy to find the kind of work for which he is best fitted. As some one has expressed it:

> Under the coöperative public school system, the grade school principal pays much less attention to biology and much more to the boy.

The second feature is the work of shop coördination. In order to explain this, the method of operation in the Engineering College at the University of Cincinnati will be used as an example.* The coöperative engineering course at the University of Cincinnati is of 6 years' duration. There are, therefore, six classes, and there is one shop coördinator for each class. The shop coördinator is a college graduate acquainted with shop practice. He spends every morning at the university and every afternoon in the shops. His function is to make a direct weekly coördina-

* This feature is just being introduced and its effectiveness cannot be stated at the present time.

tion of the work of the shop with the technical work of the university. This afternoon, for instance, he may be at the shops of a local manufacturing company, where he will observe the student apprentices at their work. He will know what they are turning out, their "speeds, feeds, and cuts," the angle of the tool, how the batch of work is "ticketed," how the work is set up, the power drive—everything important in connection with the operation. Next week these young men will be grouped together with their classmates for two periods in class, when he will explain the functions of the particular articles on which the students were working in the machine which the local manufacturing company builds.

He will take up all questions of speeds, feeds, cuts, accuracy, etc.; figuratively speaking, he will take from the student apprentices the blinders which would restrict their vision except for this explanatory work. The "ticketing" of the batch of work is gone into, the system of shop routing is explained. Ultimately, all problems of shop organization, shop accounting, cost keeping, shop planning, power transmission, heating, ventilating, lighting, etc., are discussed during the 6 years' course. In conjunction with this, a card system is employed, by means of which everything the student does in the shop that exemplifies a theory taught in the university is called in detail to the attention of the teacher of the theory, so that when the student comes to that particular theory the exemplifications which he has had in his practical work in the shop are called to his attention. It will be seen, then, that out of the student's own experience is drawn much of his course in mechanisms, thermodynamics, machine design, strength of materials, etc.

In the coöperative work in the public schools this close and immediate tie is taken up in a modified way. There will be a shop coördinator for such trades as have a fairly close relationship, rather than by classes, as in the university.

Now in order to show the application of the fundamental principle of the coöperative system mentioned above, but with a deviation in detail to meet a special condition, let us consider the department store. Such a course has been devised and is about to be put in operation. The fundamental principle is strictly held to, but the details are modified to suit the conditions. In any department store, the clerks are not usually busy until 10 o'clock in the morning. A store can get along easily with one-half its clerks from 8 until 10 o'clock. It is

intended to divide the force into two sections, one-half of whom will receive instruction this week from 8 until 10 o'clock, while the other section is working; the following week the sections will be changed about. In this particular instance the students do not go to the public schools; the teachers go to the store. It is evident that it is easier to transport 20 teachers than to transport a large number of student-clerks. A number of rooms in the store, such as carpet rooms and lace rooms, are set aside during these two hours for the class work, the chairs being removed at 10 o'clock, and sufficient space being reserved for any business which way be necessary up to that time.

It is contended by department store owners that salesmen should know the psychology of salesmanship, and have a fairly expert knowledge of the things they are selling. They should receive, besides, a certain amount of general education. The psychology and the general education are to be taught by trained teachers from the public school system. In order to teach the practical end, the following method has been adopted. Consider, for instance, the shoe department. If one pair of shoes costs $1.85, and another pair costs $1.95, the salesman should know where the difference of 10 cents' value lies. Let us assume that this particular department store buys shoes from a firm in Brockton, Mass. When it makes its next contract for shoes, it will insist that the firm selling the shoes send an expert demonstrator to its store, to explain in detail all the different successive operations in shoe making and all the different elements which make differences in cost. The tanning firm from which the shoe manufacturer buys his leather will be required to send to the store an expert to exemplify practically to the students the different grades of leather in a hide, methods of preparation, and why one kind of leather is used in one part of a shoe, and another in another part of a shoe. It has been found that the shoe manufacturers will very gladly enter into any scheme of this sort. He would in fact be a very short-sighted manufacturer who would not. This same general idea is followed in all of the other departments, such as jewelry, linen, silk, furniture, etc.

Other instances might be cited of experiments now in operation based on the simple principle of the coöperative system, with changes in detail, but the scope of this paper will not permit their consideration here.

It is intended to call particular attention to the fact that under

this plan the public school deals with the whole general problem, deals with it without physical equipment. It is, in other words, a return to the old apprenticeship system with something added; namely, definite instruction in efficiency by trained teachers, and additional instruction aiming toward better citizenship.

The writer realizes that many manufacturers will say that a scheme which brings two sets of students into any work on alternate weeks—assuming that the alternate-week system be used—is not practicable, and will result in demoralization. The only answer to this is that three years of operation at Cincinnati, and shorter periods of operation elsewhere, have demonstrated that the criticism is untenable. Two alternating students work very well together and there is no confusion. Emphasis is laid upon this, because before the coöperative system was started this objection was the greatest obstacle in the way of its adoption. Men who were skeptical about this phase of the work have become enthusiastic over the smooth working of the system after a fair trial.

It is with hesitancy that a certain feature of industrial education is touched upon at this point, in view of the many propoganda that are being made in favor of it. There is a belief that there will be a tendency to swing too far in the "practical" direction, and that the impetus now being given to trade education may result in a too material training of the youth of the country, injuring to a certain extent that phase of education which should tend to the training of all civic units for civic service. In other words, the fact that every unit is a citizen of a republic, and that the republic depends for its stability upon the character of the mass, must not be overlooked. Fundamentally and primarily, as stated at the beginning of this article, the training of men to become citizens of a republic must be a vital part of any worthy system.

The object of all education is to make good citizens, and the first duty of a good citizen is to earn his own living. The second duty is to be a good citizen in the civic sense. All education, whether industrial or otherwise, should be directed toward these two very important phases of our national life. Here again is a very strong reason why the whole problem of industrial education should be solved in the public schools, where there will be the check of conservatism as to the character of the work, and where the whole solution will be under the direction of men who are servants of the public, and who naturally and

by tradition look upon the problem with the widest and safest vision.

The writer has purposely omitted going into definite details, curricula, etc.; for instance, as to how a student should be instructed in arithmetic, algebra, and chemistry. At the present time these do not constitute the problem, and even if they did, enough work has been done in different kinds of schools to demonstrate clearly the character of instruction which ought to be given. The problem confronting us is the method of comprehensive action for the whole situation in the United States. We cannot adopt the German system, for the simple reason that the social stratification in Germany is horizontal, while in this country it is vertical. That is to say, the problem in this country is to find the highest elevation which any man's ability will permit him to reach, and to get him to that elevation. It must be evident that with this system in general operation in the public schools, admission to the university would be by natural selection. In the future, when the system becomes general, only those men will receive the higher education, who will make the best use of it, and hence will give the greatest return to the State.

The writer believes that the American Institute of Electrical Engineers, a national body of men whose personal interests are dovetailed so closely with the whole industrial welfare of the country, can do nothing better than to stand for a movement involving a broad, systematic, and thorough industrial and civic training under a national system.

Discussion on "Fundamental Principles of Industrial Education." New York, April 16, 1909*

Harry Barker: All of us, interested in educational problems, welcome any broad exposition of the aims of industrial education, and of the development of courses that will appeal to the good sense and democratic ideals of the American people. It is fortunate that Dean Schneider has diverted our attention from the details of mere "industrial training" to the broader aspects of "industrial education." When we discuss specific curricula we easily get into fruitless disputations over the distribution of attention between cultural subjects and practical training.

We admit the need of improving the schooling of the coming generations, not only of mechanics and artisans, but also of business and professional men. The first group seems to need a better grasp of the possibilities of some trade which will yield a living, a better introduction to certain ideals of that living, and a cultivation of the imagination to illuminate and inspire the ideals. The younger members of the second group need a closer contact with many practical things and more knowledge of workers, and the conditions under which work is actually done.

The broader the view we take of industrial education, the more comprehensive becomes that term, until finally it stands as one expression of a tendency of the whole system of American public school instruction. The educational method of the future seems likely to become, as President Eliot has so well said, a teaching of how to do by doing—not by reading—an education that is more than memory training. In making courses for future mechanics and artisans, the effect on possible courses for future business and professional people may well be considered at the same time. There seems, in fact, no need for a strict classification of the education of such classes.

The poorer a child's parents are, the more utilitarian needs his education to be, that he also may quickly become a breadwinner. This means a most practical equipment for advance at some trade, and a corresponding lesser emphasis on the so-called cultural side. The more money and time there are available for the child's schooling, the greater the attention we can pay to this side, and the later perhaps can we postpone the utilitarian training.

Two ideas should stand out clearly. First, with the poorer children we should not utterly neglect art, literature, civics and the awakening of the imagination to throw color about dull work. Secondly, with the children in better circumstances, we should insist on attention to more manual or industrial training, not stating exactly how or when this should come. This will help to make better citizens, broadminded workingmen, and more sympathetic business and professional people.

*C. E. Dooley's contribution to this discussion is contained in his paper "The Training of Non-Technical Men," pages 1087-1093, Proceedings A. I. E. E., August 1909.

A few words on the relation of this training to that of the future professional men, especially engineers. Most such young men must attend college. Under the inflexible entrance requirements which we must unfortunately expect for many years to come, these popular industrial courses cannot develop literary, historical, and mathematical subjects to such an extent that a student completing a machinist's course, say, would be admitted to the freshman year of a mechanical engineering school, no matter what his ability and inclination. If ambition urge him on, he must be able to continue under some coöperative plan to complete the studies that satisfy the entrance requirements. The work and study for becoming a desirable wage-earner will help to make him a good engineer, but it cannot prepare him for higher education through the usual channels. Therefore, teachers in all industrial courses need to be especially alert, so that a child who exhibits exceptional ability, inclination, dexterity, and ambition, may be encouraged to aim at something higher than a mere occupation. Perhaps in many cases the poorer boys can be helped to begin a college preparation without the loss of a year or more in first completing the regular trade course. A comparison of actual and proposed industrial courses and of standard college entrance requirements seems to show that this is the only way in which the statement near the close of Dean Schneider's paper may be fulfilled. He says:

> It must be evident that with this system in general operation in the public schools, admission to the university would be by natural selection.

One possible effect of the spread of industrial courses will need to be guarded against; that is, the drawing into wage-earners' courses of boys who do not need to sacrifice the greater benefits of present and future high-school training—boys who do not really have to earn as they study, or who do not have to become wholly self-supporting at, say, 17 years of age. It seems a reasonable rule to impose that every student should be kept from making greater concessions to immediate circumstances than are imperative. The great number of applicants for entrance to the few industrial school courses now in operation seems to indicate such a tendency.

There is one point on which Dean Schneider ought further to inform us. He states that 17,000,000 school-children have to go to work as soon as the law allows them. This is 95 per cent of his estimate of the school-children of the country. In the short time available I have not checked these figures thoroughly, but Dean Schneider's source of information does not seem to agree with the U. S. census reports. In 1900, of all persons in this country 10 to 14 years old, 80 per cent were in school, and of all persons 15 to 17 years old, 42 per cent were in school. The deduction is that less than half the school-children went to work at the minimum legal age. It is noteworthy, too, that these percentages are found 10 to 12 per cent more favorable, to the children in manufacturing states and less favorable in

states like Virginia, South Carolina, Oklahoma, and Arizona, where there is less manufacturing. In this connection it is interesting to note that in 1900 only 24.3 per cent of all persons in gainful occupations in this country were in manufacturing and mechanical pursuits. This incidentally supports Dean Schneider's contention that the courses must be broad rather than special, so far as the public school teachers are concerned.

The best educators demand that all possible attention shall be paid to preserving the "play-spirit" of those who come under any scheme of industrial education. It is hard to give them as much training as is needed without filling their days and months so full of work that they lose the opportunity, and later the ability, to utilize those relaxations and recreations that are wholesome and refreshing for the body and the mind. Are we justified in taking a boy of from 13 to 16 years of age, and working his mind and body for six days a week throughout the year, with a scant vacation of two weeks? What comes from the lack of air, sunshine, and fun during the years of adolescence? In reply to such cautions we are often told about the relief and relaxation that come by the mere insertion of class hours between shop-periods. That is undoubtedly true to a certain small extent, but by a little more attention on such a point, the production of a still better kind of "civic units", as Dean Schneider says, will be possible.

The spread of "industrial education" may be slow, but there is an immediate field of work needing attention. Success here will pave the way for the industrial courses. Our schoolboys need *systematic* watching to see what abilities are cropping out; they need to be put to work to prove or disprove the existence of natural bents. Those now best able to exercise such surveillance are teachers in public high schools, but some of us feel that they are not quite properly prepared for giving the most intelligent attention and advice. A great and increasing number are college graduates with some knowledge of the natural sciences, and often having had intimate contact in college with engineering students. Such teachers will be able to grasp the desired methods easily, and they have some excellent pedagogical associations through which they may be reached.

Arthur D. Dean: A great deal of interest is now being taken in the subject of industrial education. It presents a newer phase of education. In the past we have been confining ourselves to the "citizenship" idea, a civic idea which has always held true and always should hold true; but we are bringing in, as we should, the self-supporting and economic purpose in education.

President Butler thinks it is a grave error to set vocational training and liberal training in sharp antagonism to each other. He says:

The purpose of the former is to pave the way to some appreciation of the latter, and to provide an economic basis for it to rest upon. The

equally grave error of the past has been to frame a school course on the hypothesis that every pupil was to go forward in the most deliberate and amplest fashion to the study of the products of the intellectual life, regardless of the basis of his own economic support.

Dean Schneider says that education is very largely, but is not entirely, a public function. I believe that it is the duty of the state and of local communities to carry on industrial education. Of course private institutions can train their own workmen, but it is primarily the duty of the state to educate its own children. Dr. Draper, Commissioner of Education of the state of New York, says that this plan is the only one which holds out an equal chance to everyone; it cannot, in the nature of things, be left to private enterprise. It cannot be dominated by any forces which are in the least exclusive. American workmen are not willing to depend on philanthropy; they will not widely accept the training schools set up by the manufacturing corporations. They are entitled to much the same rights as those that are already granted to the professional and employing classes. They know that they are entitled to equal educational opportunity and will exact what belongs to them. Whatever is done they want done so completely as to command the respect of the best skilled workmen. They will tolerate no false pretense about mechanical skill, but they will be glad to shorten the time in which their boys may become real journeymen. In any event, they know very well, at least their leaders do, that when these things are so they will have to accept them. All this can come in no other way than upon the basis of, and in association with, the public schools.

The New York State Education Department is encouraging local communities to avail themselves of the new law relating to industrial and trades schools. The state of New York is committed to the policy of industrial education. The Legislature of 1908 passed a law providing for the establishment and maintenance of general industrial and trades schools in cities and in union free school districts. For each of these schools which has at least 25 pupils and employs the full time of one teacher, the state will make an annual allotment of $500.00 to the Board of Education and an additional $200.00 for each additional teacher.

Those that are charged with the carrying out of this law are not committed to any one plan. There are several excellent plans. Dean Schneider's plan is only one; the public trades school, without coöperation with the factory is another. It is too early to prescribe rigid general rules to which all must conform. For the present each community will have to be treated as an individual community, and for some time educational necessity will demand plans flexible in nature, plans that can be modified as the need appears for changes based upon a large experience.

I have spent the last 9 months travelling over the state of New York and have met hundreds of teachers, many manu-

and public spirited men and women. I have yet to
onents of the new movement. The State Education
nt recognizes that these industrial schools must be
ough to provide: first, for those that can attend school
second, for those that work a part of the time in order
living but can afford to go to school part of the time;
those that work all day but can attend school at

ial education is a new thing to some people in
. A good many people are asking what the funda-
are in education and urging that we get back to
ome school superintendents suggest that the part-time
possible solution of a serious school problem. They
e fact that large numbers of children are burdened
with the necessity of bread-winning and that they are
o it with no suitable general training, and not sufficient
physical maturity. A certain number of manufactur-
t seem to favor the part-time idea; they say that they
ant to be bothered with boys and girls who will work
day in the factory and go to school the other half.
chnical difficulties in the way of maintaining a half-
em in all lines of industries are evident. I can see
plan might be successful in Fitchburg, and I can see
ight be a failure in the papermaking and textile in-
n New York state.
ly in the future we will rise to our opportunity either
state laws, force of public opinion, or individual willing-
he part of manufacturers, to have a system of con-
schools similar to those of Germany—a system that
it possible for the children to be dismissed from factory
r a few hours a week in order that they may attend
hools and receive training which will make for true
p in an industrial democracy. When this is done I
see it a part of the scheme of public school education.
blem of industrial education is a public affair. Of
includes the interests of the manufacturers, but pri-
is in the interests of the workers. I am perfectly
see the public schools take the boys and girls a part of
and to have the shops take them a part of the time;
the schools give shop mathematics, business English,
ounts, mechanical drawing, etc., while the private
ve the necessary shop practice. As a public official,
know what the boys and girls are doing in the factory;
be sure that the work that they are doing is educational,
shop schemes are educative ones, that the boy is being
d in his shop training. I expect that the coöperation
the shop and the school will be so complete that the
hool authorities will receive reports of what the boys and
doing in the factories, how long they are kept at certain
s, the variety of their work. I believe that true co-

operation between the public school and the private shop will be worked out only when local or state school officials are allowed to go through the factory and see how well the work is being done from the standpoint of the development of the young worker. The schools can provide the theoretical work which accompanies the practical knowledge, but for the school to do its share without being sure that the factory is doing its share, is fundamentally wrong.

In this matter of industrial education we are in the midst of a great task. We are working out the basis and the details of the greatest industrial democracy in history. Let us lose nothing of our good humor; let us cultivate toleration of opinion; and let us think clearly, with an open mind.

C. E. Downton: The company with which I am associated has made several attempts to evolve a comprehensive apprenticeship system for trade apprentices. We have a two-year engineering apprenticeship course for technical graduates, and a four-year course for the various trades. I shall speak of the latter course this evening.

The proper training of apprentices for the trades is a serious question with us, as it is with all industrial enterprises. A concerted effort must be made to bring about a comprehensive method that will be universally recognized as practicable, else we shall, in the near future, experience a dearth of skilled mechanics and artisans which will place the manufacturers of this country at a decided disadvantage with those of foreign countries, where this matter has been given attention for years. It will not suffice to train men to handle certain machine tools; they must be given a general knowledge of a trade as well as some mental training that will enable them to use the brain to better advantage. They must be taught to guide their hands more intelligently.

The public schools do not meet the requirements of the trades, nor can they be expected to. The problem is entirely beyond the scope of any public school. The factory must therefore assist the schools through coöperative methods, or the manufacturers themselves must establish an educational system supplemental to the shop apprenticeship courses adequate to the mental development required for each particular trade associated with the product manufactured by them.

This condition can only be brought about by an effort on the part of all manufacturers, each carrying his share of the burden. It would not be fair to those willing to spend time and money in the training of apprentices to have others appropriate the benefits. The distribution of apprentices throughout a large works, left to the care of each foreman, will not bring about the desired results. The apprentices must receive closer supervision than can be expected from the average foreman, whose first interest is large production and low cost; for the tendency would be to keep the apprentice confined to one process for too

long a time. Increased production, for which we are all striving, does not go hand in hand with the proper training of mechanics; for under these conditions the boy is apt to become a specialist rather than a master workman.

To carry out an educational scheme to work satisfactorily with such a distribution of apprentices is a difficult task. The work is so varied in character, and the boys are so scattered, that the problem is so complicated as to seem beyond solution. The most sensible and the simplest plan is to provide a separate department in the works where the primary object is the proper training of apprentices. The boys should be under the direct supervision of instructors who have been chosen not only because of their skill as mechanics but also for temperamental qualities and knowledge of boy nature.

An educational scheme suitable to the requirements can be carried out without friction or decrease in efficiency of any of the producing departments. An excellent example of such a scheme is that in force at the Lynn works of the General Electric Company described in the paper presented by M. W. Alexander at the Atlantic City convention in 1908.* We have introduced the Alexander system in our works on a small scale, and it has proved very satisfactory. We have not begun the educational part, but we are contemplating doing so in the near future. We have provided school rooms for the purpose.

Chas. P. Steinmetz: In former discussions the Institute has dealt with the problem of college education, and if the college of to-day is superior to the college as it existed in America eight years ago the Institute probably can claim some credit for it. This year the plan was to cover the educational work outside of the college and that preceding the college. While a number of interesting and important suggestions have been received they do not appear to be a conclusive review of this feature of the general educational problem. They cover a wide field, far wider than any the Institute has ever attacked before, so that in discussing the subject perhaps it will be advisable to analyze the situation first.

Society says that safety requires every citizen to have at least a certain minimum amount of knowledge of the subjects important in daily life, and proceeds to supply this knowledge in the education given by the elementary schools. Every future citizen must be made to acquire at least this minimum amount of knowledge. But preferably the citizen should know a good deal more; he ought to be informed on many other subjects. To those who can afford the time this education is given by the high school and the college, and we speak of men who have gained this broader and wider knowledge as educated men. This, however, is not sufficient for the work of life, but in addition to the general education there must be a training in some particular industry.

* TRANSACTIONS A. I. E. E. Vol. XXVII. Part II. p. 1459.

If a boy learn the art of brick-laying or boiler-making, we do not call that acquiring an education; it is called learning a trade. If, however, the boy is sent to an electrical engineering school he is supposed to be getting an education. But becoming familiar with electrical engineering is not getting an education; it is learning a trade; and does not differ essentially from the learning of any other trade, as that of machinist. Calling it a profession, is merely giving it a different name. The work of our electrical engineering colleges is twofold: they devote the first two years to completing the general education, and the last two years to teaching the trade—or profession, if this term is preferred, of electrical engineering.

When discussing the education of an engineer, there should be kept in mind these different and somewhat antagonistic phases in the preparation for life's work: 1. The education which gives general information on all subjects on which the human being should be informed, which has the effect of broadening the mind. 2. The training in one particular subject, whether called trade or profession, which of necessity must be narrowing, as it deals with one subject and devotes all attention to that. Though this special training lowers the ability to judge impartially of the relative importance of things, it is necessary because it is impossible for one individual to master the details of more than a very small part of human knowledge. Therefore, in order to reduce as much as possible the harmful effect of its narrowing action, we must have as broad a general education as possible. Now, since we do not call the learning of the trade of boiler-maker or blacksmith an education, we have no right to call the special instruction in electrical engineering, in architecture, etc., an education. The term education can properly be applied only to general and broadening instruction, not to the specific training in a profession or a trade. Though this training is necessary for life's work, it is rather antagonistic to true education.

I agree that it is time now to realize that reading, writing, and arithmetic are not sufficient, that there are other subjects concerning which the intelligent mind—whether it merely receives the elementary education or the education is to be continued to a college course—should know something else, food, shelter, clothing, etc. If this is called industrial education it is legitimate, and probably the educational system of the country should be modified. But the teaching of a trade is not within the purpose of the elementary school: the time of study in the elementary school is altogether too short to withdraw any part from the broadening effect of general education to devote it to the learning of a trade or business. If this be the purpose, if industrial education means converting the elementary school or the high school into a trade school, or a combination of trade schools, it is a serious, a vicious matter.

Let us take up the second subject, the learning of a trade,

or the studying of a profession, whichever it may be called. In this the college fails to a considerable extent by being too theoretical; the ordinary apprenticeship fails by being only practical and not theoretical. In learning a trade we must consider that there is both the practical side and the theoretical side. Theoretical knowledge and practical experience are both required by the brick-layer or the boiler-maker as well as by the electrical engineer or the architect. The weak point of our present methods of preparing for life's work is the failure to supply the proper proportion of both theoretical knowledge and practical experience. In the college education, energetic attempts are made to supply the deficiency of practical experience, either by introducing more or less inefficient shopwork in the course, or preferably by an alternation of the theoretical training in the college with practical experience in the factory. The same conditions probably also apply to all the other professions and trades. But the college takes care only of a small fraction of the educational work of the country.

Thus far consideration of all the other educational agencies has been almost entirely neglected by the Institute. It is important for the future development of the country that the various agencies of education should coöperate—the college, the trade schools, correspondence schools, apprenticeship courses, etc., and that the proper assistance is given them by us.

It is only a very small minority of young men that can get a general, extensive education in high school and college. The majority have to learn a trade to earn a living and are confined to the limited education of the elementary school. Where they desire to advance, to get a broader education and learn a profession higher than the trade which they have acquired, they have to look to the trade schools, industrial schools, and correspondence schools. By these an enormous work is being done in the industrial cities of the United States. It is a work that cannot be done by a college. At present we have no systematic provision for the education of young men whose necessities have thrust upon them the need of earning a living. Here the supervision of the Institute should be exerted, and it is the field which I consider offers the greatest promise for the Institute's activities.

W. S. Franklin: Professor J. McKeen Cattell has recently said* that perhaps it would be well for us to "scrap" our entire educational system and organize it anew. Being inclined to the same opinion, I would ask Dean Schneider why he says, parenthetically, that no displacement of the present high-school course is contemplated in his scheme of coöperative industrial education. For my part, I think that the greatest obstacle in the way of establishing industrial education is the school system that is now fastened upon us. The expression "educational mill" has been used by one or two of those who

* *Popular Science Monthly*, March, 1909.

have discussed Dean Schneider's paper; I think it would be more fitting to use the expression "educational windmill"; for I believe that nearly all serious-minded men are convinced that our present system of education accomplishes but little. I do not mean to apply this term educational windmill to a narrow group of educational institutions, but to all alike; and I will attempt to justify the expression by referring to some of my own work.

I have been simplifying my instruction every year for many years. Last spring I taught elementary mechanics to 190 freshman engineering students in the simplest way I knew; in particular, I discussed minutely the practical aspects of work and energy, giving what was intended to be a thorough drill in problem and laboratory work. At the end of the half-year I gave this problem:

A cart moves due northwards at a velocity of 6 ft. per second. A mule pulls straight northwards on the cart with a force of 90 lb., and a man pushes straight downwards on the cart with a force of 180 lb.; how fast is the mule doing work and how fast is the man doing work?

Of the class, 43 per cent calculated that the man was doing 2 h.p. and the mule was doing 1 h.p. Now I do not consider it the fault of my own teaching that 43 per cent of the class should be so lacking in appreciation of the practical aspects of a problem of this kind. I believe the fault lies in the fact that most of our instruction in elementary school, in secondary school, and in college is too highly elaborated; in brief, that we teachers persistently ignore the plain and simple and fundamental aspects of things. Educators have apparently reached the point where they go through motions and ignore results. I certainly believe that a first necessity in the establishment of an effective system of industrial education is to ignore what professional educators have to say on the matter.

I am one of those who believe that our common-school system should be revised from beginning to end. And I think that the vague idea as to the desirability of "keeping children in school", which has been conceded by nearly every speaker this evening, is ridiculous. This concession does not seem to me to be based upon a clear appreciation of the value of a common-school training; it appears rather to be a concession to popular pride in the common school as a thing which sooner or later is destined to educate all of us so thoroughly that no one will be obliged to work. That very certainly is the spirit of the common-school system of to-day, and it is the essence of the popular belief in the system.

It seems to me that anyone who has a keen appreciation of the tremendous importance of industrial education must thereby have acquired a degree of contempt for our present system of education. Of course no one in his right mind wants to put boys into shops and work them to death—although many men not in their right minds not only wish to do this but actually

accomplish it on a grand scale—but young boys must be given something to do that will develop them; that is certain, and many of our most thoughtful men and women are beginning to realize that the common school does not meet this condition.

What, essentially, is any great industrial establishment, the Bethlehem Steel Works, for example, but a school. For unless men are trained to do the work our industries cannot continue. Every workshop and every industrial and trade establishment in the country, to the extent that it is a school, should be used as such. I believe that the proper utilization of the conditions of trade and industry, especially in our populous centers, in a rational system of practical education is the most important educational problem of to-day, and I believe that its solution lies in an intimate coöperative organization of the common-school system with every industrial and trade establishment where boys can be properly employed. In a practical sense nothing has educational value but work, and nothing else but the actual conditions of our practical life can shape the coming generation for their work in the world. It is, I believe, extremely absurd to attempt to elaborate an artificial substitute for industrial conditions in the detached manual training school or in the detached industrial school, except to a very limited extent and as a rudimentary beginning in the initial training of the youngest students.

Many men seem to be shocked at what they consider the materialistic point of view of those who advocate steel-mill and cotton-mill versus windmill education; but let these men consider whether a materialistic point of view may not after all lead us infinitely further and higher than any fanciful system of windmill education.

John Price Jackson: The great educational departure that is being tried at Wisconsin has not been touched upon yet at this meeting, though in many ways it is one of the greatest experiments in education in progress. Under the direction of the state and the University of Wisconsin, Director Louis E. Reber has established extensive courses which are intended to reach all classes of people in all walks of life. Thus the state sends qualified teachers to manufacturing establishments to teach classes of an industrial nature, or to other communities to teach such subjects as may be called for; correspondence, special classes, educational trains, etc.—all are utilized. I do not know the value of this movement, but there seems to be no doubt that it is in the right direction.

The great bulk of our young men and women, after they leave the public grammar school, cannot go to any other school; a few only are able to go to higher grades. My feeling is that, in the early years, the schools must adhere closely to the "three R's" which form good practical education. The pupils must be taught the proper use of the mind, hand, and eye; how to build up a good body; and to appreciate some of the higher expressions of

life and thought. *After* such preliminary preparation, *not with* it, may come specialized training. Many boys because of environment cannot reach the industrial school. Therefore, to reach those who cannot go to a trade school, send a teacher to the factory, or store, or railroad—give them, as well as the more fortunate, an opportunity to develop. Dean Schneider's coöperative plans are excellent for the purpose, but they must be applied with such judicious care that they will not interfere with the early school work.

I have said a word or two in my written discussion, appended hereto, in regard to Dean Schneider's "alternate week" system for college men: it is only applicable to a limited extent. Thus, in hard times especially, industrial or commercial concerns cannot take all the pupils who must later enter the industries or commerce. The school system must give every boy a chance, and, therefore, it seems to me that in its remodeling care must be taken to do the work in a way so facile that it will meet the different conditions of the various communities and classes with which it has to deal. This will require trade schools, schools in the industries and elsewhere, correspondence courses, and other methods, but little time or thought can be wisely given by the pupil in his early years to other than the general foundation training needed by all children.

Otis Allen Kenyon: Education of the young is the greatest and most important problem with which we have to deal. We may possibly go so far as to say that the answer to the "eternal question" as to the real purpose of life is to produce a better generation than the present ruling one. It is a field of endeavor in which every man and woman can take an important part. The education of an individual begins the day of birth and extends over the entire period of active life. For this reason it is most important that all of the early training should be in the fundamental principles, which may be considered as tools with which all subsequent progress can be accomplished; that is, we must prepare our youth for self-education.

Reading is the first requisite, since I consider that most knowledge is acquired by reading. Broadly considered, training in all fundamentals is simply learning to read. Especially is this true of languages and mathematics. I was astonished some months ago to learn that a large percentage of our American born children did not learn to read, and especially so when I found that New York state stood near the top of the list in illiteracy. The natural conclusion was that the city of New York, with its immense foreign population, was the cause of it. Not so. New York City stands exceptionally high. Illiteracy occurs largely in the rural districts of the state, where there has been no foreign influx for years.

In the Scandinavian and German countries such a thing as illiteracy is practically unknown, and I had previously thought such to be the case here. In New York we have a fine educa-

tional system in theory with laws to back it up. The law says that all children between certain ages shall attend school, and having these excellent laws we rest contented, leaving their enforcement to the discretion of local authorities. Therefore, illiteracy is the most powerful where it finds least resistance.

This great problem should be attacked at the bottom and all children given a good start in spite of their local surroundings. Their industrial training will take care of itself if necessary. In former times before industrial training schools were heard of, the hand-workers learned their trade as very few do now. For example, old books made on hand-presses reveal work which we cannot equal to-day with the finest machinery. It is true that a comparatively small percentage of the people were skilled artisans in those times, and, undoubtedly, they were a superior class of men such as we now find doing only brain work. Nevertheless the fact remains that the bread-winner must learn his trade whether he will or not; and if equipped with a fundamental education, he will go further and faster.

Our educational system is not what it should be and the trouble seems to be one that is inherent in our form of government, namely, the lack of centralization of power, and, therefore, a lack of uniformity. The fact is that we have no system.

I believe in individualism in the highest educational institutions, but in the lower ones there should be more and more uniformity as we descend the scale. To say that a boy has a high-school diploma means nothing until we have been told what state the school is in, and even then it really means little unless the name of the city or town is also given. It is a fact that our own colleges recognize diplomas from certain high schools and not from others. With such a state of affairs it is little wonder that the best European schools absolutely discredit all diplomas obtained from our public schools. The first step, then, is to obtain a uniform standard of education throughout the country, so that a diploma of a certain grade has a meaning which needs no special interpretation or explanation.

Granted that we have a uniform and well-organized school system, we can then take up the details of what to teach and how it shall be done. What to teach should be decided by the central organization and should be fixed for each grade. How to teach should be left to the teachers, since no uniform method will please all teachers nor suit all pupils, and no good teacher can work well when using a method with which he is not in sympathy.

The most important thing is to have good teachers. Our teachers should not be reservoirs of knowledge; they should be independent thinkers and natural educators. The whole purpose in education should be to awaken a thirst for knowledge and to encourage independent and analytical thinking. Any youth starting out with the thinking habit and a working knowledge of the fundamental principles of his profession will advance to a limit imposed only by his innate shortcomings.

Dean Schneider's idea of coöperation between employers and educators is an excellent one. The public school cannot teach specialties; it must teach the elements which are used to build up the specialties. However, if the school can follow the pupils into the shops where they are learning their trade and show them how to apply their theory, it can do great good.

As Dean Schneider points out, by far the greater share of our children leave school early to earn their living, and, therefore, the public school should consider them first. They should not be taught many different subjects, but rather be thoroughly drilled in the most fundamental ones. It seems to me that our schools attempt to teach too many subjects, the result being a superficial education which is sometimes worse than none at all. It is absolutely harmful in some cases to teach things which can never be properly comprehended. Many pupils are ruined by education instead of being bettered. They are taken beyond their depth and given a smattering of this, that, and the other; and when they are through they think they are too well educated to learn a trade. Coöperation between schools and employers is a good thing; but what we need most is coöperation in the schools themselves to produce an efficient and uniform system of public instruction.

Dugald C. Jackson: I wish particularly to call attention to the following sentences found in the paper.

> Under present conditions most of the children who leave the public schools go at once into the industries or stores, and there is no connection whatever between school and shop. The children go to work not because they want to, but because they have to become little bread-winners. Here, then, are many children working in some capacity, and a series of public schools carried on only for those who are fortunate enough to be able to continue in their school work. Since the public school system is the only organized institution capable of dealing with all these children at work, and since the children are also learning a trade and earning money sufficient for their simple wants, it seems that the only complete solution of the problem is a system of coöperation between the schools and the factories for efficiency training and civic training of the young people after they have found their work.

This is a clear statement of the present situation, and it emphasizes the fact that industrial training in this country may rest with the public-school system. At this point I wish again to urge the importance of having the organization and development of industrial training put under the direction of adequate educational bodies. When I say *adequate*, I mean bodies that are not only strong in the educational branch, but that are also well informed as to the industrial requirements. I know of no bodies that meet the conditions so satisfactorily as the faculties of the great engineering schools. While it would be a misfortune to have the engineering schools take industrial education into their curricula, such faculties may wisely be held responsible for the organization and development of the work. This is the procedure which is now going into effect in the state of Wisconsin, where the organization and development of industrial

education have been placed by the legislature under the direction of the University of Wisconsin, and the work is being carried out by a department of "university extension" of which the director is a well-known engineer who was for a long time dean of one of the engineering schools. This seems to be already bringing favorable results, though the work has been in progress only a short time.

Dean Schneider indicates that the German continuation school idea, which is supported in Germany under the requirements of imperial law, approaches his ideas of the necessities of the present case. In that case the teachers go to the students, as desired by Dean Schneider, but the coöperative or alternating plan discussed by Dean Schneider is not put into effect. Dean Schneider's plan seems to contain the desirable elements of the German continuation school idea, and it adds thereto other elements. To carry out his idea of *in loco parentis* for the public schools would require endless tact on the part of the teachers, and would doubtless call for a higher average grade of men for teachers' positions, in case the plan could be put into effect. But such changes in the teaching staff would be well worth while, provided they brought the results of increased efficiency and improved citizenship for the youths who must bear the brunt of artisanship for the coming generation.

The "coördinator" plan proposed by Dean Schneider is a novel one. It seems likely to be useful; but better results would probably come from the work proposed to be done by the coördinator if he could give his instruction in an extra half hour at noon or at some similar time during the boy's shop assignment. This might be practicable only in the case of establishments employing a large number of the coöperative apprentices. It will be observed that Dean Schneider himself makes a suggestion in harmony with this criticism, in the discussion of the shoe division of a department store.

I believe that this paper presents ideas which are worth the careful consideration equally of the employers of skilled workmen and the public school boards of industrial cities. The ideas are also of vital interest to the engineering profession, for without skilled workmen to execute their projects engineers would be helpless; and the engineers should properly give careful attention to any plans which promise a better training for artisanship.

A. R. Dennington (by letter): As a factor in industrial education the correspondence system deserves attention, not only for what it has done in the past but also for its inherent possibilities. Considered from the standpoint of adaptability, instruction by correspondence is almost ideal, as the student pursues his usual vocation at the same time that he is studying the principles of the underlying theory. It may be implied by some that in the past the student has spent his time studying some line of work entirely different from his daily task and

that as a result the correspondence schools were instilling the spirit of discontent and making employes less efficient. Instead of the machinist studying mechanics or something related to his work, he would be spending his leisure time studying bookkeeping or chemistry, with the intention of giving up all his practical experience and making a new start on the foundation of the few unrelated facts which he had learned. It is of course true that an occasional man is able to achieve signal success by such a course, but he must be regarded as the exception rather than the rule. Most persons who take up correspondence instruction have passed the period of life when they can easily make changes in employment, and when such persons do change there is a certain economic loss. It is reasonable to expect that if the energy expended in preparing for the changes in employment had been expended in making the workers more efficient in their original line of work, the measure of their success would have been greater.

There seems to be an almost universal idea in this country that the work of some other individual is more desirable than one's own, and this makes the man who has spirit and ambition a ready listener to any proposition pointing toward imaginary opulence. The man who has spent years in one line of work is unable to judge accurately of conditions outside of his trade. He may appreciate the crowded condition of his trade and may realize the difficulties in the way of advancement, but he does not realize that the same conditions exist in the occupation he thinks of taking up and that individual success in either case involves more than average ability.

The common defects of correspondence instruction as pointed out above are not insurmountable, and will as time goes on be more or less completely eliminated. Statistics show that the greater number of persons who have been materially benefited by a course of correspondence study have read on subjects related to their previous work. They have used their studies as elements in the process of evolution. The usefulness of the correspondence school depends upon its ability to increase the efficiency of the worker in his employment rather than to incite him to make a change. He should be taught the truth that any line of honest endeavor is honorable, that it merits special study and will reward it. It should then be the aim of the correspondence course to make the plumber a man who will do his work with careful attention to underlying sanitary engineering principles; to enable the farmer to produce more from the same soil without impoverishing it; to give the engineer or fireman in a power plant the knowledge which combined with his practical skill will make possible the more economical production of power. In short it is the province of the correspondence school to improve the artisan and thereby cheapen the cost of production while increasing the economic and industrial value of the producer. Training of the kind outlined is narrow in the sense

that it is applied with the idea of developing and directing thought along one line, but experience shows that even one small division of any science or art offers problems worthy the most thorough study.

The absence of any personality in the instruction is an inherent defect in the system. This disadvantage cannot be measured because of the diversity of the effect of the personality of any teacher on a number of pupils, and also because of the different individuality of the teachers. Most of the printed lessons represent the composition of several individuals, and an entire course in any branch represents the work of a small army. There can therefore be no pronounced individualism in the courses of instruction.

In correspondence instruction as in regular teaching work the attitude of the student is very important. Many persons take correspondence courses with the idea that their part in the contract is merely to remain passive and receive in some mysterious manner the dreamed of knowledge and power. There is also a noticeable tendency on the part of some to regard the number of pages covered or the number of subjects skimmed as the only essentials to increased efficiency. These tendencies must be considered in forming any judgment of the results obtained by correspondence instruction. In time they may be reduced, but the change must come rather slowly, as many people are willing to accept the superficial and artificial as the thorough and genuine education.

Herman Schneider: In order to satisfy Mr. Barker's question about the figures used in regard to the children of school age in this country, I quote from a report compiled by Professor Ayer of the University of Cincinnati from the Report of the United States Commissioner of Education:

In 1906–7 there were 24 million children of school age in this country. Of this number

76 per cent were enrolled in the schools;
54 per cent were in average attendance;
18.5 per cent completed the elementary courses;
12.6 per cent entered high schools and academies;
3.4 per cent graduated from high schools and academies;
1 84 per cent entered colleges and universities;
0.67 per cent graduated from colleges and universities.

Confusion may exist in regard to the figures I have given previously, because in some instances I state the total number leaving school when the law permits them to do so, and in other cases I state the number leaving school without industrial training. The report of the Commissioner of Education contains the interesting information that out of 24 million of school age about 63,000 children receive commercial or industrial training in the United States.

Mr. Barker is of the opinion that under this system boys would have no time to play, go fishing, roam the field, etc. As a matter of fact, without this system, as shown by the figures

given, the majority of the children would be in the shops throughout the year. It is specifically stated in this paper that the coöperative system contemplates taking the children *after* they have found their work, and giving them further schooling. Under the coöperative system the children would be at school at least every other week; they certainly would have play hours and time to roam the fields, which they would not have were they not in school at all. The coöperative system is fairer to the boy than the present industrial system, which shuts him up in a factory nearly all the time.

Mr. Barker further questions my statement that the coöperative system leads by natural selection to the university, saying that a student under the coöperative system would not be prepared to enter the university. Under the Fitchburg scheme, a boy from the coöperative course could enter college after one more year of High School. If he should show an engineering bent in his shop and school work under the coöperative system, he would in all probability be urged to go to an engineering college, and opportunity would be offered by an extra year's high-school work to enter any technical school. If then the technical school has a coöperative system—as a number of them will have—it ought to be evident that the way is open for any ambitious young man of virtually no financial means to obtain an engineering education, provided he ought to have it.

Mr. Dean says that the coöperative system is not generally applicable, that it could not be used, for instance, in the dairy industry of the state of New York. I would call Mr. Dean's attention to an article in the current number of the *World's Work*, which describes a coöperative scheme whereby the University of Chicago coöperates with the large farms in Indiana and Illinois under a plan by which the students work on the farm during the open season, and attend the university during the closed season. There is a definite co-ordination of the theory of the university with the practice of the farm, and this includes the dairy work. It can be shown that dairy work under a coöperative scheme is just as feasible as machine work under similar conditions. The details of coöperation, however, must be made to suit the peculiar conditions.

Mr. Dean also speaks of the difficulty of getting teachers for this system. This difficulty exists in any industrial education system, if industrial education is to be *real* industrial education. The trouble with most industrial education is that it is not industrial, and that it is not taught by teachers who know industries. Teachers can be found if they are sought. The statement is not more of an argument against this system than it is against any other system of industrial education.

Mr. Dean further suggests that if the coöperative system is put into the public schools—and he agrees that it should be—the schools should be given authority to plan the courses in the

shops, and that he would insist that his department plan the courses through the shop. I have been operating a coöperative course in engineering for several years, and am not quite sure that I am as yet capable of planning the shop work of my students. The shop superintendent knows more about this than I do. Since the public schools would have to deal with hundreds of trades, it would take a man of greater ability than exists anywhere to plan all these courses in the shops. The shop superintendent knows more about it than any educator. As a matter of fact, these courses are worked out by consultation between the school authorities and the shop people. Coöperation means coöperation; it does not mean dictation by either manufacturer or school.

Professor Franklin asks what is meant by my statement that no displacement of the present high school courses is contemplated, stating that in his opinion the present high school courses should be "scrapped." I agree with him that most of the work of the high schools should be scrapped, and I think it would not make a very valuable pile of junk. What I mean in my paper is that no displacement of the present courses preparing students for the university is contemplated.

Professor J. Price Jackson says that the coöperative system would not be feasible during hard times when the students would be thrown out of work. When such times arrive, it ought to be obvious that under the coöperative system the students are in school at least every other week. If there is no coöperative system, they are idle every week. Between the two, it seems to me that the coöperative system has the advantage. During the periods of depression the student could, if it is desirable, be held in school all the time, and his job reserved for him at the shop until work is resumed. It seems to me that the coöperative system is specially well adapted for periods of depression, in order to have a check on the youth when there is no work.

The purpose of this paper is to point out two things: first, that the problem of industrial education is so big and comprehensive, that it must be solved by the public school system; second, that this must be done by coöperation with the industries for the education of children *after* they have found their work. No arguments have been deduced in this discussion to show that these two propositions have not been established.

The paper attempts to show further that the details for every industry, and for every city, would vary. For instance, the scheme which fits the University of Cincinnati in its engineering college might not fit the Massachusetts Institute of Technology. The details of a scheme that fits the machine industry of Cincinnati might not fit the same industry of Fitchburg, Mass. The details which fit commercial life will not necessarily fit industrial life.

There are many ways in which this coöperation can be carried on. For instance, in the building trades in Chicago there is an

agreement between the master builders, the union and the public schools, whereby the apprentices go to school during the slack months in the winter to obtain efficiency instruction. They are not, however, taught the trades in the schools. This is a coöperative system.

The one idea which I am trying to impress upon educators and manufacturers is the necessity for coöperation between the manufacturer and the school for efficiency training and for citizenship training; in other words, to fit the student for the thing he is to do, and for the man he is to be. There is a fundamental difference between a coöperative course in an engineering college and a coöperative course in the trades. In the engineering college, the university selects a number of young men, sends them for a trial period to the shop, where they are finally selected by the manufacturer. In the trades the idea is to take the young men already on their jobs and give them more education. In other words, to take the school to the boy in the shop, and not to take the boy in the school to the shop.

Charles S. Howe (by letter): There are two methods of training industrial workers: through the apprenticeship system and through the trade school. The apprenticeship system is the most obvious and natural method. As the journeyman will work in the shop after he has learned his trade, it seems proper that he should acquire that knowledge in the shop. Manufacturers usually believe this to be the best way because the boy is then under shop conditions while he is learning; he works with experienced men, he gets the shop atmosphere; he learns to do his work so that it will come up to the commercial standard. All this is of great advantage to the young apprentice, and if the conditions are favorable, there is no place better than the shop for learning a trade. It however has some disadvantages.

1 The superintendent may not have a very broad view of the duty of the shop toward its apprentices. If he has not, he will see that each apprentice learns to use some one machine well, and that he remains the greater part of the time at work upon this machine instead of acquiring a knowledge of all the machines in the shop.

2. The smaller shops are limited to one particular kind of work. As a rule it is the coarser kind and hence the apprentice learns only this sort of work. He can not acquire the other parts of his trade in that particular place. Yet under the apprenticeship system he gets at this one place all the knowledge that is required to make him a journeyman.

3. Under the apprenticeship system the foreman is the teacher, but his principal duty is not to teach the apprentices it is to turn out as much work from the shop as possible. When he has spare time he spends it in showing the apprentices how their work should be done. Where the management is sympathetic, where it is the purpose to train the boy in all branches of

the trade, and where a special foreman is employed to teach apprentices, no better system than the apprenticeship method can be found for training the journeyman in any trade.

There are many large establishments which have an admirable apprenticeship system, but the great majority of factories of all kinds do not have such a system, and hence we are not training enough skilled workmen in any of the trades. To meet the demands of the future and to give the boys and girls a better industrial opportunity, it has been proposed to establish trade schools, either under the administration of school boards or under special boards authorized to expend public funds. A few such schools have been in existence for a number of years, and their graduates have been successful. These trade schools, if properly designed and planned, will be in buildings which resemble a shop, will have the same machinery which a first-class shop would have, and the teachers will be men who have been foremen in shops, but who have left their regular work to enter the school because of the higher pay which will be offered. The work turned out will also be of the same kind as that of the shop and will be subject to the same standards of efficiency because it will be sold in the market in competition with other goods. The amount of goods so sold, however, will never be large enough seriously to affect market prices.

The trade school method will train more journeymen than the apprenticeship method. If properly conducted, I believe it will train them just as well as the best apprenticeship system, and in the majority of cases, it will train them better. It will meet the demands of the manufacturers for more skilled workers and it will give the boy or girl who wishes to learn a trade the most complete opportunity of so doing.

V. Karapetoff (by letter): I am favorably impressed with the spirit permeating Dean Schneider's article. There is so much freshness and optimism in it, that one immediately feels new courage and energy for promoting industrial education. The principle of coöperation between schools and industrial establishments ought to be encouraged by all means. The question as to how far it ought to be carried may be discussed later on, when sufficient information and experience will have been acquired.

I am particularly pleased with two ideas in the paper: one is that it is our common duty as citizens of a state or a country to educate the coming generation for a maximum of efficiency; that the duty does not end with paying the school tax. Industrial establishments can help the cause of education by direct coöperation as outlined in Dean Schneider's paper much more and with less sacrifice than by contributions in money.

The other idea is that of interweaving theory and practice so that neither becomes a burden to the student. Practice alone prepares a narrow artisan by its multitude of petty rules and empirical " ways of doing things "; theory alone is barren, being

like a form without contents. With the plan suggested, the student is shown how certain results are achieved; at the same time the observed facts are coördinated in his memory by the underlying theory.

G. M. Basford (by letter): Those who are watching the interesting and promising development in the direction of industrial training, and especially those who are in position to realize the great importance of the movement cannot fail to be hopeful of exceedingly important results. This problem is so complicated as to require many different plans for meeting various individual needs. Of all the plans so far suggested, that of Dean Schneider seems to contain the largest number of elements necessary to successful achievements. For large manufacturing establishments producing heavy machinery and involving progress in methods and equipment, progress which at the present time is exceedingly rapid, it is obviously necessary that the school should be taken to the shop. I believe that the plan of Dean Schneider is exactly what is required for such establishments, with one modification, that the part of the development which is generally spoken of as education should in such cases be given at the works of the company. This is going one step further than Professor Schneider has gone.

Alexander C. Humphreys (by letter): The following statements appear in Dean Schneider's paper: "The chief end of education is so to instruct the individual that all individuals as civic units will combine to make the best possible State." . . . "The object of all education is to make good citizens, and the first duty of a good citizen is to earn his own living."

I fully agree that these *should be* the end and object of education, but I am not inclined to believe that we have convincing evidence that these are the end and object of the public school system of these United States. Certainly the results do not furnish such evidence. Some system of industrial education for the training of the masses should be developed to meet this deficiency. At least, the public school system should be remodeled so as to prepare along the lines of maximum efficiency for later specific training for the industries.

As Dean Schneider points out, our public schools cannot be expected to give specific training in all trades, but much can be done in the way of preparation that will apply in a general way to all trades, and more specifically to a few trades. In my opinion, the first step to be taken is in the direction of greater thoroughness in the fundamentals, and especially the "Three R's".

Dean Schneider quotes some one as saying "Under the coöperative public school system, the grade school principal pays much less attention to biology and much more to the boy". I wish this statement could be made truthfully of all elementary public school instruction.

Dean Schneider mentions shop mathematics. The term is suggestive of practical efficiency, but I would like to extend its scope by making it include the simple mathematics required in shop, field and counting house. The every-day mathematics should be better taught in the schools, and especially elementary arithmetic in the elementary schools. I sincerely hope the author is warranted in his belief that the coöperative system can be so developed as to make it generally applicable throughout the United States. Certainly there are great difficulties in the way, and especially in obtaining the coöperation of those engaged in the industries. I fear one trouble that would develop would be the tendency of the professional teacher to attempt to teach where he is not qualified so to do by practical experience. The system to be a final success would have to be the product of evolution, for as yet none of us is capable of preparing the detailed specifications of such a system—we must be satisfied to hold to the good and reject the bad as the practical conditions are met and overcome. In the line of manufacturing, if in any case the green boys and girls could be used to advantage, I should not apprehend any final trouble from the alternating employment of two sets of students. One favorable feature to put against the unfavorable features would be that of incentive to best effort through honest rivalry.

One thing we may all agree upon—in the training for citizenship, and, incidentally, for the industries—our public school system is sadly deficient, and all of us—and we all are concerned either directly or indirectly—should be willing to work as well as talk for reform.

Ralph W. Pope (by letter): There can be no question as to the benefit to the individual, of an industrial education. Even though the person learning a trade may never be required to practice it for earning a livelihood, knowledge of a handicraft will be a source of satisfaction to him throughout his life, especially if he has a natural taste for the particular branch which he has studied and practised. There appears to be, however, considerable doubt as to whether the graduates of industrial schools will follow through life the trades they may have partially learned. A strong social prejudice exists against manual labor, skilled or unskilled, as well as against the confinement and discipline of factory life, which no educational system can overcome. In a comparatively new country the demand for labor of all grades can only be met by immigration. High wages in themselves will not divert the rising generations from their tendency to obtain a livelihood in some other way than working at the bench, not even with the possibility before them of promotion to the grade of foreman or superintendent. Unless some method can be devised to counteract this tendency, the final outcome of industrial education so far as furnishing a supply of skilled apprentices is concerned will simply add to the already overburdened curricula of our public schools.

That this condition of affairs is not peculiar to the present day is indicated by the following excerpt from an essay by Ezra Sampson a minister, editor, and a captain in the revolutionary army. It appears in a school reader published in 1820:

> In the proud and fastidious times in which we live, manual labor of the useful kind is accounted a thing too vulgar for those of the better sort. Many a young gentleman would feel himself dishonored by doing anything called work; and many a young lady would blush to be found employed in an occupation really useful; even though in circumstances imperatively demanding their industry.

It is well to profit by past experience, and for this reason I will refer to the fact that about 1682 Thomas Budd introduced into West Jersey, (the southern part of the present State of New Jersey) the Quaker system of education through which the boys were to be taught:

> Some mystery or trade as the making of machinery, joynery, turnery, the making of clocks and watches, weaving and shoemaking. The girls were to be taught in spinning in flax and wool, the knitting of gloves and stockings; sewing and making all sorts of needle work, and the making of straw work, as hats, baskets, etc.

This system apparently was gradually abandoned, yet I have been told by a neighbor in Elizabeth, N. J. that in his school days (about 1845) the girls were taught shirt-making, and he remembers the teacher's instructions to "fell the gusset to the sleeve, and the sleeve to the gusset." If these "girls" ever made shirts for the family the domestic art has long since been abandoned. About twenty five years ago manual training was again introduced in the public schools of Elizabeth, and has since been abandoned. I asked a pupil of that era if he knew of any instances where the trade learned had been followed after graduation. He could recall but the single example of architecture which is of course recognized as a "respectable" profession. It is evident that a sufficient number of industrial schools have now been in operation for such a length of time as to permit the collection of statistics bearing upon this question, which is: of the number of students learning trades, what proportion have actually followed them in after life?

Sydney W. Ashe (by letter): Dean Schneider's paper discusses the problem of industrial education from a manufacturer's viewpoint. The solution of this problem becomes more difficult where it deals with the employees of an operating company, whose success depends upon all departments moving in synchronism. Where one is attempting to raise the educational status of the employees of an operating company, the work must not in any way interfere with the moving mechanism of the company but must be carried on at such times, afternoon and evenings, as the men are free from their regular hours of duty. The work must be flexible, as the ages of the men range from 16 to 60 years, and their education ranges from that of the grammar school to the college

graduate. The work must be attractive, for with the meetings of various other societies, especially in the large cities, the men will not take advantage of such courses and will not attend.

The course must be progressive each year, as there are a certain number who constitute the backbone of the course, who attend almost every lecture, and who must therefore be given first consideration. Most of the men are handicapped by a lack of advanced mathematics which must also be considered. Educational courses for their employees are now being carried on by the New York Edison Company, the Edison Illuminating Company of Brooklyn and the Edison Electric Illuminating Company of Boston, all of which the writer had the privilege of instituting. The method used in carrying on this work was described in detail in the *Electrical World* of May 16, 1908; briefly, it consists in giving an extensive course of experimental lectures on electrical engineering.

The first year a course is given extending from the elements of magnetism through direct and alternating currents. At certain intervals in the course, radical departures are made from the regular program, Table 1, by the insertion of lectures such as on turbines, for the purpose of arousing fresh interest. The second year the subject of fundamentals is dropped for the time being, and the application of the fundamental principles to the company's methods is taken up. A course of such lectures extends from the coal pile to the consumer. A third season, more advanced principles may be taken up.

TABLE I.

CALENDAR OF DATES AND TOPICS. 1909.

1. Thursday, Jan. 7. Magnetism.
2. Thursday, Jan. 14. Electromagnetism.
3. Thursday, Jan. 21. Electrolysis. Electrolytic corrosion. Storage-batteries.
4. Thursday, Jan. 28. Electromagnetic induction. Theory of the dynamo.
5. Thursday, Feb. 4. Practical management of shunt motors.
6. Thursday, Feb. 11. Practical management of series motors.
7. Thursday, Feb. 18. Steam engines and steam turbines.
8. Thursday, Feb. 25. Underground conduit systems.
9. Thursday, March 4. Voltage regulation.
10. Thursday, March 11. Ohm's law applied to distribution problems. The three-wire system.
11. Thursday, March 18. Ohm's law applied to electrical measurements. Ammeter, voltmeter and galvanometer methods.
12. Thursday, March 25. The physics of light.
13. Thursday, April 1. Arcs and arc lamp mechanisms.
14. Thursday, April 8. Incandescent lamps.
15. Thursday, April 15. Principles of recording wattmeters.
16. Thursday, April 22. Elementary principles of alternating currents.
17. Thursday, April 29. Alternating currents continued.
18. Thursday, May 6. Transformers.
19. Thursday, May 13. Alternating current motors.
20. Thursday, May 20. Synchronous converters.

The writer had constructed a special form of projecting lantern with large condensing lenses which could be quickly connected

during a lecture from a horizontal to a vertical lantern, also a special projecting galvanometer having a transparent scale, which could be placed directly before the lantern. The galvanometer can also be used with shunts as an ammeter; with resistance, as a voltmeter; with a thermo couple attachment, it will indicate temperature; and with a speed tachometer it indicates speed, all of these being readable on the screen. For alternating-current work new forms of projecting wattmeters, ammeters and voltmeters have been built for the writer. With this combination it is possible to perform on the lecture table and screen, any experiment which can be performed in the laboratory. In two lectures, for instance, it is possible to perform all the experiments of simple electrical measurements given in 20 afternoon sessions in college, provided the constants be properly chosen. The writer, for instance, has projected a voltmeter reading on the screen and in 25 minutes gone through all of the calculations and with a standard cell and potentiometer calibrated the voltmeter directly before the men. This, however, is an extreme case, although the Wheatstone bridge, the Thomson double bridge, and similar apparatus are demonstrated.

In connection with the lecture course a syllabus is printed and distributed to the men, giving for each lecture an outline of the ground to be covered (see Table II.) This syllabus also gives directions for study so that auxiliary reading may be carried on from week to week. In addition to this weekly bulletins are posted calling attention to the lectures. After the lecture the men are free to ask questions, to come forward and examine the apparatus, or to make any suggestions in connection with the course. The lectures are given weekly, afternoon and evening, so as to afford all an equal opportunity to attend.

This work was started three years ago by the Brooklyn Edison Company, which has continued it each year since. Two seasons have now been given to it by the New York Edison Company and this year is the first season for the Boston Edison Company. This year the work was expanded by the New York Edison Company, which has been giving a course of laboratory work and a course of talks by various professors and designing engineers as well as the experimental lectures.

As this work has now been carried on for three years it seems proper to ask whether it has been a success and wherein have the various companies benefited. Increased loyalty of the men to the companies has been the first great benefit of this work. The personal interest of the managements has created in the men a very kindly spirit and a feeling of pride in the home company. The men have felt that the company has been endeavoring to raise their technical standard so that they could advance, and in time occupy positions which now have to be filled from outside talent. The question may be asked as to whether this work would not in time serve to make the men

discontented, whether they would not feel that as they had attended all the lectures for a season, they should immediately be promoted to such an office, for instance, as general manager of the company. While it is true that a little knowledge is sometimes a dangerous thing, our results show that a little knowledge in this case has served more to whet the appetite of the individual, and make him realize his own limitations. The men have become more earnest, more inquiring, less mechanical, have read the technical papers more, have kept the librarian busy, have purchased many technical books and have kept themselves busy.

By having a better understanding of the work in which they are engaged, the men are better able to handle their apparatus, a mistake in the use of which, due to lack of knowledge, might sometime result in damage far greater than the cost of a season's lecture course. The men have also been taught to appreciate the danger which is always present, and to use due caution at all times, particular points of danger being emphasized. As 90 per cent of the accidents which occur are due to negligence or lack of knowledge, it is realized how important it is that a man should have as thorough a knowledge as possible of the particular apparatus with which he has to deal.

The men have also been taught to think quickly, which is a valuable asset in times of emergency, especially in the lighting field where continuity of service and close voltage regulation are all important.

Considering the many adverse factors attendant upon work of this kind, its success has been phenomenal. The attendance at the lectures of the New York Edison Company last year totaled slightly under 6000 men. The Brooklyn Edison Company this year had an average attendance 60 per cent better than last year. Out of 13 evening lectures given so far this year, out of a course of 20 for the Boston Edison Company, the average attendance has been 166 men. A slight fee has been charged in Boston for the entire course, and by slightly increasing this fee another season it is hoped to make the course entirely self-supporting. In New York and Brooklyn the course this season has been free to all employees. In Boston the same method was considered but it would have been impossible to have accommodated in the lecture room all who would have attended. As it was, for the first six lectures the seating capacity was taxed to its utmost.

TABLE II.

LECTURE NO. 5. FEBRUARY 4, 1909.

The Practical Management of Shunt Motors.

Preliminary.
Slide showing simple dynamo.
Slide showing windings of shunt motors.
Magnetic Circuits of Field of Shunt Motor.
Parts of enclosed motor.
Field winding has high resistance.
Armature winding has low resistance.
Resistance values determined with ammeter and voltmeter.
Way of connecting field winding to circuit.
Short-circuit current.
Polarity of field coils.
Change of direction of current through field coils causes change of polarity.
Way of determining whether or not field coils are properly connected.
Self-induction of field coils; danger of opening field circuit.
Magnetic Circuits of Armature of Shunt Motor.
Loop of wire and development of armature.
Magnetic circuits of armature.
Formed coils and spacing of same.
Various armature windings.
Change of polarity of armature circuit caused by changing direction of current flow.
Magnetic circuits of armature and field, cause of rotation.
Starting of a Shunt Motor.
Diagram of circuits.
Excitation of field coils.
Precautions to be observed to prevent opening of field circuit through body.
Connecting lamp board in series with armature causes rotation when field coils are excited.
Use of starting box.
Starting current of motor shown on screen.
Function of overload release.
Function of magnet arm on starting box.
Changing Direction of Rotation.
Change of field terminals.
Change of armature terminals.
Change of both armature and field terminals.
Speed Variation.
How to connect in field rheostat. Value of its resistance.
How to tell whether resistance is " all in " or " all out."
Variation of speed, measurement of values, change of field current.
How to tell from starting box when field current is open at starting.
Precautions against starting motor with all of the field resistance in circuit.
Theory of speed variation.
Location of Trouble.
Motor will not operate. Why?
Defect may make itself evident at first glance.
Bearings, clearance, brushes, broken terminals, etc.
Note: With light fuse in circuit turn on starting box first few points and let it come back.
If it flashes, but machine does not rotate, field circuit may be O. K.
If starting box smokes or machine runs very fast, field circuit may be open.
When machine is operating, if commutator flashes as it goes round it may be a high bar, a short-circuited coil, or a hard spot in the brushes.

DIRECTIONS FOR STUDY.

See Swoope, pages 393 *to* 414 *inclusive.*
See Thompson's Elem. Lessons, pages 448 *to* 455 *inclusive.*
See Sheldon, pages 185 *to* 207 *inclusive.*

Franklin Phillips (by letter): Twenty years ago at least, a change was observable in the skill of mechanics working in the engineering trades. This change was slow at first, but has rapidly increased, owing to the passing away of the old-time mechanics who were trained by long apprenticeships to become skilled workmen. Their ranks were still further depleted through the promotion of these men from the bench and machine to positions of foremen and managers of works, those having ability to handle men and the ambition to seek a higher paid position thus being able to obtain the managerial prizes.

The advent of the handy men and the introduction of automatic machinery have led many manufacturing concerns practically to abandon the apprentice system. This has led to a con-

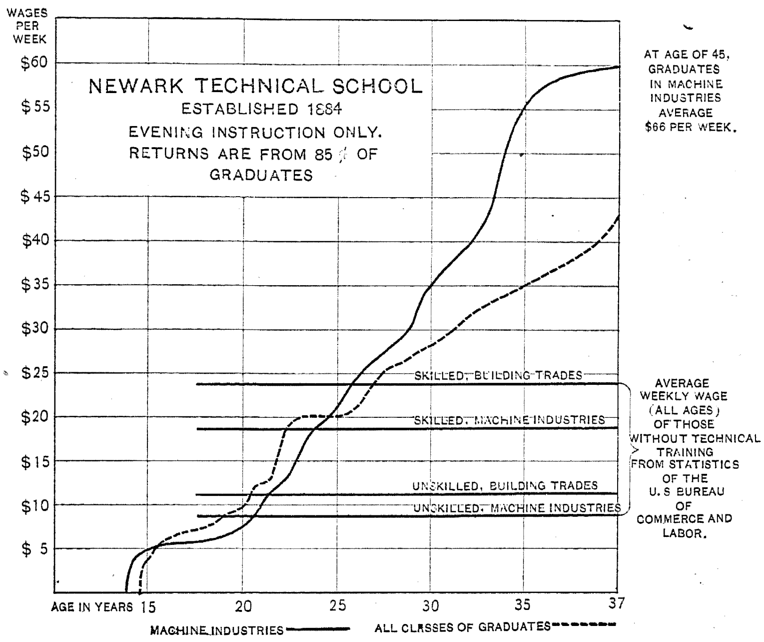


FIG. 1—The money value of industrial training.
Compiled by New Jersey Commission on Industrial Education.

stant diminution in the ranks of skilled workmen, until now the whole country has become sensible of its dependent position as a manufacturing nation and is looking about with its hands in the air crying for help. A service of three years with that able instructor Professor Sweet (and later a four years' apprenticeship, in the Whitworth School, followed by work at the establishment with which I am connected) may have prejudiced me too strongly in favor of the old-time apprenticeship, but I often recall the words of my good father when I had complained to him of the irksomeness of running a screw-cutting lathe or

some equally dull operation, he had soothed me with the words—"My son, you must remember that there is no royal road to knowledge", and so I believe to this day there is no short-cut method of making skilled mechanics.

Engineering establishments must get back to the old-time system of apprenticeship, and whether the trade unions like it or not, (they at present put limitations on the number of apprentices employed), sufficient numbers of apprentices must be installed to meet the needs of each particular establishment. This plan works out automatically, as but a limited number can be handled by any one establishment at a given time. The part-time system of Dean Schneider may have undoubted advantages in some localities and would seem particularly well adapted to cities or towns where there are a number of industries producing nearly the same kind of an output, but may not be at all adapted where the industries are small or widely varied.

To my mind the manual training school does not touch the subject at all, and is as elemental as regards teaching a trade as the primary department is to the full college education, or the boy's kite flying as compared with the skilled operations of Wilbur Wright in his flying machine. I recognize the fact that something must be done towards the education of the youth, both girls and boys, of this country, who are between the ages of twelve and sixteen, those who leave the grammar schools or the early grades in high school, and cannot go to work for legal reasons (in New Jersey) until sixteen years of age. These, therefore, must be educated, and preferably in some useful work.

My experience as a teacher in the old-fashioned practice and as an observer of the work of graduates from the establishment with which I am connected, who were also graduates of the Newark Technical School, leads me to believe that the most value and the quickest results can be obtained from the establishment of the technical night school. I refer the members of this society to the lately published report of the Commission on Industrial Education, State of New Jersey, to the chart shown in that publication [Fig. 1], and the printed matter of Appendix C. under the title of "The Money Value of Industrial Training." Professor Colton, dean of that faculty, shows that it costs but $42.00 per capita per year to educate the pupils, while the average wage of the graduates at the age of 45 has risen to $60.00 per week from a basis of $20.00 per week, the average wage of a skilled mechanic.

Do not think for a moment that in educating your apprentices you will increase the available stock of more highly educated machine hands or vise hands, for you will not see them again in this capacity, and this condition will prevail until the demand for foremen, superintendents and office engineers is met and the surplus, if any, for lack of employment must seek the old positions. My thought is, that the trades can be best taught in the engineering establishments, while the schools, either day

or night, to my mind preferably the latter, for the time being, will teach the things requisite and not dwelt upon during the course of daily work.

John Price Jackson: The writer highly appreciates such experimental work in pedagogics as is being performed by Professor Schneider and some others in this country. He believes that much good is coming therefrom, and that certain pedagogic principles are being developed which have in the past lain more or less dormant. The writer, however, is not willing to support the tenets of Professor Schneider's faith in all particulars. It is a very grave question whether the average boy can to the best advantage take his college training week in and week out. Indeed, the number who can do this is probably so small as to make the system narrow in its application. Assuming that the college does run in part on a basis of a few days in college, then a few days working for some industrial establishment and so on, it is readily seen that the teaching force and schedule arrangements of studies become difficult and expensive. It must further be noted that students who take their college work in this way, have, in a sense, two masters, and that they lose a large part of the most desirable element of college life which goes with class organization, active part in class and other collegiate student functions, and, to a certain extent, the effect of close personal contact which the college boy has with his fellows. It is recognized among college pedagogs that first of all the boy must daily do good, sound, hard, practical work, and it is also clearly understood that the associations of college life are in many ways useful in the development of the character of the man who is to be of the greatest value to his community.

I am inclined to favor the proposal of Mr. Fred Taylor of the American Society of Mechanical Engineers, who is now offering to take men immediately after the freshman year into his shops for a term of one year. Following this, he expects them to complete their course in college, going to his shops for the long vacation periods. He believes that in this way, he will not disturb the continuity of the college work, nor seriously handicap his shop organization, but that he will obtain the boy at a period of his training, when he is most susceptible to the proper industrial spirit. I agree fully with Mr. Taylor's view, but would prefer to have the boy spend the year in the shops between the junior and senior years in college, and after graduation spend another period in the shops, or other industrial establishment for further perfection and development of his practical ability.

Professor Schneider has insisted that it is impossible for a boy to gain good practical experience in college. To a certain extent, this may be true; but by employing men as teachers who have had successful engineering experience, by bringing in industrialists as professorial lecturers for a few weeks at a time; by having frequent individual lectures by men of prominent success in the industrial world, and by having the technical

school of the university directly under the supervision of a committee of successful industrialists, it is undoubtedly possible to so arrange the teaching, and courses, and life of the student that he can be filled with a proper professional spirit and pride in his life's work, and gain a true measure of his own value and worth in practical usefulness. All of these measures are being tried to a certain extent in the college with which the writer is closely associated and are seemingly working to excellent ends.

The author's contention that men, who are by nature unfitted for the engineering professions, are too frequently graduated from technical courses, has some foundation. Nevertheless, if the men at the head of the technical schools use sufficient pains to inquire into the characteristics of their freshmen and sophomores, and to eliminate those unfitted, such criticism need not long continue.

One of the greatest lacks to-day in our technical courses is undoubtedly the absence of sufficient regularly scheduled training in shop or industrial systems, and in the handling of men. This criticism has come to the writer from managers, presidents, and many others prominent in forwarding industrial enterprises. To correct this weakness, the writer has prepared and introduced into the curricula of one college, a course which is intended to give an excellent foundation, and at the same time prepare along the more specific lines required by the manager or superintendent. The course is identical with the other engineering courses in the freshman and sophomore years, including in the latter year, industrial history. In the junior and senior years, in addition to the fundamental engineering subjects of applied mechanics, materials of construction, heat and heat engines, structures, design, shop practice, applied electricity, geology, etc., are given courses in modern language—5½ years, including entrance. Shop systems, shop economics, factory planning, contracts, principles of economy, labor problems, and logic are included. If the criticisms of employers of technical graduates are well founded, this course should perform an important and valuable function, and should mark a direction toward which colleges should aim.

Willard S. Atkinson: Industrial education, in my opinion, should begin in the schoolroom. I have seen many instances of men who gave every evidence of developing into the best of mechanics through shop training, come to a full stop in their development, on account of lack of knowledge of the fundamentals underlying their work, or lack of mental training which would enable them to acquire such knowledge for themselves. To my mind much of the time now devoted in our public schools to subjects of purely academic interest, or entirely useless ones, could with profit be assigned to the study of elementary physics, mathematics and chemistry, and, it goes without saying, mechanical drawing. The embryo mechanic or industrial worker, whatever be his field, will find that there

will come a time, in the course of his development into a high-class worker in his field, when the knowledge thus attained of these subjects will seem to him the handiest tools he possesses. In this age of huge industrial enterprises and of immense and costly shop equipments, any attempt to duplicate the essentials of the aggregate of these must appear very meagre indeed. So why not devote all the time that can be made available to the boy, while he is at school, to a thorough knowledge of the underlying principles of the sciences on which the modern enterprises are based? Then when he leaves school and applies for a job, he at least has a foundation under him on which to build, and a mental training of the right kind that will help him to think. For in this age of automatic devices, and huge and intricate machinery, the industrial worker's value to his employer is becoming less and less dependent on his manual ability, and more and more on his knowledge of the work in hand and his ability to think.

The humble petition
of Joseph Jenks

To the Hon^rd Governo^r with the rest of the
Magistrates and Deputyes now
assembled at the Generall Court
in Boston the 10th of 3d m^th
1646

Humbly sheweth unto this hono^rd Court that whereas the Lord hath beene pleased to give mee knowledge in making, and erecting of Engines of Mills to goe by water for the speedy dispatch of much worke with few mens labour in little time my desire is to improve this talent for the publike good and benifitt and service of this Country; to which end my intention and purpose is (if God permitt) to build a Mill for making of Sithes; and alsoe a new invented Saw mill, and divers other Engines for making of divers sorts of edge tooles; whereby the Country may have such necessaryes in short tyme at farre cheaper rates then now they can; Now yo^r petitioner doth humbly beseech this Honoured Court that you would please to Grant mee this priviledge, and to order that noe other person shall sett upp or use any such new Invention or trade for the space of fowerteene yeares without my licence, which hath beene the usuall priviledge and liberty granted by the high Court of Parlayment in England to men that doe ~~first sett upon workes of this nature~~; least after your petitioner have expended his estate, study, and labour, and have brought things to perfection, another when hee seeth it, makes the like, and soe I loose the benifitt of that I have studied for many yeares before; which will tend to my great damage if not my utter undoeing; But if this Hono^rd Court shall please to order it as aforesayd, yo^r petitioner shall bee incouraged forthwith to sett on the worke and shall have cause ever to pray —

The magistrates consideringe the necessity of raysinge such manufactures as are mentioned in the petition, & being sufficiently informed of the ability of the petitioner to performe such workes [illegible] (for the encouragement of the petitioner) that his petition be granted, so farr as [illegible] any thinge new [illegible] so as the power of the Court be reserved to restrayne the exportation of such manufactures & the prizes of them to moderation if occasion so require

Jo: winthrop: Gov^r Consented to by y^e Deputs.
Edward Rawson.

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The Magiftrates confid[r]ing the neceffity of raifing fuch manufactures as are mentioned in the Pet: & being fufficiently informed of the ability of the Petition[r] to pform fuch workes: doe conceive it fitt (w[th] relation to the Deptyes concurrence herein) that his Petition be granted, fo far as concerns any fuch newe Inventions: & fo as it fhalbe allwayes in the power of this Court to reftreyn the exportation of fuch manufactures & the prizes of them to moderation if occafion fo require.

Jo: Winthrop: Gov[r] Confented to by ye Deputs
Edward Rawson.

Translation of the Petition of Joseph Jenkes and Grant of the First Patent in America for an invention of Apparatus, May 10, 1646, O. S.

Records of the Colony of Massachusetts Bay, Vol. 59, p. 26.

The endorsement at the bottom of the Petition by Gov. Winthrop, representing the Magistrates, and Edward Rawson, representing the Deputies, constituted the Grant, or original official record of the action taken by the two houses of the General Court in respect to the Petition.

A paper presented at the 26th annual meeting of the American Institute of Electrical Engineers, New York, May 18, 1909.

THE PATENT SYSTEM IN ITS RELATION TO INDUSTRIAL DEVELOPMENT*

BY FREDERICK P. FISH

In an address at Springfield, Illinois, delivered in February, 1860, Abraham Lincoln said:

> I have already intimated my opinion that in the world's history certain inventions and discoveries occurred of peculiar value, on account of their great efficiency in facilitating all other inventions and discoveries. Of these were the art of writing and of printing, the discovery of America, and the introduction of patent laws.

While other episodes in the history of the human race might well be added to the list, there can be no doubt that Lincoln was right in including the introduction of patent laws as an incident of prime importance. As he states later in his address —" The patent system . . . added the fuel of interest to the fire of genius in the discovery and production of new and useful things."

Progress has largely consisted in the recognition that things were wanted, and in the supply of them. Each success in meeting a want has led to an extension of the desires of man in a constantly widening circle. The inventors large and small have performed two functions. They have recognized the wants of the community in many cases before those wants were definitely apprehended, and they have supplied what was required or desired. They therefore stand in the front rank of those who have advanced the race.

It does not detract at all from the obligations which we owe the inventors, that they have dealt primarily with material, as distinguished from spiritual and intellectual progress. As a

* Through the courtesy of Mr. W. J. Jenks, one of our members, I am able to present to you a copy of an application for a patent made by the founder of his family in this country, Joseph Jenkes, to the Legislature of Massachusetts in 1646, and a copy of the grant made on his application for what is undoubtedly the first apparatus patent recorded in American history. It is interesting not only as illustrating the form of patent grants more than 250 years ago but because of the admirable quality of its English. A photographic facsimile and a line-for-line transcript in the quaint language and spelling of that time appear side by side as a prefix to this paper.

practical matter and in the long run, the latter are impossible without the former. Those who have advanced the industrial interests of man by increasing his control of the forces of nature, and teaching him how to apply those forces to shaping and transforming the raw materials which are available for our use, have supplied a necessary foundation not only for the physical needs of life, but also for that intellectual, moral, social and imaginative development which we all recognize as upon a higher plane than material well-being.

Much may be said as to the evils arising from the complexities of modern life, and in favor of the simpler conditions of former times. It may reasonably be argued that the extraordinary material progress of the last few years has resulted in a development to some extent one-sided. If such is the case, we need only to be patient, for the underlying forces of society will surely bring about the necessary readjustment of conditions when there has been time for the instinctive and automatic operation of those forces to correct the situation to the necessary extent. And the race will never lose the added comfort, the control of disease, the increase in the length of life, the startling economies in production and the wonderful facilities for intercommunication and transportation, bringing all parts of the world so closely together in thought and effort, which have resulted from our more recent inventions and discoveries.

There can be no doubt as to the validity of the view that has always prevailed, that material progress, although not in and of itself the highest goal to which we should aspire, is of underlying importance to our welfare.

It is no wonder, therefore, that the inventors and discoverers, who have enlarged the power of man to utilize for his own well-being the forces and materials of nature, have been generally recognized as great benefactors. In Mr. Walker's standard book on patents, he quotes the following from one of the early works of Lord Bacon:

> Now among all the benefits that have been conferred upon mankind, I discover none so great as the discovery of new arts for the bettering of human life. For I saw that among the rude people of early times, inventors and discoverers were recognized as gods. It was seen that the works of founders of states, law-givers, tyrant destroyers and heroes cover but narrow space and endure for but a time, while the work of the inventor, though of less pomp, is felt everywhere and lasts forever.

Not only was the work of inventors and discoverers in the old days rewarded by recognizing them as gods after they were

dead, but it was appreciated during their lifetime, and was encouraged by the ordinary forms of compensation by which all services to individuals and to the community are recognized. Inventors and discoverers were rewarded by the respect and admiration of their contemporaries as well as by more material advantages. As was but natural, such rewards were in the first instance bestowed by the immediate beneficiaries out of gratitude for services rendered. The idea of definite stimulation to such service, which is at the basis of modern patent systems, did not become consciously recognized until a late period of history.

Not until the 15th century of our era do we find any attempt to reward an inventor by the grant of a limited monopoly of his invention, and even then it is doubtful whether there was in such grant any definite conception of encouragement for future effort, as distinguished from reward for value already received. Moreover, social and governmental conditions were such that it is not impossible that such grants were based upon favoritism as often as upon any principle of justice or sound policy. When we find what was the equivalent of a patent granted in 1467 for the manufacture and sale of paper in Berne; another two years later in Venice for a privilege, exclusive for 5 years, of practising the trade of printing; another in 1507 in Venice of 20 years duration for the introduction of a process of mirror making, and a 10-year monopoly granted in France in 1551 for the same art, it is not at all clear that these isolated cases even foreshadowed our modern patent systems, in principle or in theory.

In England, however, toward the end of the 16th century, we find a rather definite recognition of the underlying basis of the patent system of to-day; but in so far as the sound, modern principle was apprehended at all, its conception was vague, and in its application it was confused with other ideas of governmental interest in, and control of industry, that were for a time, inconsistent with sound development.

The regulation and control of industries had been practised in England for many centuries. While charters had established artificial rules for the management of the different trades through the borough organizations, the guilds, and otherwise, at least as far back as the reign of Edward III what were in effect industrial patents had also been granted for the purpose of inducing foreign workmen to introduce new arts into England. In this way, Flemish cloth-makers, German armorers, Italian glass-workers and French iron-founders were persuaded to

establish new industries in England with the hope of royal patronage. These new workmen established their industries side by side with those already existing and to some extent in competition with them. The entire system was one of regulation and control, and only to a remote degree foreshadowed the patent system that was to come. Most of the grants in these early days were to a large extent local in character. They were effective because England, almost alone among the civilized nations, had a government that was really national in its character, recognized and dominant throughout the land.

When the reign of Elizabeth began, the conditions were favorable for the adoption of a real patent system. That great Queen and her advisers were earnestly interested in the industrial welfare of the country. There was a fair degree of economic unity. The guild regulations were already declining. The field was open for development upon new lines.

The earliest recorded application for an exclusive patent of almost the modern type was in the year 1558. It was for the introduction of a new art into England and was presented jointly by an Englishman and an Italian. The petition was granted in 1562 as a "reward of diligent travail" and "to give encouragement to others," the precise grounds which justify the issue of patents under the patent systems of to-day.

At about the same time, another Italian presented a petition explaining that he had invented certain furnaces and "wheel machines" which others would copy without remunerating him unless he was protected. He prefaced his application with the suggestion that "nothing is more honest than that those who by searching have found out things useful to the public should have some fruits of their efforts and labors, as meanwhile they abandon all other modes of gain and are at much expense and inconvenience and often sustain much loss." This again is a statement which expresses the point of view and the experience of the inventors of the present time.

Prior to 1570 not less than twelve patents were granted in England for chemical products and processes and six for mechanical inventions. It is interesting to find these patents dealing with the familiar subjects matter of salt, alum, saltpetre, oil, glass, cloth, dressed leather, and machines for dredging, draining and grinding. Soon afterwards there was issued a patent for sailcloth and one for playing-cards. From that time to the present, letters patent have been granted continuously in England as a reward to the inventor and an encouragement to others.

But the situation was not yet crystallized. The power to grant patents, where it was for the public interest that they should be granted, gave the opportunity to establish monopolies which were distinctly hostile to the public good. Elizabeth and her successor, James I, issued many such patents and licenses analogous to them. Many of them were given to favorites. Some of them were issued solely to increase the revenue of the King, who attempted to exact large compensation for the grant. Instead of dealing only with the question of new industries, many of them were issued for the control of those that were well established. Some were in a way the equivalent of a modern protective tariff. Others were ostensibly based upon the necessity of regulating the quality and character of the product. Some of them amounted to but little more than an excise tax. All through this period, however, the encouragement of invention and the introduction of new industries were regarded as of prime importance, and while the monopolies granted by royal authority formed a constant source of trouble, leading repeatedly to conflicts between the King and Parliament, no one questioned the soundness of a public policy that obviously stimulated industrial development. In a leading case of the utmost interest, decided in the year 1603, in which the court held that a patent granting a monopoly on playing-cards was invalid, the successful counsel, who contended strenuously that such a monopoly as the one before the court was contrary to English law, said in his argument.

> Now, therefore, I will show you how the judges have heretofore allowed of monopoly patents—which is that when any man by his own energy and industry or by his own charge and industry, or by his own wit or invention doth bring any new trade into the realm, or any engine tending to the furtherance of a trade . . . that never was used before, and that for the good of the realm: that in such cases the King may grant to him a monopoly patent for some reasonable time, until the subjects may learn the same, in consideration of the good that he doth bring by his invention to the Commonwealth, otherwise not.

It is interesting to note that in this case an argument for the validity of the monopoly was that playing-cards were merely a "vanity"; that card playing involved "the vices of deception by servants of their masters, and the employment of time that ought to be devoted to other interests and not to such enormities" and that therefore "the Queen might prohibit it by patent."

This unsatisfactory condition of things led to what is known

as the "Statute of Monopolies", passed by Parliament in the year 1623. This was the first patent statute. Prior to this, the whole subject of letters patent had been dealt with by the courts, in accordance with the principles of the unwritten common law of England.

The "Statute of Monopolies" declared invalid all monopolies then existing and relating to the "sole buying, selling, making, working or using of anything within this realm or the Dominion of Wales, as contrary to the laws of this realm," and forbade such monopolies for the future. Certain specific grants were excepted, and there was the further general exception that the statute should not extend to any letters patent then or thereafter granted for the limited terms named, which gave a monopoly of the

> Working or making of any manner of new manufactures within this realm, to the true and first inventor or inventors of such manufactures which others at the time of making such letters patent and grants shall not use, so as also they be not contrary to the law nor mischievous to the State, by raising prices of commodities at home or hurt of trade or generally inconvenient.

During the unsettled periods of Charles I and of Cromwell, there were, as might have been expected, many vicissitudes in the development of the patent law; but the principle that had characterized the English law from the beginning, that a patent might be granted as a reward to the inventor and as an encouragement to others, ultimately prevailed in England, while all the other industrial monopolies of the Tudor and Stuart periods disappeared. Such was the situation of the English law and practice at the time of the American Revolution.

In the colonies on this side of the water, there had, of course, been but little industrial development. In spite of that fact, the value of patents had been recognized and a number had been granted. The first to be issued in this country appears to have been granted by the General Court of Massachusetts Bay Colony in October 1641 to one Samuel Winslow for a process of manufacturing salt. The term of the patent was for 10 years and the grant was conditioned upon works being established within the year. In 1656 the Massachusetts Bay Colony also granted a patent to John Winthrop, son of Governor Winthrop, for the "sole privilege of manufacturing salt after his own particular method" for a term of 20 years. A number of other patents were issued in colonial times by Massachusetts, Connecticut and Pennsylvania.

The articles of confederation adopted by the colonies in July, 1776 contained no provision for the granting of patents, but the states issued patents independently as the colonies had done. For example, in 1785 one John Rumsey obtained patents from the states of Maryland, Virginia, Pennsylvania and New York for a new type of boat which during the previous year had been tried on the Potomac river in the presence of Washington.

Those who framed the constitution of the United States recognized, with characteristic wisdom, the importance of encouraging industrial development. It is interesting to note that a proposition was before the convention that prepared the Constitution, by which Congress would have been empowered "to establish public institutions, rewards and immunities for the promotion of agriculture, commerce and manufactures." This broad but vague provision was not adopted. In its place, we find in the Constitution that authority is given to Congress "to promote the progress of science and the useful arts, by securing for limited terms to authors and inventors the exclusive right to their respective writings and discoveries."

The time has not yet come when we have ceased to admire the far-sighted wisdom of the framers of our Constitution. It is one of the most interesting things in history that it has proved so nearly adequate to meet the radically changed conditions of modern times. Many of our most thoughtful men believe that our prosperity and well-being depend to a great extent upon a strict adherence to the underlying principles upon which it is based. In the hands of a long line of eminent judges who have passed upon it and construed it, it has proved its adaptability to social conditions utterly different from those existing at the time when it was prepared.

Whatever may be its merit as a whole, there can be no question as to the singular completeness and adequacy of the simple clause which I have just quoted and upon which our patent system is based. Acting in harmony with the spirit as well as the letter of the constitutional provision, Congress has adopted a scheme of patent protection the wisdom of which has been shown by the results. No one can say how ineffective the clause might have proved if the clause and the legislation under it had been construed and applied by unwise judges; but the great jurists, from Marshall down, who have developed and fixed the meaning and spirit of other parts of the Constitution, have applied to this section, with the most satisfactory results, the patriotic and far-sighted intelligence that characterized all their work.

It is an interesting circumstance that this provision of our Constitution says nothing as to the promotion of industry or the introduction of new industries. It refers only to the promotion and progress of science and the useful arts. This seems to me to have introduced a radically new idea into the view of the function and scope of a patent system.

The patent system of Great Britain, to which I have briefly referred and which was the only one that had received any development before our own, approached the matter from a different point of view. Its object was to promote progress, primarily by encouraging the establishment of new industries in the Kingdom of Great Britain or the improvement of those already established. The English patent law, from the beginning, was construed as affording to one who brought into England an industry or an improvement from a foreign land, a right to the same protection as that to which he would be entitled if he had been the inventor or the discoverer of that industry or that improvement. The reward of the law was not for inventive achievement. It was for giving to the community knowledge of an art that it did not have before. The man who by reason of facilities for travel or intelligent powers of observation acquired information as to a manufacture long carried on in foreign countries, was exactly on the plane with the man who had originated the idea. This has been a characteristic of many of the important patent systems that have been developed in countries outside of the United States and England. In great part they take the English view. They reward the man who introduces or improves the manufacture and not alone the man who originated the improvement. For the first time in the world, the framers of our constitution laid the entire stress of their effort on the recognition and reward of inventive thought. They adopted the new theory that if men are encouraged to invent by the certainty of reward, or, as Lincoln put it, if the fire of genius is fed with the fuel of interest, the industries will take care of themselves. They believed that there was no way of securing new industries or of making improvements in old industries so effective, as to encourage intellectual effort by way of invention and discovery. As already stated, they ignored the suggestion that there should be other forms of reward for the inventor or discoverer. They authorized only a single reward, a single recognition of his merit, namely, that there should be secured to him the ".exclusive right" to his

invention for a limited time. It is to be noted that they did not authorize Congress to give him a right that was less than "exclusive"; nor did they authorize any discrimination as to the personality of the inventor or the magnitude and character of the invention.

It is not impossible that the framers of the Constitution fully appreciated the nature and efficacy of the patent system which they were authorizing. If they did, it is only another instance of their far-sighted wisdom and intelligence.

We all know generally the development of this provision of the Constitution under the influence of friendly public opinion, sympathetic legislation, and strong, intelligent, and comprehensive judicial interpretation. We have to-day a patent system which is generally regarded as the best in the world. Under this system, applied science and the useful arts have been promoted as in no other country and at no other period in the world's history.

Much may be said of some assumed abstract quality in the American mind and temperament which has led to our marvelous industrial development of the last century. Many reasons may be given for the existence of the quality that has made this progress possible. It may be referred to climate, to the influence of democratic institutions, to our system of education, to the fact that we as a community are made up of the more enterprising off-shoots of many races which have here met and under our unique conditions have developed new capacities. There may be a certain amount of truth in each or all of these hypotheses. None of them, however, seems adequately to explain the result, nor are any of these considerations so related to some or all of the others as to make it by any means certain that it is for any such reason that we have advanced in industrial achievement by leaps and bounds. During the period of our greatest development, namely, during the last 50 years, we have been closely associated with all the other great nations of the civilized world. They have known of us and of our work. We have known of them and of their work. There has been a constant interchange of ideas, methods and products of manufacture between one country and another. I doubt if any of us, in view of our knowledge of and relations with the English, French, Germans and Italians, can be satisfied that we have done such great things industrially because of our peculiar situation or by reason of any of the special conditions above referred to.

There can be no doubt, however, that we have had a dif patent system administered in a different spirit. Is it no sonable to believe that this is at least the most importar vantage we have had in our industrial work and that beca it more than because of any other consideration, we hav tained our great success in applied science and the pra arts? I am quite prepared to admit that many other siderations may have played an important part in our dev ment, but it seems to me certain that that development is la due to the special character of our patent system and it ciency in encouraging invention.

What is our patent system, not merely as a statute or vision of law, but in its relation to actual affairs? When str of all the many refinements that are necessary for its pra application and administration, and when regarded ir simple form in which it is apprehended and appreciated b millions of people in our community who know of it an interested in it, our scheme of patent protection amounts si to the grant to one who has had the intelligence and sk make an improvement in the useful arts which involves i tion, of the exclusive right, for the term of 17 years, to r use and sell that which he has discovered, invented or desi Almost everyone in the country knows this much of the ch teristics of an American patent. Everyone who has an formation on the subject at all, also knows that this abs right to exclude everyone else from making, using and s the patented thing for the term of 17 years is one that ma exercised and enjoyed by the patentee exactly as he ple He may himself work under the patent. He may sell the e patent. He may sell a part interest in the patent. He grant licenses in any form he chooses to any number of pe provided his licenses are consistent with each other. He give one a right to use only; another a right to manufacture sell. He may give to one a right, exclusive or non-exclu to apply the invention in one field and to another an eq valid right, exclusive or non-exclusive, to apply it in an field. There is no limit to the inventor's absolute control c thing that he has invented, within the limits of honesty and dealing imposed by law generally on all transactions.

Moreover, everyone knows that the cost of taking a pa is ordinarily not great and that once the patent is grante remains in force for 17 years without any further expense t

patentee. He is not obliged to work under the patent, nor is he obliged to make any use at all of the invention unless he chooses. The law recognizes that such a requirement is not needed. Self-interest and a desire to get a return from the invention will be quite enough to stimulate the inventor or patentee to effort and to insure commercial development of his invention, if it has any value. At the end of the term, the patented invention is free for all to use.

If capacity for invention can be stimulated at all, there could be no more effective means of exciting it to the utmost than the opportunity of securing just this reward which our patent system gives. Every one of our industrial workers, and a vast number who are not engaged in industrial effort, knows that if one has a new idea in any of the useful arts which amounts to invention, he may monopolize that idea for the term of 17 years. Short as is this period measured by some standards, it seems and is long as compared with the period of the activity of the individual. The inventor may recognize that there is no immediate market for his invention. If, however, he feels that his idea is a good one, he will work it out and patent it with the hope, amounting in most cases to a belief, that during the 17 years of the life of the patent, the art will develop so as to make his idea of value to him. If he is a poor man absolutely unable to go into business on his own account, he feels that if he gets his patent, he has a substantial length of time during which he may seek for help in the commercial introduction of his invention, work up enthusiasm for its exploitation, find a purchaser for it or licensees who will work under it. The manufacturer who is not only concerned with the successful application of his present industrial methods but is looking forward to their improvement, or who is seeking for new things to manufacture, is ready to take hold of new ideas, whether developed in or outside of his own factory, to experiment with them at a large expense and to secure a patent on them, even though he knows that he can not immediately utilize them. He is glad to take the chance that years later the ideas which he thus controls may fit into his industry and become of profit to him for part of the 17 years during which he controls them. The capitalist, large or small, always ready to risk his money in the chance of a substantial return, will take up with a new invention even if he clearly sees that years, during which his disappointments may be great and his expenditures large, are sure to elapse before he

can hope to make the fortune which he is after, for he believes that his time for profit will come during the latter years of the 17-year term of the patent. The result is that there is the strongest stimulus to manufacturers, employees and capitalists, and even to those who are not directly in touch with industrial operations, to seek out new ideas, new devices, new methods and new structures and to patent them, because they have the assurance that if the patent is once obtained, the invention will be controlled absolutely for the term of 17 years, during which it may have value, and that if at any time during those 17 years there is a demand for it, it can be handled, as a matter of right, in that way which is most advantageous to the owner. Under these circumstances they are willing to take large chances.

Almost every one knows the uncertainties of patents and patent enterprises. Almost every one appreciates, what is probably the case, that the greater number of patents never prove to be of value. No one may find it profitable to use the invention for which a patent is taken. Many enterprises, based upon patents, fail for one reason or another. But the chances of success, even if they are no more certain than in the case of those who seek for gold in the bowels of the earth, are quite sufficient to justify even the most prosaic man in an enthusiastic effort to promote the useful arts if he has the inventive faculty himself, or to contribute his time or his money to such promotion, as manufacturer, business man, or capitalist, if he comes in contact with an invention that seems to him promising.

As a rule, the systems of foreign patent law are less liberal than ours. The term of the patent is often less than 17 years. It is sometimes more expensive to secure the patent, and not infrequently, after the patent is once obtained, the inventor can not look forward, as he can in this country, to enjoying the right to his monopoly for the term of the patent on exactly the basis he chooses. In some countries he has to face the payment of onerous fees annually or at an interval of a few years, and if such payments are due at a time when he is out of funds or discouraged, as all patentees and patent owners are likely to be even if the invention is one of great merit and sure to be ultimately successful, the patent may be lost for non-payment of the fees. In some countries, the patentee or those interested in the invention have to look forward to the forfeiture of the patent if there is no manufacture under it within a certain limited and often a very short period. It may well be that this

period will expire before the art is ready for the invention or before the invention is commercially perfected or before capital can be secured for its development. In some countries the owner of the patent may be forced to grant licenses, the terms of which are fixed without his consent, and be thereby deprived of that control of the invention which is essential to its profitable exploitation.

I believe that if any such provisions had become part of our American patent system there would have been nothing like the same effort on the part of our people to promote the progress of the useful arts as has characterized them from the beginning, and particularly during the last 50 years.

Even as it is, there is discouragement enough to our inventors and patentees. Although our patent system is, in my opinion, the best in the world, the administration of it in the Patent Office and in the courts involves difficulties and annoyances to the patent owner which in some cases seem to him intolerable. For example, no one can be sure that the patent which he gets is valid, nor as to its scope. The Patent Office does admirable work but, of course, sometimes errs, and the entire art can not be known to it. The uncertainty in the application of the law to the facts, which is naturally greater in patent causes than in most branches of the law, makes it a difficult matter to foresee how a patent will be viewed and construed by the courts and leads to more or less conflict in judicial decisions. This is a great evil, affecting not only the owners of patents but the public generally, who are unable to come to a sound conclusion as to their rights even when acting under the advice of competent counsel What will be the finding of the courts on questions of validity and scope can not be foreseen. The methods of taking testimony in patent causes are cumbersome and expensive. It seems certain that an intelligent and well-directed effort on the part of the patent bar and the courts could bring about a reform in this direction.

Such evils characterize the practical operation of all human institutions. We can only hope to minimize them.

It is not at all impossible that the establishment of a single Appellate Court to take the place of the nine independent Courts of Appeal now existing would go a long way towards helping the situation. Such a court would surely bring about harmony of judicial decisions to an extent that does not now exist. It could also criticize intelligently and forcibly the present methods

of taking testimony, and stimulate the inferior courts and the bar to deal wisely with this important matter.

The Patent Office can undoubtedly be organized to do more effective work in its examination of applications. On its judicial side in interference controversies, where it has an organized machinery for trying out cases of priority of invention, there is the same cumbersome and expensive procedure that characterizes trials in court, with the added hardship of an excessive number of appeals. This also could and should be reformed.

The difficulties of the American inventor and patent owner, great as they are, and they have been so great as to lead some to express disgust with the entire patent system, are due not to the law but almost altogether to defects, some of which can never be eliminated, in the ancillary machinery of the law; and although inventors, patentees and the owners of patents have in individual cases undoubtedly suffered hardship from the inadequacy of the procedure devised to secure and to maintain the rights vested in them under the constitution, if we look at the matter in a large way there is no doubt whatever that the patent system of the United States has to a notable extent stimulated invention and encouraged and rewarded inventors.

In criticizing the procedure of the courts and of the Patent Office, we must not overlook the inherent difficulties of the situation. The problem is so complex and the administrative machinery has grown up under such conditions, that it is almost wonderful that a more serious situation has not developed. It is to be deplored that there is so much expense, trouble and uncertainty in the administration of the patent system, but it would be much more unfortunate if the law was wrong or if there was, in general, substantial injustice in the grant of patents or the protection of rights and of industries carried on under patents. This I am sure is not the case.

It is easy to find fault with the courts and the Patent Office for apparent or real error in special cases or for ineffective and cumbrous organization and methods of procedure. All human institutions are subject to the same criticism. I am glad, however, to have the opportunity of expressing my own belief, that the Patent Office of the United States, hampered as it has been by inadequate appropriations, insufficient facilities and the pressure of work which has increased faster than the organization necessary to handle that work could be developed, has

been, on the whole, more efficient than any other administrative department of our government. Of one thing I am sure, that in no department has there been greater intelligence or greater honesty and sincerity of purpose. Further, I believe that nearly all who are competent to judge, agree with me that the work of the United States courts in patent causes has been of a high order and is entitled to great praise. There has never been a question as to the good intention of those courts and the certainty of their earnest effort to apply and develop, logically and fairly, the constitutional theory upon which our patent system is based. It has not been much of a disadvantage to the patent system or to patentees that the judges have as a rule had no special technical training. Their lack of such training has simply made their work more strenuous. While undoubtedly, as in all other branches of the law, cases relating to letters patent have been wrongly decided, the conclusions of the courts of the United States on patent matters as expressed in the recorded opinions, have been characterized to a marked degree by a sympathy with the spirit of the constitution and the law, and by an intelligent and straightforward application of principles to facts of an unusually difficult and complicated character. The record is most creditable.

But whatever discouragement to invention exists by reason of imperfections in the work of the Patent Office and of the courts, it is a matter of common knowledge that we in this country have the benefit of a patent law which stimulates and encourages invention to a marked degree. The condition of our industries as compared with those in other countries is proof of this, and proof of a higher order than any statistics that are available. What statistics there are, serve only to confirm this view. For example, in 1903, the last year for which I am able to get information as to all the countries, there were issued in the United States 31,046 patents (50,213 applications were filed in that year), while in the three other great industrial countries, England, France and Germany, there were issued in the aggregate 36,110 patents.

It is a familiar proposition that patent protection exists primarily, not for the benefit of the inventor, but for the benefit of the community. It is easy to argue that an inventor should have a property right in his invention, exactly as an author should be allowed to control for his own benefit, his ideas and form of literary expression, which he is under no obligation to

give to the public. We all instinctively feel the justice of such a contention. But the right of the inventor as well as of the author, so far as it exists in theory, is of no value to him unless protected by a definite rule of law, and the extent and character of the protection to be granted is to be determined by the community in view of its own interests. If it were the sentiment of the people that an author should have no other protection than the exclusive right to his work prior to publication, and that the right of the inventor should be limited to the monopoly of his invention so long as he could maintain it as a trade secret, the sole ground upon which law or legislation to that effect could be properly criticized would be as to its wisdom or expediency. Exactly as the whole civilized world has come to the conclusion that the author's abstract right should be recognized by securing to him a monopoly of his publications for a limited time under conditions more or less wise and liberal in the different countries, and this not merely or so much with the view of rewarding the author as to encourage authorship, so the patent systems of the world agree in giving legal sanction to property in inventions, partly because of the feeling that the inventor has some abstract right in them, but for the most part as a matter of public policy to stimulate and secure the promotion of the useful arts.

It is not alone by the specific disclosure of particular inventions that the public gains from our patent system. Inasmuch as a United States patent is particularly attractive because of the fact that the expense of getting it is ordinarily not great, and when once obtained there are no further payments to make and no danger of forfeiture for non-working or for any other reason, so that the patentee has the legal right to control his invention and utilize it as he pleases for his own advantage for the term of 17 years, most of the important and significant inventions made anywhere in the civilized world are likely to be patented in this country and thus disclosed to our manufacturers, inventors and technical men soon after they are made. The Official Gazette of the United States Patent Office prints every week the claims of all the patents issued in that week. Copies of each patent may be obtained for 5 cents each. The result is that in every art there are many who read every patent relating directly or indirectly to their work. Thus all new ideas are promptly circulated throughout the community and immediately brought home to those who are capable of apprehending them and working from them.

Every invention is potentially the cause of others. The study of a new idea, whether developed in England, France or Germany, or the United States, when explained in a patent, excites many persons not merely to improve upon the specific form in which that idea is embodied in the patent, but to work out alternative schemes for accomplishing the same result. It is not infrequent that the reading of a patent specification leads to the invention of an important improvement in a radically different direction, because of the train of ideas which arises in the mind of the reader and which might never have been excited except for the disclosure of the patent. Our patent system, therefore, does not merely stimulate invention by promise of reward, but it affords a basis for new ideas by bringing to those who have the inventive faculty just the kind of information and inspiration needed to start them on lines of thought or of investigation that will lead to the promotion of the useful arts.

It is sometimes suggested that our patent system should discriminate against foreigners. The policy of our law from the beginning has been almost without exception directly the other way. I am satisfied that there should be no change in this policy. Not only is our industrial progress stimulated by that immediate knowledge which we get from the patenting in this country of inventions made in foreign countries, as to which we otherwise might not be informed for years, but the issue of such patents gives our industries the benefit of the use of inventions made abroad much earlier than would otherwise be the case.

Sometimes we buy or take licenses under the patents granted to foreigners. Sometimes, although less frequently, the foreigner establishes an industry in this country; but in all cases we get an early knowledge of the foreign work that is of great value as an exciting cause of further invention.

The extent to which, in the aggregate, inventors and those who supply the capital necessary for the development of inventions actually make a profit out of patents in this country is an open question. It may well be that there has been a loss rather than a gain from patents to the individuals who have secured and worked under them. That the community is the gainer from the patent system is altogether clear.

Let us consider for a moment the way in which work is done and applied science and the useful arts are promoted under the stimulus of our patent system.

Those who invent may be divided roughly into classes. The first to which I shall refer comprises the individual inventors to whom once or several times in the course of their lives occurs an idea that seems likely to be of value in the industries. Such an inventor may be of any station in life or of any occupation. The idea may be suggested to him by his work or by accident. If he has faith in it, he develops it, puts it into practical form, either on paper or by actual experiment or construction, and patents it, making in the specification of his patent the full disclosure of it that the law requires. He gets nothing by his patent except the right, for 17 years, to exclude others from making, using or selling his invention except on terms satisfactory to him. Frequently he is without means. Sometimes he has business experience and capacity but often he is utterly incapable of developing or handling a business enterprise. Sometimes he is in such relations to others as to know where capital and business experience are available for promoting the particular invention he has made. Not infrequently, he has not the remotest idea where he can find either. In this matter, a great deal depends upon whether his invention is one definitely germane to an existing art, or radically new or at variance with the prevailing methods and tendencies of the industry to which it is related. Sometimes he is himself without capacity to put his new idea into commercial shape where it will be of value to the community, and he has to secure the help of engineers or inventors of a more practical turn of mind before his invention is embodied in a form which has any real utility.

The value to him and to the community of a patent issued to such an inventor, depends as much on the inventor's skill or good fortune in introducing his invention into actual use as upon the merits of the invention.

It is probable that the vast majority of patents issued are of no real importance. Many of them are for definitely worse contrivances or distinctly more inefficient ways of getting at the result than others already in existence. Many are based upon radically wrong principles, although they seem to involve ideas that are not only practically effective but brilliant in their ingenuity. Frequently an invention is made years before the art is ready for it, and even the most diligent and intelligent efforts can not advance the art to the point where it will utilize the invention. In many, and perhaps in most cases, the commercial success to be obtained in working out any specific

patented invention is a gamble only comparable to the gamble of the mining promoter who seeks to develop a new mine from surface indications.

The invention of the telephone was made by an individual inventor whose name will be forever famous in the annals of science and industry. While the result of that invention has been the creation of one of the most far-reaching and important pieces of our modern industrial and social machinery, it is easy for those who are familiar with its early history to see that it might have failed to secure any recognition except as a scientific toy. While the greatest credit is due to Professor Bell for his invention, it is almost equally to his credit that within a comparatively short time after he had solved the problem of the electrical transmission of speech, the wonderful possibilities involved in that invention were definitely foreseen by him at a time when few men had even a glimmering of its real commercial importance. In spite of Professor Bell's clearness of vision, th development of the invention might have been indefinitely postponed, in fact it might never have become such an important part of our social and industrial machinery, if he had not been fortunate enough to find, while his patent was still young, the right capitalists and the right business men to do the work of organizing, developing and financing the industry.

Sometimes these individual inventors find their capacity for originating new ideas so great that they adopt invention as their profession. Of this class of inventors, our fellow-member, Mr. Edison, is perhaps the most conspicuous. It is unnecessary to remind this audience of the wonderful fertility of his inventive genius or the wide range in which he has done marvelous work for the advancement of the useful arts.

Such professional inventors occasionally work with their own capital, and some of them show capacity for embodying their inventions in practical form and the requisite business ability to direct to a greater or less extent the commercial introduction of their inventions. But this is not always the case, and with them as with other classes of inventors, it is generally essential that their inventive faculty should be supplemented by the capital and business energy as well as the technical and inventive skill of others, in order that the joint efforts of all concerned may result in a real improvement in the useful arts.

With them as with the man who makes but one invention, it is only because of the protection afforded by the patent system

of the United States, that capital and business experience can be diverted from standard occupations to the chances of an enterprise based upon new inventions. There are too many well-known instances of the absolute failure of what seems a promising invention to make it possible to attract capital and business ability to the inventor's support unless, because of patent protection, there seems to be an opportunity to secure a return commensurate with the risk involved.

The history of the telephone industry also affords an illustration of the activities of another conspicuous class of inventors, whose work under the protection of the patent system of the United States is most effective, and of the conditions under which they make their contributions to the progress of the useful arts.

The business men and capitalists who took hold of the telephone enterprise in the early days, would never have risked their time, energy and money, if they had not believed that by reason of their fundamental patent they could exclude all the people of the United States from making, using or vending telephones without their consent, not for a period too short for effective work, but for one that was reasonably long, namely, 17 years. They undoubtedly foresaw, although very inadequately, that the business upon which they were entering would be full of disappointments; that only by the exercise of the utmost ability and energy could it be made to succeed; that a very great investment would be required; that large sums would have to be spent in engineering and in perfecting apparatus and that they would have to run the risk of judicial decisions as to the validity and scope of their patent. In going on with the enterprise, they took their chances with all these contingencies, hoping and believing that in spite of them, they would secure an adequate profit.

From the very beginning it was clear that the telephone invention in and of itself could be of value only when supplemented by many other inventions which would be found necessary in the effort to make the main invention commercially effective.

Because the business men of the organization knew that suitable apparatus must be developed, and that every added invention would strengthen their position not only during the 17 years of the main patent but during the 17 years' term of each and every one of the patents taken out on subsidiary methods and devices invented during the progress of com-

mercial development, one of the first steps taken was to organize a corps of inventive engineers whose duty it was to make every effort to perfect and improve the telephone system in all directions, first, that it might become of greater commercial value; and, second, that by securing accessory inventions, possession of the field might be retained as far as possible and for as long a time as possible. The result has been a very great number of patents taken by the engineers and inventors of the Bell company for inventions in, or relating to, telephones or telephonic apparatus. Moreover, the patents of others were studied with the utmost care and an innumerable number of outside inventors found a ready market for patents which they secured on their contributions to the telephone art. It is not impossible that nine-tenths of the patents so acquired were for inventions which proved to be of no real importance in the development of the art, but taken all together they have made the mechanism of the business and have given us the wonderful system of telephonic intercommunication that we have to-day. Without the stimulus afforded by the patent laws of the United States, it seems to me certain that the telephone art would not have attained its present situation in the social and industrial organization of to-day, in many times the number of years that have elapsed since Professor Bell's invention.

In the same way, in practically every manufacturing and technical organization in the United States, careful attention is given to the promotion of the particular art, whatever it may be, by the employment of men who have shown the requisite capacity, to make inventions relating to the special industry. Many such inventors are under definite contract to assign their inventions to their employer, receiving compensation for this sort of work either directly or in the form of a salary. If the individual workman in such an establishment who is not under contract makes an invention, it is apt to be acquired as part of a general policy which the employer has adopted and to which the employees have assented, on terms that are agreed upon.

Very many of these inventions make no real impression upon the art and are simply a source of expense and annoyance to the manufacturer. The justification of the manufacturer for the outlay, and the energy devoted to invention in his establishment is found in a general advance in the quality and character of his output and in economies and improvements in operation, the specific value of which can not be estimated, but whose general importance is very great.

I have in mind a particular business enterprise based upon patents which will in a short time, by the introduction of new machinery, have released the labor of not less than 100,000 skilled operatives in this country for other kinds of work than that upon which they were formerly engaged. I knew of the inventions upon which this enterprise is based from the time of their inception. It occurred to an experienced manufacturer, not himself an inventor, that there could be a revolutionary improvement in a certain standard machine. He employed skilled mechanics of inventive capacity to work upon an idea, not in itself involving invention, which he was able to state to them with sufficient clearness. One set of machinery after another was devised, constructed and thrown into the scrap heap. Finally, after the expenditure of more than $300,000, the manufacturer was able to put his first machine into commercial use. More than a million dollars had been invested in the enterprise before there began to be any return. Repeatedly since then very large expenditures have been required to secure further inventions needed to perfect the apparatus. Although the enterprise to-day is a great success, changes are still constantly made in the mechanism, for which changes patents are taken.

If it had not been for the patent system of the United States, the manufacturer would never have even contemplated the development of this machine. If there had been any provision in the statutes by which any patent which he acquired would have been forfeited unless manufacture under it had begun within a relatively short time, as is the law in Germany and elsewhere, he would not have touched the enterprise.

The question is sometimes debated whether or not under the patent system of the United States the inventor himself obtains the share of patent profits to which he is justly entitled. If he does not get his fair proportion of such returns as are due to his invention, the fault is not with the patent system, for the whole matter of dealing with his invention is in his own hands. No one except the true inventor can obtain a valid patent, and he need neither work under it, assign it or any part of it, nor grant licenses under it unless he chooses. In so far as there is any foundation for the contention that under modern conditions, the inventor himself does not get all that he should for his work, the basis for the contention is not the patent system or the law, but the social and industrial conditions which prevail.

It may be argued with equal force and with the same reasonableness or unreasonableness that capital gets more than its share of the return from industrial enterprises as a whole, and without regard to the patent features which are merely incident to them. If wages are lower than they should be under ideal conditions, if salaries of engineers and executive officers are less, it may well be that the inventor fails to get his share, and for the same reasons, which are based solely upon the principles upon which society is organized as the result of an evolution of thousands of years. A discussion of this broad question is outside of the scope of this paper.

No statistics are available on the point, but it is my own impression that there is no valid foundation for the suggestion that inventors are not adequately compensated. It is undoubtedly the case that in individual instances, meritorious inventors fail to secure all to which it might be said that they were fairly entitled as a reward for their work. It is equally true that other things than merit determine to a large extent the reward of the professional man, the artist, the architect and the engineer. It probably would not require very close analysis to show that in very many cases the material reward of the business man and the capitalist is not at all proportionate to his individual merit. Accident, opportunity and particularly the fitness of the particular man for the special environment in which he happens to find himself, have much to do with all forms of reward that come to men whether in the goods of the world or in reputation. This is as true of the inventor as of other men; but taking inventors as a whole, I do not believe that they have cause to complain.

To many of them their inventive capacity is part of their stock in trade. Because they can invent, they receive substantially more by way of salary and standing than would otherwise be the case, as do those who are deft in the use of tools or who show skill as designers or capacity as salesmen. In many instances, some of which have come to my personal attention, the inventor is the only one who has made any profit out of an enterprise based on patents, his associates the capitalist and the business man having suffered great loss. In other instances, both the inventor and his associates have been adequately compensated.

In dealing with this whole question we are apt to overlook the fact that by far the greater number of inventions, par-

ticularly at this late stage of the older arts, are mere details of improvement or alternative devices or methods from which, regarded specifically, no large amount of profit can be made by anyone. Such inventions are only valuable as incidents to a manufacture. They can not be segregated and appraised independently.

In spite of occasional cases of hardship such as occur in every business and profession, I believe that the ordinary social and business principles, which, irrespective of the patent law, determine the distribution of the profits of invention between the inventor, the capitalist and the business man, result in securing to the inventor, on the average, as large a portion of that to which he would be entitled under ideal conditions (such as never can be attained) as come to any other workman in our vast industrial army.

In this paper I have not undertaken to deal with the many special features of our patent system which are of great interest, nor to point out to any extent the details in which according to my own opinion the law is defective or capable of improvement. It is quite probable that the amendments to the law which I should advocate would be as unwise and ineffective, in my opinion, as most of those suggested by others. I have only sought to call to your attention some of the underlying principles at the basis of our patent policy and to show how those principles seem to me to work in practice. If I am right in my belief that we have a fair and reasonable patent system which promotes the progress of the useful arts more effectively than any other that has been devised, and that to the beneficent operation of our law is due to a large extent our recognized position of supremacy in the industries, I trust that you will all agree with me that our system of patent protection is entitled to that hearty support and friendly consideration which I believe it to have generally throughout the United States. No human institution or human law is perfect. We all know that there are some provisions in our patent statutes that might perhaps be amended to advantage. We all know that our patent laws like all other laws work hardship in special cases. We all know that sometimes they result in individual instances of injustice, although it is more than probable that there would be a great divergence of opinion as to the particular cases in which there was such injustice. Generally speaking, however, it seems clear that in their present form and with their present spirit as they have been

developed and applied by the courts, they are among the most effective agents for the promotion of our national and individual prosperity and as such are entitled to the cordial support of all. They are particularly entitled to recognition as a social and industrial force of the utmost importance by the members of this body, many of whom are inventors of a high order and all of whom are definitely engaged in lines of work which probably could not have been developed to their present state of relative perfection in a thousand years from the date of Faraday's work and the construction of the Gramme machine, if it had not been for the stimulus of the patent systems of the world, and in particular of the patent system of this country.

DISCUSSION ON "THE PATENT SYSTEM IN ITS RELATION TO INDUSTRIAL DEVELOPMENT." NEW YORK, MAY 18, 1909

President Ferguson: The subject chosen for to-night is one in which every member of the American Institute of Electrical Engineers is either directly or indirectly interested. We are especially fortunate in having among our members a man able to discuss this subject so authoritatively. He is a lawyer who stands foremost in his profession, having been general counsel to the largest electrical manufacturing company in this country for more than 20 years. He is able to treat the subject not only from the legal standpoint but from the standpoint of the business man and executive officer, as he was for six years the president of the largest electrical undertaking in this country representing an investment of over 500 millions of dollars, the American Bell Telephone Company.

Francis B. Crocker (by letter): This subject is one in which I am greatly interested. Ever since Charles G. Curtis and I jointly obtained a patent for a telephone call system in 1880, while students at Columbia, I have given much attention to inventions and patents, including the prosecution and defense of patent suits. The paper is particularly valuable because it sets forth the fundamental basis of patent rights, showing them to be well-grounded, historically, legally, and morally. They have been recognized for more than 100 years and have had substantially the same legal status in England for nearly 300 years. Their moral justification is found in the fact that they "promote the progress of science and useful arts," in the words of the Constitution of the United States.

It is a common notion, especially among those who have not given the matter much attention, that the patent system is of doubtful benefit; some think it positively obnoxious. This notion probably arises from the fact that a patent being a monopoly, is classed with all other monopolies and shares their unpopularity. The patent right is really a just reward for service rendered to the community; it is also to encourage others to render like services. As stated in the paper, ever since the enactment of the "Statute of Monopolies" in 1623, all monopolies of the objectionable kind, granted for mere "buying, selling, making, working or using" of some already known article are illegal not only in Great Britain but also in the United States, which has adopted fairly closely the legal principles of the mother country. The broad and clear statement of patent rights contained in the paper will do much to put the matter in its true light in the minds of electrical engineers who are particularly affected by patents because of the novelty and rapid progress of their profession.

I agree fully with Mr. Fish that in the main our patent laws are good and that they have been intelligently and honestly administered. Of course mistakes are made, but these arise

mostly from failure to understand the extremely difficult technical points. Even when cases have been decided against my interests, I have not doubted the probity of the Patent Office officials or of the federal judges. Considering the extreme complexity of many patent cases, it is indeed surprising that the decisions are generally right or substantially right.

Nevertheless, there are defects, one of the most serious of which is the fact that "the methods of taking testimony in patent causes are cumbersome and expensive," as stated in the paper. Mr. Fish suggests only a very general and far-away remedy when he says: "that an intelligent and well directed effort on the part of the patent bar and the courts could bring about a reform in this direction." While I believe that the better element of the patent bar would be willing to have such reform, nevertheless the volume of business would be reduced, and human nature as a whole could not be expected to put forth strenuous and concerted efforts in that direction. In order to initiate this movement specifically and promptly, I suggest that the Board of Directors be requested to appoint a committee to investigate and report upon what reforms are desirable and how they may best be brought about. On this committee there should be representatives of the patent bar, manufacturers and operating companies (who have to pay the bills), also engineers who do not act as patent experts as well as those who do. This would insure a breadth and balance of views most likely to bring about the best results. The Standards Committee of the Institute has always had a similar all-around representation, and 11 years' experience has proved that excellent work is done by apportioning it to those who understand it best. Under these conditions the various interests are safeguarded and the conclusions are respected.

This same committee could also consider and report upon the establishment of a single United States appellate court in patent cases, as well as reforms in the Patent Office, especially in obtaining prompt and competent examination of applications, simplication of interference and appeal proceedings, etc. The entire revenue of the Patent Office and the past accumulation should certainly be expended to benefit that branch of the public service. There is no reason why the Patent Office should be run at a profit while the Post Office shows an enormous deficit.

Personally I consider the great bulk and complication of expert testimony the worst of all these present evils. Frequently the record consists of many ponderous volumes which could be reduced to one of moderate size and contain just as much or more of the real facts. This bad practice is commonly laid to the dishonesty of expert witnesses. In some cases this charge may be true, nevertheless the bulk and irrelevancy of the testimony would not be much reduced if every expert were perfectly honest. It is *partisanship* which is largely responsible.

Every day we hear arguments on political or social questions which do not affect the disputants to the extent of a single cent, and yet they will wander from the point, fail to admit the other side of the question, however obvious, quibble and struggle to maintain their own proposition even after they are proved to be clearly in the wrong. If on the other hand a question is left to anyone to decide, he will usually consider it in a conscientious, judicial manner and give his best opinion, endeavoring to eliminate obscure or immaterial matter. If two experts were appointed by the court from among specialists of experience and standing, the facts could be brought out in less than one-quarter of the time and space now required to mix them up. Most of the points would be settled with little or no controversy. In case the two experts failed to agree on certain matters a third could be called in; even if his decision were wrong, the case would be so boiled down that the court and counsel could see the precise issue.

The need of this reform is particularly pressing for electrical cases, because judges and lawyers have the greatest difficulty in understanding the electrical technicalities involved. I think that the Institute is the body most interested in and qualified to lead in bringing about the reforms proposed.

Albert G. Davis (by letter): It is difficult to discuss Mr. Fish's paper. The points which he makes seem so self-evident, after he has made them, and the paper is so convincing and so exhaustive, that criticism is impossible. During the last two or three years there has been a tendency on the part of a certain class of inventors and persons interested in inventions to criticise our present patent system. No two of the critics seem to agree on any particular remedy, and many of them violently contradict each other, but they all are in general agreement that something ought to be changed. They seem particularly to be troubled by the idea that the patent situation is being handled by the patent lawyers, and to feel that this is all wrong. The law has been handled by lawyers, that is, by a special class of persons who have given their whole time and attention to the subject, ever since the Babylonians inscribed their judicial decrees and contracts on clay and baked them into bricks. It will probably continue to be so handled as long as engineering is handled by engineers, or as long as the art of healing is handled by physicians.

The patent lawyers of the country, or at least those of them whose standing is such that their views are worthy of consideration, are a unit in their desire to simplify the patent procedure wherever such simplification can be effected without risk of injustice to litigants, and to see that the meritorious inventor receives adequate protection and adequate reward. This desire is only natural, because the minute it ceases to exist, that minute the patent system will begin to decline, and the occupation of the patent lawyers will be gone.

A good lawyer is a man who is trained in the idea of justice. All his instincts are in favor of justice. A good patent lawyer knows the benefit that has accrued to this country from the patent system, and realizes that this benefit is due to our patent system being the best in the world. He knows that bold and far-seeing men, individuals as well as officers of corporations, are engaged in research and development that must cost hundreds of thousands or millions of dollars before any practical results can be brought forth. He knows that this would not be done and could not be done if it were not for the patent system.

The complexity of our present procedure and the delay and expense attendant on any thoroughly contested patent litigation are not the fault of the patent bar, nor are they the fault of the judges, nor, broadly speaking, are they the fault of the system. The truth is that the questions necessarily arising in an ordinary patent infringement suit are exceedingly complex, so that patent litigation is necessarily one of the most difficult and, therefore, one of the most expensive sorts of litigation in which a man can engage.

I think that most competent patent lawyers will agree that most of the discontent comes from patentees or patent owners who have failed to enforce rights because, as a matter of fact, they did not have any rights to enforce; that is to say, people who have not really made meritorious inventions, but merely thought they had. Any patent lawyer of experience will agree with me, I think, that a man who has a meritorious invention and will hold on to it until the art is ready for it, whether that man be a millionaire or a workman at two dollars a day, will receive his reward. He has a piece of property which is of value and which can be sold. He may be foolish enough to allow himself to be swindled out of it, just as he may allow himself to be swindled out of a piece of real-estate, but there are always plenty of manufacturing companies and individuals willing to put money into a meritorious patent if they can only be convinced that it is meritorious.

The present American patent system has been built up by a steady growth and development through a century and a quarter, keeping what is good and rejecting what is bad. As it stands to-day it is the result of the combined wisdom of generation after generation of able lawyers and learned and conscientious judges. It is a better system to-day than it was 50 years ago, and it will be a better system 50 years from now than it is to-day. Like any other product of the human mind it is capable of improvement and it will be improved, but the improvement will not be produced by any one radical change coming at one time, but rather by the same slow growth by which all real legal and other reforms are effected.

Arthur Von Briesen (by letter): [This communication is not original with Mr. Von Briesen; it is a memorial circulated by the Merchants Association of the City of New York.]

Hon. Secretary of State,
The Senators for New York,
The Chairman of the Committee on Patents of the Senate, and the House of Representatives.

This Memorial is respectfully presented to your Honorable Bodies by the undersigned members of the United States Bar largely practising in patent causes. The law recently enacted by the British Government has already injuriously affected the interests of American patentees and manufacturers and calls for action on the part of our Government. Retaliatory measures have been suggested. By Section 41 of the Payne Tariff Act it is proposed to retaliate by subjecting inventors who are not citizens of the United States, when they seek patents in this country, to the same rules and restrictions which prevail in their own countries. But it has been found that such a provision in our law would do violence to International Treaties, particularly the Treaty of 1883, Article 2, which provides that in this country the citizens or subjects of foreign countries who are members of the Treaty shall enjoy the same privileges regarding patents that are enjoyed by our own citizens. Hence the enactment of a law such as Section 41 of the Payne Tariff Bill would constitute a breach of a solemn treaty and it would otherwise create administrative, judicial and commercial difficulties of such magnitude that the provision in question is by us deemed impracticable. In other countries similar restrictions exist which are not found in our law, the result of which is that the citizens of such countries receive much more liberal treatment and greater rights in this country than our citizens do in theirs. We therefore respectfully petition your Honorable Bodies to take measures which will lead to a new International Conference at which the question arising from the enactment of the said law of Great Britain may be thoroughly discussed with the view of bringing about a complete understanding between the nations, and the entire elimination of all needless and harmful restrictions. In such a Convention the representatives of the United States Government should be instructed to lay stress upon the intolerable conditions created by the enactment of the British Law and by the presence of similar restrictions in the laws of other nations. The objectionable nature of such provisions has indeed led to the question whether it might not be best for the United States to withdraw from the International Treaty regarding the protection of industrial property by giving one year's notice as called for by the Treaty. Should we withdraw from such a treaty, we can then be in position to enact retaliatory measures. But why not press upon the representatives of other nations the importance and value of the simple American system, which grants a patentee protection for a certain number of years, without placing any restriction whatsoever upon him, requiring neither the payment of taxes from time to time, nor the enforced manufacture of patented devices, nor the enforced granting of licenses. The American system in its simplicity has worked the greatest benefit upon our people. An inventor very frequently is

ahead of his time. When his patent is granted, it may be laughed at, and frequently is, and it has taken some of the most meritorious inventors as much as ten or twelve or even fourteen years after the grant of that patent, before the value of their mental creation became recognized. During all this time the energy of the inventor is at work towards pushing his creation to the front; no discouragement is placed in his path by any restriction created by law, and as a result the industrial development of this country has been greater than that of any other Where heavy taxes are required to be paid from year to year,—and in many countries taxes increase from year to year,—inventors, unless they are very rich, are enforced to abandon their patents and relinquish that splendid energy which accompanies the unrestricted patentee throughout the life of the patent. It may well be said that thousands of patents which lapsed for non-payment of taxes in foreign countries would have proved of great value, had the inventors been permitted to continue their privileges to the full end of the original term.

Analogous obstacles are created by the laws that require patented devices to be manufactured in the country in which the patent is granted to a foreigner. The intercourse between nations has become so intimate and so thoroughly interlocked that provisions of this nature work more harm than good in stifling, for the imaginary benefit of the home country, the enterprise of intelligent inventors, who bring into play that splendid mental and intellectual competition which, where a patented machine comes from a foreign country and seeks to cover the market, induces the native inventor of higher grade to create organisms of greater importance and greater value than those that come from abroad.

That license restrictions should not be put upon patentees who under the Constitution are to be granted exclusive rights, is a point that has been thoroughly discussed in the case of *Continental Paper Bag Co. v. Eastern Paper Bag Co.*, decided by the Supreme Court of the United States in June, 1908, and reported in 210 U. S. Reports, page 405, where the court after showing the intent of our Constitution and the object of the law to enter into contractual relations with inventors uses the following language;

"The inventor is one who has discovered something of value. It is his absolute property. He may withhold a knowledge of it from the public and he may insist upon all the advantages and benefits which the statute promises to him who discloses to the public his invention." (p. 424); and then says:

"As to the suggestion that competitors were excluded from the use of the new patent, we answer that such exclusion may be said to have been of the very essence of the right conferred by the patent, as it is the privilege of any owner of property to use or not to use it, without question of motive." (p. 429) * * * "In some foreign countries the right granted to an inventor is affected by non-use. This policy, we must assume, Congress has not been ignorant of nor of its effects. It has nevertheless selected another policy. It has continued that policy through many years. We may assume that experience has demonstrated its wisdom and beneficial effect upon the arts and sciences."

Your petitioners therefore pray that measures be taken towards bringing about at the earliest possible date another International Conference regarding the questions touched upon in this memorial, with the view of bringing the laws of all civilized nations into more thorough uniformity so that no restrictions may be placed in the path of meritorious inventors who are citizens of other countries and who desire to have their inventions duly protected in this country.

Thomas B. Kerr: If the matter of taking testimony were left to me, I would decide it promptly and decisively. I would not permit my opponents to cross-examine my witnesses. That would relieve my clients of their objections. I have read Mr. Fish's paper several times and have got a new view of the patent law from it. I have been at patent law all my life, and have known no other practice. This has subjected me to this kind of criticism: One day I met Mr. Knox, the present Secretary of State at a club in Pittsburgh. We had a patent case together, and I had explained to him that we had to have an expert to testify as to certain scientific matters, and I had come to him for his opinion in regard to certain questions of law, collateral to the specific patent questions involved. He was discussing lawyers generally to some lawyers and laymen. He looked at me and said:

> There is one branch of the profession which should not be admitted to an equal standing with the others. I mean the patent lawyers. If it is a question of mechanics they must hire an expert; if it is a question of law they must hire a lawyer.

I told him that that was the first time I had ever heard defined the status of the patent lawyer and that as I understood society, the most important man to be found was the middle man. The function of the patent lawyer is between the law and science, between the engineers and the court, because the engineers talk in a language that the court does not understand. The patent lawyer must get an inkling of the inventor's meaning, and convey it in simpler language to the court. Judges have confided to me that their greatest difficulty in patent cases was in understanding the testimony of the experts.

I never fully appreciated the Constitution of the United States until I read Mr. Fish's paper. In recent years the only part of the Constitution that I have read more than two or three times, is that upon which the patent system is founded, but I never appreciated the wisdom of that provision until I read Mr. Fish's paper. What possible improvement could be made upon it from a practical standpoint? How can you pay for something with nothing more perfectly than the Constitution pays the price, or prescribes the price which the country shall pay for inventions. It gives to the inventor something the public does not own, and yet it is so perfect a provision that out of it not only does the inventor gain his reward, but the country gains the prosperity which Mr. Fish has spoken about. I agree with him that American progress in commercial, industrial, and scientific lines is largely due to that one provision of the Constitution.

Now what handicap or harm comes out of it? Does it prescribe or put any limiting conditions upon the inventor? Does it in any way dampen his ardor or ability? No! It simply clears the track and says, "Now, run your race." For an invention, properly patented, is but an opportunity given to man, and it is of no value to him unless he puts back of it the same industry, purpose, and judgment which is necessary to success in any other line of endeavor. The fathers who wrote that provision laid the foundation for the most splendid patent system ever devised.

Mr. Fish pleads for support of the patent system. I do too. It has its weaknesses, like every other human institution. But it is the best patent system extant. The engineers especially have an interest in it, because in the future nature, under stress of greater intelligence, boldness, and earnestness of investigation, will yield her secrets much more freely than in the past. There are open to all men the rewards which come to the successful inventor, but the patent system is needed to give him the opportunity to profit by his labor.

Livingston Gifford: I think that the sentiment of the bar is absolutely unanimous in favor of the patent law as it stands to-day. I say, unanimous in a general sense. As Mr. Fish says, every individual has some fault to find, something that he would suggest as an improvement, but everything suggested as an improvement by any individual is generally condemned by almost everybody else. So I think that taking the general sentiment of those who know, the conclusion is that the patent law as it stands is satisfactory.

On the other hand, the consensus of opinion is that the system of administration—the taking of testimony and hearing and deciding cases—is all wrong. It is antiquated and requires some changes to adapt it to present requirements. It has existed for probably three-quarters of a century, ever since the patent law commenced. It is worn out and does not meet present-day demands. It is complicated, cumbersome, and does not reach results. But when all this has been said, how is it to be made any better? We have practiced under it for over one-half a century, and everbody is condemning it all the time. Yet I have never heard a suggestion from anyone of any improvement that anybody else would agree to as being an improvement. As soon as an attempt is made to curtail the system, one runs into difficulties just as great in other directions.

It is very easy, and the common practice of everybody under all circumstances, to say, "have a committee appointed". But Mr. Crocker does not suggest any change for a committee to consider. It would be useless to appoint a committee and expect them to revise a system which has existed for three-quarters of a century, without suggesting in advance any scheme of improvement.

It seems to me the proper thing is to follow the suggestion of Mr. Fish; that is, to have a court to take and hear all patent

cases as a court of last resort, so that the court can become acquainted with all of the difficulties that arise. That court, perhaps coöperating with all interest in patents, may work out a solution of the problem and cure the evil in the long run. I see no other possible way of accomplishing it.

E. W. Rice, Jr.: Mr. Fish speaks of the work of the inventor as that controlling the forces of nature and teaching man how to apply those forces to shaping and transforming the raw materials which are available for his use, as distinguished from intellectual, moral, social, and imaginative development.

I have no doubt Mr. Fish will agree with me that one of the chief characteristics of the inventor is his imaginative quality: I do not think a man can invent anything of value who has not the imaginative quality developed in very definite manner.

Mr. Fish's statement

> It has not been much of a disadvantage to the patent system or to patentees that the judges have, as a rule, had no special technical training.

is of great interest to me, for at one time, I thought it was a very great disadvantage not to have had this training. I still think it would be desirable in an appellate court, such as Mr. Fish has suggested, for one of the members to have a first class engineering training. Nevertheless, my experience with lawyers and patent lawyers has astonished me. They show such wonderful capacity in grasping abstruse engineering problems, and in restating them in such a clear and simple manner that the so-called expert is often confused. I think, therefore, that even without an engineering expert sitting as a member of the court, the system is fairly safe. That has often happened in my experience.

It is self-evident that one of the great advantages of our patent system is the stimulation of invention. Mr. Fish has referred to the rewards of inventors. It is a most difficult subject, one with which I have had to deal personally, and on many occasions I think we would all like to see inventors reap the richest rewards of their efforts. However, I believe that inventors, as a class, do receive a full measure of reward, compared with similar efforts in other lines. It happens under our patent system that a monopoly is given only for a certain kind of ideas. Now there are other ideas and efforts just as important and valuable which are not patentable. The manager of a works or the executive of a business must use great originality and great skill in managing the business, but he has no means of obtaining a monopoly on his ideas or methods.

It is also true in many cases that inventions, as Mr. Fish states are only valuable as incidents to the manufacture, and cannot be segregated and appraised separately. It often happens that an inventor makes an invention which he believes will save his employer a large sum of money, and he feels that he ought to be rewarded accordingly. But the difficulty has been accurately to determine the saving in the particular case. Others have

contributed to make that invention possible, and others must help to make it profitable; and it is impossible to segregate and value the individual contribution.

As to the improvements of the patent system, I think it would be difficult to offer any suggestion beyond the one Mr. Fish has made as to a single court. We all agree, I think, that it would be a great improvement if some method could be devised to limit cross-examination. I have felt very keenly at times that my time has been wasted by opposing counsel. I think all engineers and inventors brought into patent cases, who are not patent experts, regard the waste of time given to the taking of direct and indirect testimony, as something almost unbearable. They would rather be engaged in the work of designing or inventing new apparatus. Therefore, any suggestion which will improve the practice in that respect will be welcomed.

Chas. P. Steinmetz: After the very comprehensive paper of Mr. F. P. Fish, I cannot see how I can add anything, still less criticize the paper; but shall merely discuss one particular feature of the patent situation from my point of view as an inventor. I have made a few inventions myself. The direct financial gain which I have received from inventions amounts to $500. Indirectly, in salary and reputation, however, I believe I have received at least the value of my inventions; so that I can find no fault in my personal experience, and my treatment as an inventor. Still, I recognize that considerable fault is found with the operation of the patent laws by inventors and others.

In criticizing the operation of our patent laws, I believe it very important to realize the point of view from which the criticism is made; because what may appear as severe criticism from the manufacturer's point of view, may be desirable and necessary from the inventor's point of view; and inversely.

To illustrate: The requirement that the invention should be manufactured within a short time limit of one year or two years would offer very little handicap to the manufacturer who can arrange to have somebody put to work on it; but it would be fatal to the inventor, as it would practically deprive him of the chances of reward, since most inventors are not able to carry out the manufacture of their inventions on their own capital. Therefore, if this were a legal requirement, the inventor would have to turn over his invention to a manufacturer at any price, or lose it altogether. So, you see, we may criticize from one point of view a feature which is entirely justified from another point of view; for you can realize that the manufacturer may desire to have such a clause, as it would turn the inventor over to him, helpless, while the inventor should for the same reason strongly oppose it.

I propose to discuss this paper from the standpoint of the inventor, and not that of the manufacturer or the capitalist; because I believe the inventor who has created something which did not exist is the one to be rewarded, and to reap the benefit,

while the capitalist, who makes the exploitation of the invention possible, by financing the business, or the manufacturer, are entitled to interest on their investment, to the return of the money they put out, and an additional return appropriate to the risk of the investment.

Developing an invention is not a safe investment like putting money into real estate, but is rather a gamble; and, therefore, the returns of capital should be very much higher to compensate for the risk involved. We do not realize the risk of investment in an invention because it is human nature only to see success, and forget failure. For instance, we consider the success of the Bell telephone, but we forget the Keeley motor. At the present time, everybody knows that the Bell telephone was a great invention, and the Keeley motor a humbug, yet in the view of the technical adviser, at the outset both appeared as wild fancies and capital could in neither case get reliable technical advice. So we must consider the time when capital was approached, and not the viewpoint of the historian whose judgment is guided by experience. Five years ago certain inventions in aeronautical matters would have been ridiculed. Nobody would dare to do that to-day, after the recent success in that field.

My opinion is that the patent laws should be drawn and executed in such matter as to give the greatest benefit and reward to the inventor. The ways of exploiting an invention by an inventor are three-fold. 1. The inventor may proceed to develop the invention for commercial use with his own capital; 2. He may interest capital in his invention; in short form an exploitation company. 3. The inventor may turn over the exploitation and development of his invention to an existing manufacturing company engaged in similar work.

As to the first point, the development and exploitation of the invention by the inventor: at present, in the field of electrical engineering, the cost of development up to commercial exploitation of most inventions is so high, as to be far beyond the means of most inventors; and therefore, this is not feasible. Furthermore, the inventor is frequently not a good designer, and usually an inferior business man and administrator; therefore the chances are against his handling his own business satisfactorily. Besides, this method is not economical, as it deflects the time of the inventor from further creative work. Therefore, this method is feasible only in such rare instances, that it hardly needs to be considered. The second method: to interest capital in the development of the invention, the invention must represent a tangible value; it is the market value of the commodity on which depends the attraction to capital. This depends on how far the invention can control the industry or part of the industry to which it is related, and how safe the control is; that is, how good the chances are that the patent may be sustained finally by the court. Even with this assured, it still is an uncertain value,

because it may never develop, but unforeseen difficulties may kill it, or just as it begins to develop and become profitable, somebody may invent something similar to it, which the courts consider as non-infringing. Or, again, when the commercial exploitation is ready, some other inventor may do the same thing, in a simpler and better manner and destroy the former invention just when it was about to start on a paying basis. The inventor needs the capital to develop his invention. The capitalist does not need the inventor. The stronger of the two parties is the capitalist; and therefore the control must lie with the capitalist and not with the inventor. It is not a question of morals, but the result of an economic condition, which in this case operates against the inventor. Very much more favorable is the situation of the inventor with regard to an established manufacturing company in similar lines, because in this direction the inventor has more the upper hand. He can hold back the invention until it is needed by the manufacturer, and the manufacturer has to come to terms, or as is more commonly the case, until the manufacturer realizes that this invention offers a certain financial advantage in his field, and therefore is willing to share the advantage with the inventor. As in this case, the conditions are more favorable to the inventor, it is the usual method of exploiting an invention.

The value of an invention, that is the reward which the inventor can expect, depends on its market value as a commodity. The market value depends on the control which the invention offers to the industry or branch of an industry, and the certainty of this control. That is, it depends on the broadness of the patent covering the invention, and the probability or certainty of the courts sustaining this patent. This is the feature which the inventor very often does not realize. He may in some individual cases be handicapped by broad inventions of others, from exploiting his detailed invention, and be led to the idea that broad patents are objectionable. But he does not realize that any limitation of the patent is an advantage to the exploiter of the invention, but not to the inventor. For instance, an engineer may get the conception, and make the invention that by combining several alternating currents, acting in different directions at different times, he can get rotation; and thereby transmit power. The patent of such a conception may be broadly covering the production and transmission of power by the combined action of several alternating currents, displaced in phase and position, or it may be limited to the construction of the particular motor, having two sets of field poles energized by two alternating currents displaced in phase. In the broader case, it would cover the use of two- or three-phase, secondary short-circuited member or energized by direct current, separate polyphase currents or currents derived by splitting the phase of a single current, or many other combinations which the inventor did not and could not think of, because the one who

conceived the general idea, cannot think of all the particular applications to which it can be put. Such a broad patent would have a value assuring the inventor an adequate compensation for his broad idea. If, however, the patent is limited to a specific application which the inventor carried out, it makes the patent worthless, and deprives the inventor, who had the broad and fundamental conception, of the value of his work, because the first conception must necessarily be the crudest, the least refined, and least efficient, and any further development or modification thereof, after the general method has been shown, would, naturally, usually constitute a better way of doing it; and if the inventor is barred from the control of these lesser improvements, the fruit of his work and the results would accrue to those who had improved in detail on his particular principle. That would not be fair, and would be very harmful to the value of an invention, because it would make it possible for the benefit of a fundamental invention to be reaped by someone who had simply made a modification or shown another way of doing it, of which the original inventor did not happen to think.

That is why I say the broadness of the invention is one of the most essential things from the point of view of the benefit of the inventor. Not so much from the point of view of the manufacturer, however, for the manufacturer may find it more convenient to limit it, for then he would not have to come to terms with the inventor of the fundamental idea. So the advantages are different from different points of view.

When the invention is finally adjudged by the court, the ownership of the invention has usually left the original inventor, and we are very liable then to imagine that it is of no moment to the inventor, as he has no further financial gain in the question of whether the invention is sustained or destroyed by the court. We do not realize that the destruction of a patent by the courts reduces the market value of all other inventions just as much, and any patent broadly sustained increases the market value of all inventions, and it is of special advantage to the inventor to have the value sustained, even though there is no direct financial gain from that particular patent. The failure to maintain a patent in court means the destruction or depreciation in value of all inventions, and that is the most serious blow to the inventor. To the large manufacturer in the industrial field it is rather immaterial. The destruction of fundamental patents, may destroy his monopoly of certain fields, but at the same time it would relieve him of the necessity of buying the control of an invention, and thus it would merely mean for the large manufacturer a readjustment of business methods, but would be neither a gain nor a loss. Many branches of business are monopolistic without being protected by a patent. Many are held as a monopoly where the patents expired years ago, and still the production of that commodity is carried out almost exclusively by the same manufacturers as before. Business

methods, and not patent rights, give their control. To the inventor the destruction of a patent can only be a disadvantage. This is one of the most dangerous features. It is fortunate that the courts in general have been rather inclined to sustain patents. There is a great handicap in that direction, however, because electrical engineering has progressed enormously, so that what was a great invention ten years ago, is so self-evident to-day, that we do not realize how much of an invention it once was. One of the greatest difficulties after the lapse of many years is to imagine one's self back to the time when inventions were made; when what was a great invention has become common knowledge to every schoolboy, and we find the invention expressed in the terminology of the bygone days, when it commonly was impossible to describe ideas in the exact terms of to-day, as these terms did not yet exist in the language.

That is what you engineers and inventors must guard against. It is to our advantage, and nobody else's, to have the patent law allow us to make the claims of every invention sufficiently broad to cover all modifications of our fundamental idea which may be made, and to have the patents broadly sustained by the courts, because these two features constitute the market value of our work, and we are the only ones benefiting by it.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y. June 28, 1909.

CENTRALIZATION OF POWER SUPPLY

PRESIDENT'S ADDRESS

BY LOUIS A. FERGUSON

The most prominent characteristic of present-day industrial development is the tendency to consolidate. It is so apparent on every hand that this may well be called the Age of Centralization. The economies that may be effected by large production and centralized direction are being appreciated more and more, and extremely large industries have been, and are being, built up by combining many individual plants. In the main, such combinations have resulted in material benefits to the country and its people.

The lessened cost of wholesale manufacture permits a lower selling price, thus placing the commodity within easier reach of a larger number. The greater economy of production resulting from manufacture in large quantities and with improved methods is and will continue increasingly to be a material factor in that very important problem of the conservation of resources, which is so prominently before this and other countries at the present time.

In the industry of electric power production the advantages of centralization seem thus far to have failed somewhat of full appreciation, but the awakening is apparently at hand. In a few noteworthy instances centralization was planned and put into effect some years ago, and the work already done will blaze the way for others. When the economies of concentrated management and large production are more generally understood the growth of centralization will be rapid.

From whatever standpoint we view the problem the advantages stand out boldly. The investor will be more favorably impressed

with large issues of securities which require no detailed information. These securities will be easily marketed and on much more favorable terms than is possible with the smaller issues of individual companies. Large as this saving is, by far the most important one results from the economies effected by combined investment and output. The load-factor of our central stations; that is, the ratio of the average load to the maximum load, absolutely controls the earning capacity of the investment. The cost of generating and distributing electricity consists primarily of fixed charges on the investment and of operating costs. The fixed charges are independent of the hours use of the equipment; therefore, if the load continues heavy for long hours daily the fixed charges are distributed over a larger output and the cost per unit of output is thereby lessened. In other words, with long hours' use a greater output follows, and greater earnings are produced without any increase in the investment. Obviously, therefore, the greater the proportion the fixed charges bear to the total cost the greater will be the effect of the load-factor. In modern large power systems, as in water-power plants, the operating costs are now so low that they represent a comparatively small part of the total costs, the fixed charges constituting by far the larger part. The importance of a good load-factor is thus apparent and everything that will improve this factor should be sought.

The yearly load-factor for any one class of service is determined largely by the seasons, the habits of the people, and similar conditions which ordinarily do not change. Improvement in load-factor must therefore be obtained largely by combining different classes of service, the maximum demands of which occur at different times of the day or of the year. Also, the larger the number of customers in any class the better will be the load-factor.

The ratio between the sum of the maximum demands of various classes of service to the actual simultaneous maximum demand is termed the diversity-factor, and the more non-coincident peak service that is combined in one system the greater will be this factor. It is here that centralization presents its most marked effect in increasing the load-factor and thereby the economy of production. The addition to a central station system of business having a high load-factor, such as power business for all-the-year-round service, amusement parks, auto charging, and refrigeration for the summer, utilizing the otherwise idle investment, is desirable not only in itself but also because it reduces the cost of the entire output.

The whole community through this reduction in cost is thus benefited by centralization, since the central station company is enabled to sell its product at a lower price to the small consumer because of the improved condition of manufacture effected through the supply of its product to the large consumer.

In the power station the larger output of combined loads will permit the use of large generating units which have a high performance efficiency, and which require a comparatively small operating force. This results in further reduction of costs. The investment per kilowatt, not only for the apparatus but for the land and building as well, is also less for large units, and the total reserve required by the centralized system will be but a small fraction of the combined reserve required where the load is furnished from individual systems. The saving in investment in this single item of reserve alone is an important factor in the problem. One of the leading causes for the success which has attended the development of the electrical industry in this country has been the standardization of equipment, and the American Institute of Electrical Engineers has been the active leader in this important work. It is still a far cry to the ideal but all steps taken should be in the forward direction.

Centralization leads toward standardization. In the present state of the art, generation will be by three-phase alternating current and the predominating demand will determine the frequency, be it 25 or 60 cycles or some other frequency. If, however, the demand in any community for an auxiliary standard frequency becomes sufficiently large, even though it remain a small part of the total output, then generation at this auxiliary frequency is justified; but connecting links of ample capacity should be provided between the sets of generators of different frequency so that the advantages of the diversity-factor and of common reserve may still obtain. The voltage should also be so chosen that the transmission cable may be interchangeable between the two systems.

An incidental advantage of centralization which, however, is of great sociologic importance in a community, is the resultant cleanliness due to the absence of countless smoking chimneys. This fact is being actively recognized by civic improvement bodies, and such recognition is a valuable asset to a central station company in the prosecution of its industrial power campaign.

One of the early and prominent examples of centralization is found in England where the Newcastle-upon-Tyne Electric

Supply Company furnishes energy to one of the most important manufacturing areas in the United Kingdom. A special effort has here been made to develop particularly the industrial power business, and the general adoption of electricity has had a marked effect on the industrial conditions in that district. The tramways are operated from this system as well as electrified steam roads. In many of the smelting mills the waste gases are used to produce steam in boilers and the energy thus derived is used in turbine stations and is fed into the general system, the same smelting mills being in turn customers of the supply company. The total load is not so very large compared with that in some of our American cities, but this system is mentioned in such detail because it typifies what can be done in the way of centralization. There is still a good deal of independently produced power in the district, but it will undoubtedly eventually be won over by the supply company. Even though the difference in efficiency between the smaller turbine units and the larger units is decreasing, the diversity-factor will in most cases enable the supply company to underbid independent producers.

The diversity-factor is the very foundation rock of centralized energy supply. It is the birthright of the central station; the fundamental reason for its existence; and its resultant value belongs to the central station company.

In some American cities, as already stated, the total load is considerably greater than in the example just cited, but the centralization possibilities are only just beginning to be developed.

The system of the Pacific Gas & Electric Company with its hundreds of miles of transmission lines, its many customers among the mines dotting the hills, and the many cities receiving electrical power from the company, is perhaps the nearest approach to the Newcastle system. At Niagara the transmission lines are reaching out in all directions, and we may hope to see one of the great industrial centers of the future in that locality. At the present time there are many independent companies, and the full benefits of centralization will not be realized until the stations are loaded up to the point where they will be compelled to arrange for interchange of energy.

Chicago, because of its strategical location regarding transportation, is certain to remain one of the world's greatest markets and industrial centers. If the imagination were allowed full play the possibilities in power requirements of the future in the Chicago districts would appear almost boundless. A vigorous

start toward centralization of power production has already been made. Street-railway power houses have been shut down and the energy for the operation of the street railways is purchased in whole or in part from the central station company.

The supply of power for electrified steam railways is one of the possibilities of the near future, and engineering plans and expenditures of past years have been made with a view of taking care of wholesale industrial business on a comprehensive scale.

In a large number of cities two prominent public services—street railways and electric lighting—were originally inaugurated by one company, and with the natural growth and development of the two systems the electrical generation has in many cases been unified so that one set of generators furnishes energy for both railway and lighting. However, in most of these cities the industrial power business has not been vigorously developed, either because of lack of appreciation of its value or timidity on the part of the supply company in making sufficiently low prices to obtain the business.

There are also very few, if any, communities or territories in which the centralization of power supply for all purposes has been effected and in which electric power is utilized in such manner that the best economic results are realized. By combining all systems of neighboring cities within a radius of 30 or 40 miles into a single centralized system, a great saving in production would result and in most cases an improvement in service as well. The utilization of water-powers will help in this direction.

The monopolistic character of the electric power industry is now being recognized by the intelligent public, and monopoly privileges with proper regulation are being granted as the best safeguard for the interests of the people. The legal way is thus being paved for the beginning of centralization. Such centralization will naturally include the power for electrified steam railways, the universal coming of which is but a matter of time. In most instances the total amount of power required by a railroad for its terminal and suburban service, even where counted in the tens of thousands of kilowatts, is but a small part of the total power required for all purposes within the same area.

If this railway power were generated in a separate plant the load-factor would be far below that of the central station and the necessary reserve capacity would require a high fixed charge to be carried by the plant. With less reserve the reliability of the service might suffer disastrously. No railroad could thus generate its

own power as cheaply or as securely as it may be purchased from a properly directed central station company. This applies also and to an even greater extent to power production for industrial uses or to individual generation for any purpose.

In cities where the power production is not centralized, a multiplicity of systems of voltages and often of frequencies is usually found, and the duplication which spells waste is abundant. Without the economies resulting from centralization, the price of the output will necessarily remain higher than otherwise and the community must pay the cost. With the lower efficiency of generation the fuel resources are wasted and the entire country will ultimately suffer as a result. It has seemed to me that a condition of such transcendent importance to the electrical profession should receive liberal attention and consideration from our Institute. We, as an organization, have a large part to play in the development of the art of electrical application as well as in electrical science. There is scarcely any branch of the electrical industry which does not ultimately affect or which is not in a measure affected by this development.

The progressive work of the electrical engineer will determine with what success we can reach out into these larger fields. The mechanical engineer has given us a steam turbine of greatly improved economy; it is for us to improve still more the method of transmission and conversion and the application of electricity in the arts. High-voltage transmission made possible our entry into this field, but, while very great advances have been made, there are still some unsolved problems in this field, particularly with regard to underground transmission. There are also many problems and countless opportunities in connection with the utilization of electrical energy. The successful solution of these problems is of the utmost importance.

It is for the commercial electrical engineer (which really includes nearly all of us) to educate the layman and particularly the power user to an appreciation of the superiority and desirability of electric service and to emphasize the material and ethical advantages to a community of centralized power production. It is for the technical electrical engineer to furnish him with the sinews of war, with the ammunition to conquer in the fields where waste, where useless dissipation of energy prevails. We should bend every effort to advance the cause of centralization with its resultant economic gains. Those connected with the manufacturing interests especially should not tolerate for an instant a selfish backward step within their fold.

As we meet here in convention to gain a broader knowledge and to discuss important problems let us think of these opportunities which our profession affords in this grand movement for world betterment, this guiding of the forces so that Nature's bounty to man will bring to him the highest and greatest good. The thought carries with it inspiration and enthusiasm and a determination to do our part, individually and collectively, in the successful upbuilding of an industry which means so much to the economic development of the country's resources. Some day, in the centuries yet unborn, the broadening perspective of time will show more clearly the full significance of these days' accomplishments. Surely, gentlemen, nothing could be more inspiring to greater effort than the knowledge that we have in our hands the means for this achievement.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

THE CONVECTION OF HEAT FROM SMALL COPPER WIRES

BY A. E. KENNELLY, C. A. WRIGHT AND J. S. VAN BYLEVELT

It is well known that when an electrically heated wire is supported in air, it dissipates its heat by conduction, radiation, and convection.

Conduction takes place longitudinally through and into the metallic terminals of the wire, and would take place laterally through the air, if the latter were kept at rest like a rigid medium. The metallic conduction at the ends may be eliminated by confining observation to parts of the wire not near the ends. The lateral conduction through the air is also negligible, because the air does not remain at rest but expands and flows convectively. Consequently we may safely ignore conductive thermal loss.

Radiation loss from the wire is a function of the surface area of the wire, its temperature, the nature of its physical surface, and the temperature of surrounding substances. For a given wire with a given surface condition and in a given environment, radiation loss is regarded as a function of temperature only.

Convection loss from the wire is a hydrodynamic phenomenon, involving the flow of air past the surface of the wire, and the amount of heat which this moving stream of air can carry off. Very little seems to be known quantitatively about convection. It is the object of this paper to describe a research made on the convective loss of heat from small copper wires, of less than 0.7 mm. or 27.5 mils diameter, and to present some quantitative results arrived at from the observations.

In order to study convection loss to advantage, it is desirable to experiment on small wires; because, for a given temperature elevation of the wire, the radiation increases in proportion to the

surface area; whereas the free convection is independent of the surface area, to a first approximation. Consequently, a wire of small surface area per unit of length, *i.e.*, a thin wire, will have relatively small radiation loss by comparison with its convective loss. On the other hand, when radiation loss is the subject of investigation, it is desirable to employ conductors of large surface per unit of length.

We may divide convection into two classes:

1. Free convection, when the wire is supported in free still air; that is, air substantially free from draughts, or movements except such as may be produced by the heated wire alone.

2. Forced convection, when the wire is in motion relatively to the surrounding air; that is, when the wire is either moved bodily through the air, or is held in a stream of moving air.

An electrically heated wire is therefore subject to free convection when it is supported at rest in still air, substantially free from draughts or wind motion. It is subject to forced convection when it is either moved bodily through the air, or when it is placed in a wind.

In order to separate convection loss from radiation loss, it is desirable to operate the wire under test at constant temperature and at a constant temperature elevation above surrounding solid bodies. The radiation loss will then be constant, and changes in the surrounding air will give rise to changes in heat dissipation that may safely be attributed to changes in convection alone.

The experimental methods employed in this research were:

1. To vary the atmospheric pressure surrounding the test wire, under conditions of free convection, and to measure the change of electric power required to maintain the wire at constant temperature elevation.

2. To subject a wire to varying wind velocity, under conditions of forced convection, and to measure the change of electric power required to maintain the wire at constant temperature elevation.

Free Convection at Varying Atmospheric Pressure

General plan. A short length of the copper wire under test was supported horizontally* in an air-tight tank and supplied

* It has been found that, other things being the same, a wire supported in a vertical position becomes slightly hotter than when supported horizontally, apparently owing to the action of the enveloping stream of convectively heated air.

with a steady current from a storage battery. By adjusting the current strength, the wire was kept at constant resistance, and, therefore, at constant temperature. The pressure of air in the tank was varied by means of an air-pump.

Copper wires. Three sizes of small, bare, soft commercial copper wire were employed, the particulars of which appear in the following table.

Electrical connections in free convection tests. The connections employed are illustrated in Fig. 1. A storage battery E, of from 4 to 16 volts, supplied current to the main circuit R^1, R, A, h. The resistance h was an adjustable carbon rheostat. The current strength was measured on a standard laboratory ammeter A, the scale of which had recently been calibrated. The resistance R is that of the length of the test wire supported horizontally inside the pressure tank $B\,C\,D\,F$. The constant resistance R^1 was of German-silver wire, selected as approximately equal to the resistance of the test wire R. Pressure wires were taken both from the constant resistance R^1 and the test wire R to the coils G^1 G of a differential galvanometer. If G and G^1 are the respective resistances of the galvanometer coils, we have at any differential balance:

TABLE I
PARTICULARS OF COPPER WIRES USED IN FREE CONVECTION TESTS

Test-wire No.	Diameter		Cross-sectional area	Surface per linear cm.	Length of wire under test	Resistance of wire @20° cent.	Resistance of wire @ 0° cent.	Linear resistance @ 0° cent	Resistivity @ 0° cent.
	Cm.	Inches	sq. cm.	sq. cm.	cm.	absohms	absohms	abs-ohms/cm.	abs-ohm-cm.
1	0.01143	0.0045	1.026×10^{-4}	0.03591	30.88	0.5133×10^{9}	0.4736×10^{9}	153.4×10^{5}	1574
2	0.02616	0.0103	5.374 "	0.08219	118.6	0.3784 "	0.3491 "	29.44 "	1582
3	0.06907	0.0272	37.47 "	0.217	102.5	0.0467 "	0.04308 "	4.203 "	1575

$$\frac{R}{R^1} = \frac{G+\rho}{G^1} \qquad \text{numeric} \quad (1)$$

where ρ is an adjustable resistance capable of being inserted in either galvanometer circuit.

Method of measuring. The resistance of the test wire having been determined carefully with a feeble measuring current, a

temperature elevation was determined on for the test wire. To this temperature elevation corresponded a test-wire resistance:

$$R_T = R_0\,(1 + 0.0042\,T) \quad \text{absohms† or c.g.s. mag. units of resistance.} \qquad (2)$$

Where R_T is the resistance of the test wire at the working temperature T degrees cent., R_0 its resistance at 0 degrees cent., and 0.0042 the assumed temperature coefficient of resistivity for copper* used throughout this research. If the temperature of the air in the tank was t degrees cent., the temperature elevation of the test-wire was $\theta = (T - t)$ degrees cent. The correct value of ρ in Fig. 1 was taken from equation (1), to give dif-

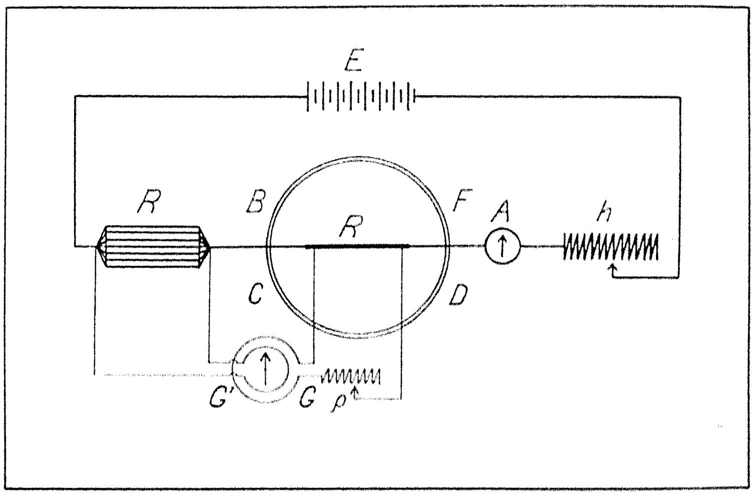

FIG. 1.—Electrical connections in measurements of free convection.

ferential balance at R_T; so that as long as the galvanometer indicated that balance was maintained, it was evident that the temperature elevation $T - t = \theta$, of the test-wire, was constant. The current strength in the main circuit $E\ R^1\ R\ A\ h$ was then kept at such a value, by adjustment of h, that the differential balance was maintained. This current strength was measured on the ammeter A, as the air-pressure in the tank was changed. The apparatus used is illustrated in Fig. 12, from a photograph.

† An absohm is the c.g.s. magnetic circuit of resistance. See TRANSACTIONS, A.I.E.E., Vol. XXII, p. 534.

* Standardization Rules of the Am. Inst. of El. Engrs: Third Edition, 1908, p. 32, Sec. 360.

The galvanometer. The galvanometer used was of the Edelmann type, in which a suspended ring magnet swings in a copper cylinder, so as to secure suitable electromagnetic damping. The resistance of each of the windings $G\,G'$, Fig. 1, was in the neighborhood of 1800 ohms. At the distance of 140 cm. between scale and mirror, 1 mm. deflection was produced by approximately 0.14 microampere. The periodic time of the galvanometer was about five seconds. The damping ratio of successive opposite elongations was 2.55; or the Naperian logarithmic decrement of successive opposite elongations was 0.9361. Four complete swings, executed in 20 seconds, sufficed to reduce the deflection to 1/1790 part, or, for practical purposes, to rest. The galvanometer was read by a telescope and scale, and in nearly all of the work it was used as a zero instrument. The resistance of each coil was measured after each test. The differential balance was also checked, and, if necessary, adjusted before each test.

The German-silver constant resistances. The German-silver resistances R', Fig. 1, were made up of grids of German-silver wire, so as not to heat appreciably with the strongest testing current used. Two such resistances were employed in different series of tests, one of 0.4248 ohm at 18 degrees cent., and the other of 0.04572 ohm at 18 degrees cent., between pressure wires. The working current values employed were ordinarily such as to produce from 0.4 to 2.0 volts potential difference between the pressure wires of these resistances.

The pressure-tank, or cylinder. The pressure tank, inside which the test wire was supported, was a vertical cylinder of the form indicated in Fig. 2, constructed of $\frac{1}{2}$-in. ($1\frac{1}{4}$ cm.) steel plates riveted. Its internal diameter was 60 inches (152 cm.) and its height 21.8 ft. (6.66 meters), the ends being rounded off hemispherically. The test wire was supported horizontally, about 4.36 feet (1.33 meter) from the bottom of the cylinder. A steam-driven air pump, in an adjoining room, was connected to the bottom of the cylinder, so that air could be pumped either into or out of the latter. Access to the interior of the cylinder was obtainable through two manholes, one of which enabled the wires to be admitted through an air-tight gland.

The atmospheric pressure in the cylinder was measured by means of a glass U-tube manometer containing distilled mercury, one leg of this tube being connected to the cylinder and the other left open to the outside air. The pressure in the tank at any time

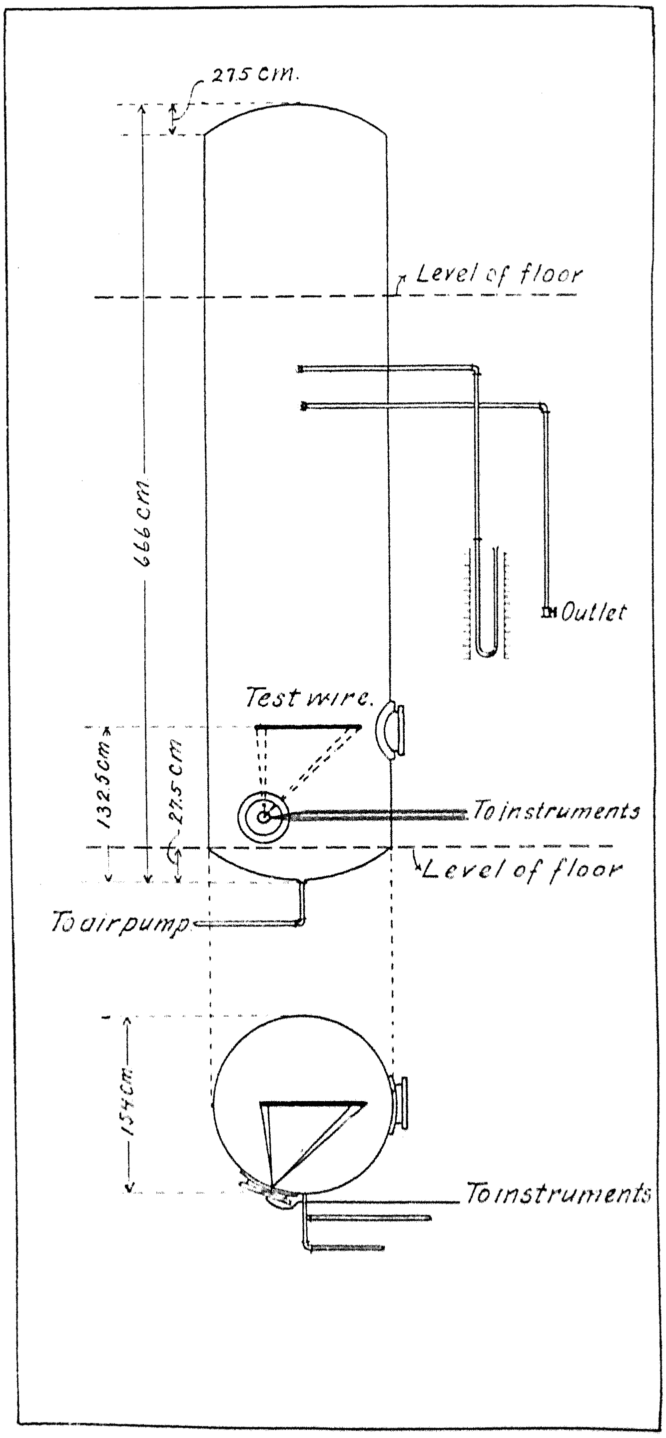

FIG 2.—Plan and elevation of tank

s taken as equal to the barometer reading in the open air, s the difference of level in the U-tube.

Specimen series of measurements. The following table gives pecimen series of measurements made on test-wire No. 2 h ρ = 100 ohms in the circuit of galvanometer coil G, and copper wire R:—

TABLE II

SAMPLE SERIES OF OBSERVATIONS IN FREE CONVECTION TEST

I Air ressure. cm.	II Air pressure. megabars	III I absamp.	IV I^2 absamp.^{2}	V $P = I^2 r_T$ Linear power. abwatts /cm.	VI P_r Linear radiation. abwatts /cm.	VII P_c Linear convections. abwatts /cm.	VIII P_c/θ Linear convection per degree cent. elevation
		$\times 10^{-1}$	$\times 10^{-1}$	$\times 10^5$	$\times 10^5$	$\times 10^5$	$\times 10^3$
28.8	0.3821	2.84	0.8066	3.182	0.321	2.861	4.87
30.4	0.4030	2.83	0.8009	3.160		2.839	4.83
40.45	0.5365	2.909	0.8462	3.340		3.019	5.14
48.35	0.6420	2.979	0.8874	3.502		3.181	5.42
57.61	0.765	3.060	0.9364	3.694		3.373	5.73
65.15	0.864	3.083	0.9505	3.750		3.429	5.84
65.2	0.866	3.083	0.9505	3.750		3.429	5.84
72.8	0.961	3.10	0.9610	3.792		3.471	5.91
72.85	0.966	3.10	0.9610	3.792		3.471	5.91
165.0	2.193	3.42	1.1696	4.614		4.293	7.31
155.2	2.06	3.36	1.1290	4.454		4.133	7.04
143.8	1.908	3.299	1.0883	4.294		3.973	6.76
143.5	1.907	3.30	1.0890	4.297		3.976	6.77
141.7	1.882	3.375	1.1391	4.494		4.173	7.11
128.9	1.712	3.334	1.1115	4.387		4.066	6.94
116.15	1.542	3.32	1.1022	4.348		4.027	6.85
116.2	1.543	3.318	1.1009	4.344		4.023	6.85
101.5	1.349	3.20	1.0240	4.040		3.719	6.33
101.1	1.344	3.198	1.0227	4.037		3.716	6.33
101.1	1.344	3.194	1.0202	4.024		3.703	6.30
92.75	1.231	3.139	0.9853	3.887		3.566	6.07
92.6	1.229	3.181	1.0119	3.951		3.630	6.30
75.55	1.003	3.092	0.9560	3.772		3.451	5.86
75.6	1.003	3.10	0.9610	3.792		3.471	5.90
75.6	1.003	3.118	0.9722	3.835		3.514	5.98

G = 1805 ohms, G' = 1729 ohms.

R' = 0.4248 ohms, R = 0.4681 ohms.

Temperature of tank t = 18.1 degrees cent. = 291 degrees bsolute.

Temperature of wire $T = (t+\theta)$ = 76.9 degrees cent. = 349.9 legrees absolute.

Temperature elevation of wire θ = 58.8 degrees cent.

Linear resistance of wire at working temperature 3.946×10^6 absohms per cm.

Barometer 29.78 in. (75.64 cm.) at 19 degrees cent.

Column I gives the pressure of the air in the tank in cms. of mercury. Column II gives the pressure in megabars. In Column III, the current strength is given in absamperes. In Column IV appears the absamperes squared. Column V shows linear power dissipated as $P = I^2 r_T$ in abwatts per cm. The linear radiation appears in Column VI. Subtracting the linear radiation from the linear power, we obtain the linear convection in abwatts per cm. The last column gives the last-named result divided by the temperature elevation, or the linear convection per degree centigrade of temperature elevation.

Air-pressures in absolute measure. In Column II of the foregoing table, the air-pressure in the tank is recorded in megabars.* The c.g.s. unit of pressure, 1 dyne per sq. cm. has been called the "bar"; so that a megabar is 10^6 dynes per sq. cm. According to the recently published data of the Bureau International des Poids et Mesures,† a column of mercury 760 mm. (29.92 in.) high, at sea level, in latitude 45 degrees, exerts a pressure of 1.0132 megabars. Consequently 1 megabar represents the pressure of a column of mercury of 750.09 mm. (29.53 in.) under the same conditions. For most practical purposes, therefore, a megabar may be taken as 1 atmosphere. It is actually 0.987 of an atmosphere of 760 mm., at sea level and 45 degrees latitude.

Allowance for linear radiation. The accepted formula for superficial power P_r radiated from a metallic cylindrical surface at a temperature of T_a degrees absolute to surrounding solid substances at a temperature of t_a degrees absolute, is called Stefan's formula:

$$P_r = \sigma (T_a^4 - t_a^4) \qquad \text{abwatts per sq. cm.} \qquad (3)$$

where σ is Stefan's coefficient, which depends upon the surface condition of the radiator. For a "black" body, or a body that emits heat by radiation to the full maximum without any reflection of incident heat, the value of σ has been found by Lummer and Kurlbaum to be 5.3×10^{-5} abwatts per sq. cm. and per degree absolute. On the other hand, a colored body or non-black

* Richards and Stull. Dec. 1903. New Method for Determining Compressibility. *Carnegie Institution Publ.* No. 7, p. 43.

† Les Récents Progrès du Système Métrique. 1907, pp. 30–31.

body, which reflects some radiation and therefore radiates imperfectly, has a smaller Stefan coefficient, which it is necessary to determine experimentally in each case.

In the tests published by one of the writers, in 1889, on copper wires electrically heated in air, the conclusion was reached that, with an ordinary unpolished surface, the wires radiated 0.0786 watt per sq. cm. at 126 degrees cent. to surrounding bodies at 26 degrees cent. Substituting in equation (3), $T_a = 399$, $t_a = 299$ and $P_r = 7.86 \times 10^5$ abwatts per sq. cm., we obtain for bright copper wire $\sigma = 4.5 \times 10^{-5}$ abwatts per sq. cm. and degree absolute.

In allowing for the linear radiation from the wire used in this research, the value taken for σ is 5×10^{-5} abwatts per sq. cm. $\times$ degrees absolute. The accuracy of this choice is not of great importance in this case, because the radiation was always small with respect to the convection. Thus in the case presented in Table II, it will be seen that the computed linear radiation from the wire was 0.32×10^5 abwatts per cm. with the above accepted value of σ; whereas the total linear emission varied between 3.18×10^5 and 4.49×10^5 abwatts per cm. Consequently if the Stefan coefficient σ were reduced 50%, or the linear radiation to 0.16×10^5 abwatts per cm., the change thereby effected in the linear convection would be only about 4%. In the case of the larger wire No. 3, the corresponding error would be approximately doubled, and in the case of the smaller wire No. 1, it would be roughly halved. Consequently, if it should be found in future that the Stefan coefficient for such copper wires is materially less than 5×10^{-5} abwatts per sq. cm. and degree absolute, the results for convection arrived at in this paper will not be greatly affected.

If we denote by P the linear dissipation of the test-wire or the power dissipated per linear cm., by P_r the linear radiation, and by P_c the linear convection, we have

$$P = I^2 r_T \qquad \text{abwatts per cm.} \qquad (4)$$

where I is the heating current in absamperes, and r_T, the linear resistance at the working temperature T, in absohms per cm.

$$P = P_r + P_c \qquad \text{abwatts per cm.} \qquad (5)$$

$$= \rho\,\sigma\,(T_a^4 - t_a^4) + P_c \qquad \text{abwatts per cm.} \qquad (6)$$

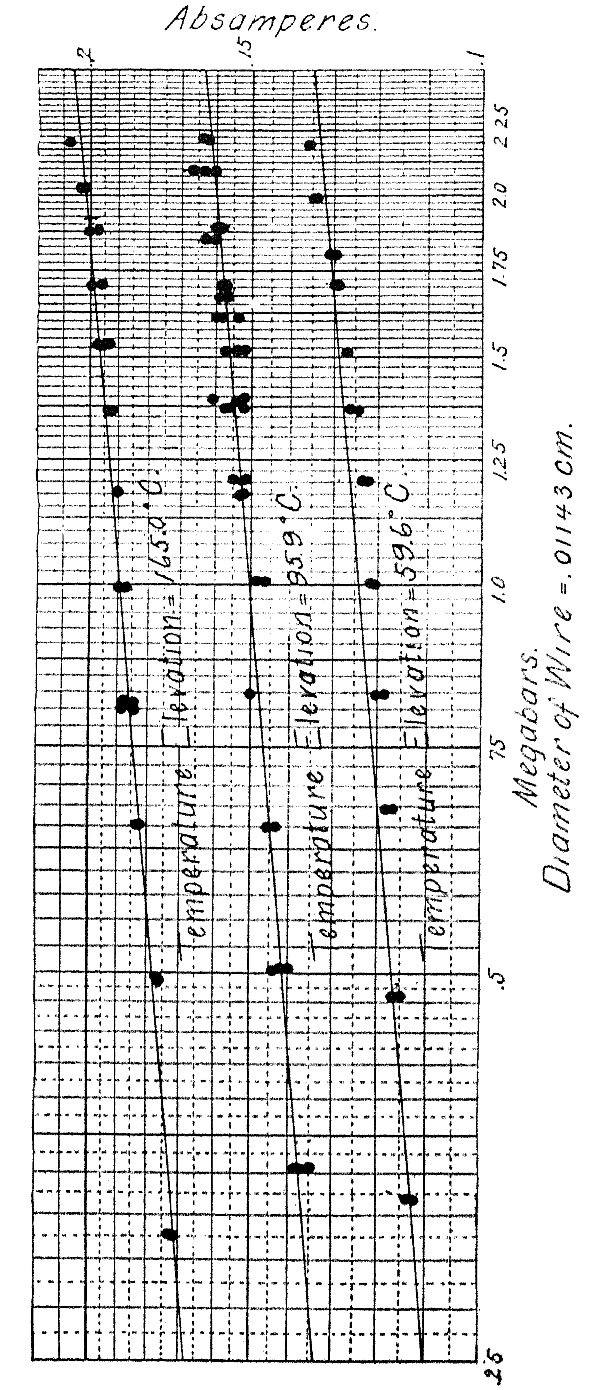

FIG. 3.—Current strengths for varying air-pressures.

where o is the circumference of the wire or its surface per linear cm.

$$P_c = I^2 r_T - o\,\sigma\,(T_a{}^4 - t_a{}^4) \quad \text{abwatts per cm.} \quad (7)$$

Results of measurements with free convection. At different times, thirteen series of observations were made on the dissipation of heat in the tank under varying atmospheric pressures, with the three test-wires specified in Table I. The results of these series are presented graphically in Fig. 3 for wire No. 1, in Fig. 4 for wire No. 2, and in Fig. 5 for wire No. 3. These figures are drawn on logarithmically scaled paper, with ordinates of current strength in absamperes through the test-wire, and abscissas of air-pressure in megabars within the tank. The temperature elevation of the wire θ, in degrees cent., is marked on each line. It will be seen that the observations conform fairly well to straight lines in each case, and that these lines are parallel for different temperature elevations on one and the same wire. This condition represents an equation of the form:

$$I_\theta = B_\theta\, p^m \qquad \text{absamperes} \quad (8)$$

where B_θ is a constant for each series of observations, and I_θ is the current strength required to be kept flowing through the test wire in order to maintain its temperature elevation θ degrees cent., under an air pressure of p megabars. The tangent of the angle of inclination of the lines with the abscissa axis being equal to the exponent m, we find that for wire No. 1, $m = 0.09$; for wire No. 2, $m = 0.12$, and for wire No. 3, $m = 0.13$; so that the exponent varies with the diameter of the test wire.

In Fig. 6, the results for the three sizes of wire are collected and expressed to logarithmic scale, with ordinates showing linear convection in units of $10^5 \times$abwatts per cm. and abcissas showing tank air-pressure in megabars. Again, the observations conform fairly well with straight lines, or indicate a relation:

$$P_c = A_\theta\, p^n \qquad \text{abwatts per cm.} \quad (9)$$

where P_c is the linear convection, or power dissipated in convection per linear centimeter of the test wire, p is the tank air-pressure in megabars, A_θ is a constant for each wire at a given

temperature elevation θ degrees cent., and n is an exponent equal to the tangent of the slope of the line. If there were no

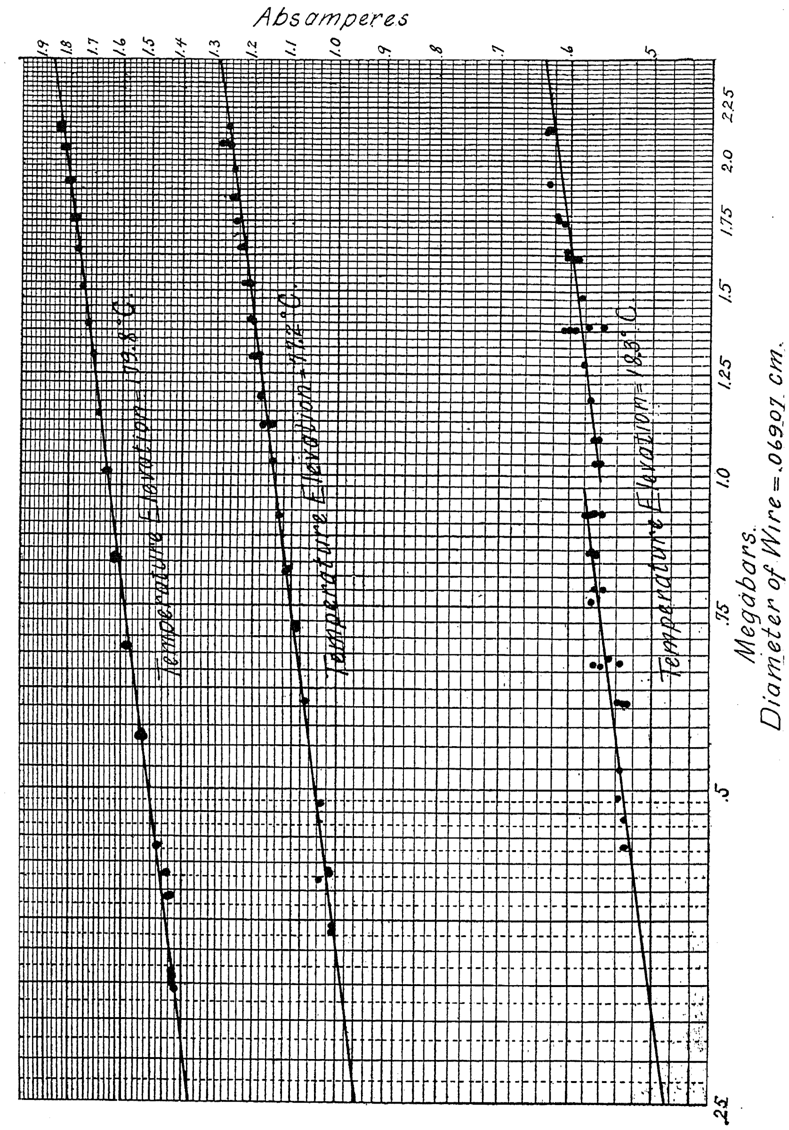

Fig. 4.—Current strengths for varying air-pressures.

radiation to allow for, it would follow that $n = 2m$ for each wire, or the slopes in Fig. 6 would be respectively double those in Fig. 3, 4 and 5. We find, however, for wire No. 1, $n = 0.19$

for wire No. 2, $n = 0.268$, and for wire No. 3, $n = 0.38$, numerical values which approximately satisfy the relation:

$$n = 0.9 \sqrt[3]{d} \qquad \text{numeric} \qquad (10)$$

where d is the diameter of the wire in cm.

If we analyze the values of A_θ in Fig. 6 for the standard air-pressure of 1 megabar (nearly 1 atmosphere), we obtain the results given in the following table.

The foregoing table shows that we may express the linear convection for any one of these wires by the approximate formula:

$$P_c = c\, \theta\, p^{\,0.9\sqrt[3]{d}} \qquad \text{abwatts per cm. and degree cent.} \qquad (11)$$

TABLE III

LINEAR CONVECTIONS PER DEGREE CENT. TEMPERATURE ELEVATION AT THE AIR PRESSURE OF 1 MEGABAR

Diam. of wire cm.	Temp. elevation θ degree cent.	Linear convections at 1 megabar A_θ abwatts per cm.	Linear convection per degree cent. elevation at 1 megabar $c = A_\theta/\theta$	Mean c
0.01143	59.6	2.96×10^5	5,000	5,300
	95.9	4.95 "	5,200	
	165.	9.2 "	5,600	
0.02616	15.1	0.98 "	6,400	5,600
	37.8	1.8 "	4,800	
	46.4	2.45 "	5,300	
	46.8	2.47 "	5,300	
	58.8	3.58 "	6,100	
0.06907	18.3	1.4 "	7,800	8,500
	77.2	6.55 "	8,500	
	179.8	16.7 "	9,300	

where p is the air-pressure in megabars (between 0.15 and 2.5 megabars), d is the diameter of the copper wire in cm. (between 0.01 and 0.07 cm.), θ is the temperature elevation of the wire above ordinary room temperature, in degrees cent., (up to 180 cent.) and c is a quantity which is in the neighborhood of 6000 abwatts per cm. and per degree cent. at 1 megabar, and appears to increase slowly with the diameter of the wire, perhaps in a linear relation:

$$c = a + b\,d \qquad \text{abwatts per cm. and per degree cent., at 1 megabar.} \qquad (12)$$

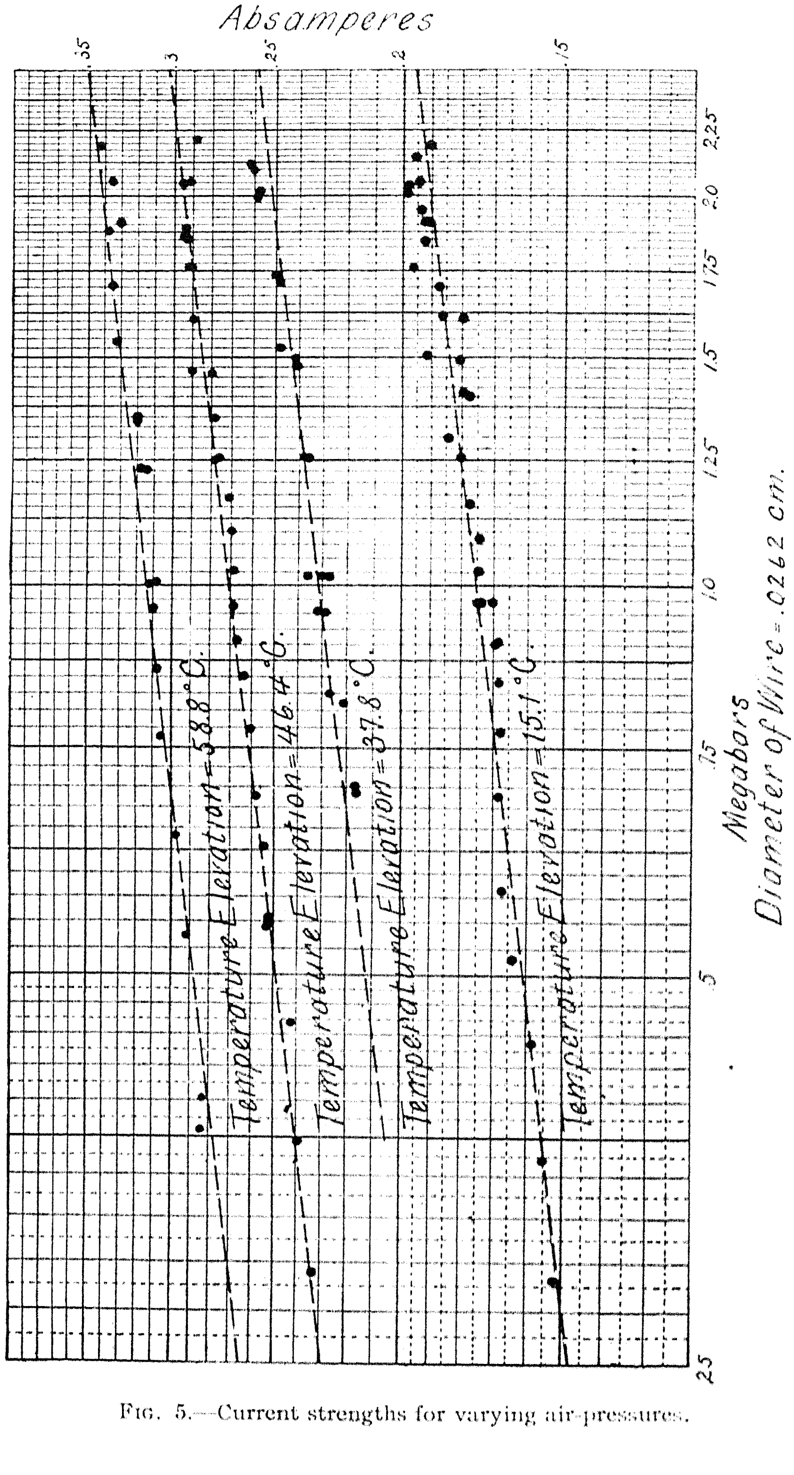

FIG. 5.—Current strengths for varying air-pressures.

As a first approximation we might take:—

$$c = (4+64\,d)\,10^3 \quad \text{abwatts per cm. and per degree cent., at 1 megabar.} \tag{13}$$

where d is the diameter of the wire in cm.

It is difficult to measure linear free convection from a wire

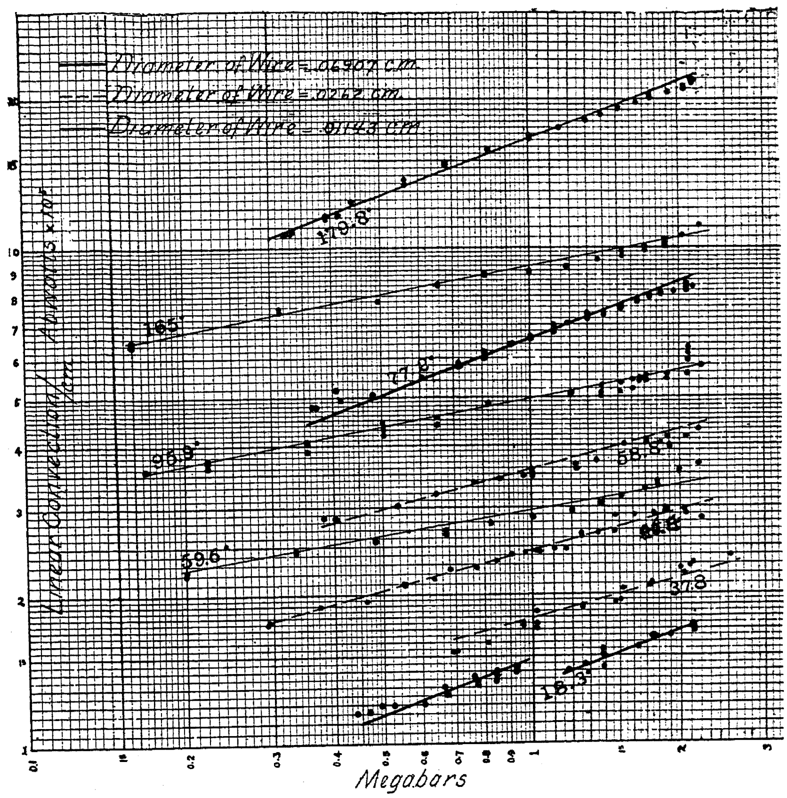

FIG. 6.—Linear convection for different wires at varying air-pressures.

with precision, for the reason that the stream of heated air rising from the wire is very unstable, and is markedly influenced by the presence of slight air-currents, or draughts, in the neighboring atmosphere. Such a wire forms, in fact, a very sensitive draught detector. On the other hand the precision of forced convection measurements is distinctly higher, except at very low wind velocities.

The complete expression of the linear free convection from a thin copper wire supported horizontally in substantially still air was, therefore, from the foregoing results:

$$P_c = (4{,}000 + 64{,}000\, d)\, \theta\, p^{\,0.9\sqrt[3]{d}} \quad \text{abwatts per cm.} \qquad (14)$$

Measurements of Forced Convection

General plan. In order to measure forced convection, the test wire was mounted between the prongs of a long fork, which was fastened at right-angles to a shaft driven by a motor at a regulated speed. The test wire was thus forced to move transversely through the air at a known velocity. This was taken as equivalent to a wind of the same velocity moving past the stationary wire.

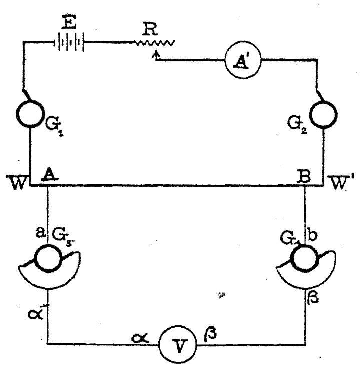

FIG. 7.—Electrical connections used in measurements of forced convection.

Electric connection was maintained with the moving wire through pairs of slip-rings and brushes, one pair of slip-rings carrying the testing current through the moving wire, while another pair served to maintain pressure connections with taps near the ends of the wire. The wire was maintained at a constant temperature during each series of tests by keeping its resistance constant; that is, by keeping constant the ratio of the potential difference between taps to the testing current, (E/I). The square of the current, multiplied by the linear resistance of the wire at the working temperature, then gave the linear dissipation in the wire at the observed velocity. The linear dissipations reduced by the computed linear radiations, gave the corresponding linear convections at the observed velocities.

Electrical connections. The electrical connections are indicated in Fig. 7. $W W'$ is the test-wire in the main circuit of the storage battery E, (from 8 to 15 volts), A' the ammeter, R the controlling resistance, and G_1 G_2 the slip-rings, with their

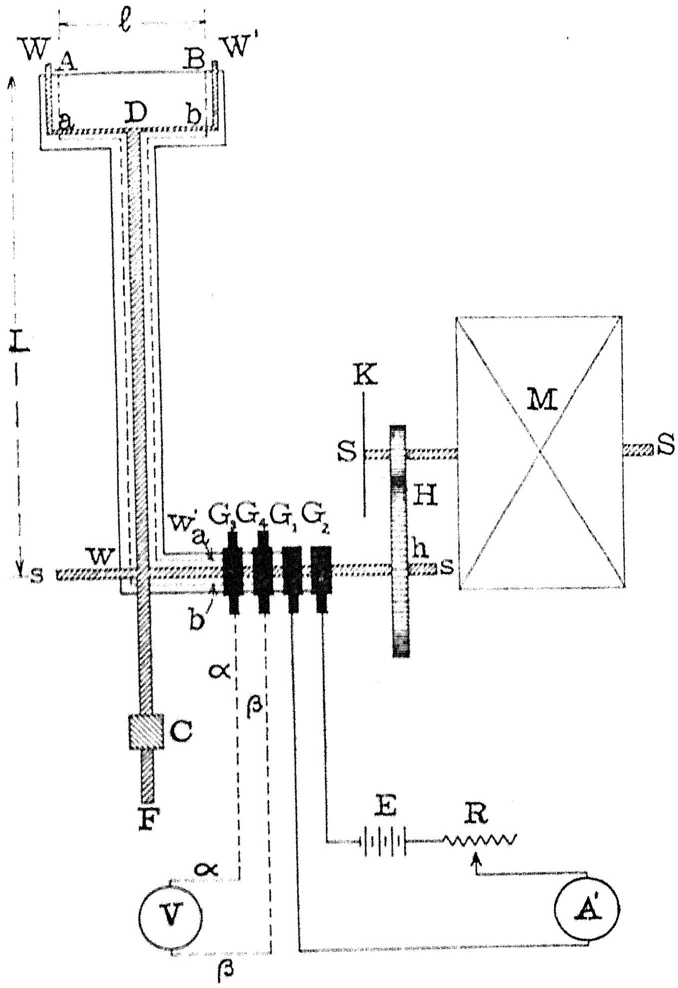

FIG. 8.—Mechanical and electrical connections used in measurements of forced convection.

copper gauze brushes. At the points A B, taps were soldered to the test-wire, and brought into connection with the voltmeter V through the wires Aa, Bb, the slip-rings G_3 G_4, and their copper-gauze double-brushes.

Mechanical and electrical connections. In Fig. 8, the test-wire $W W'$ is shown in place in the fork $D F$, the details of the support being indicated in Fig. 9. This fork is made of a steel tube of 1.9 cm. ($\frac{3}{4}$ in.) external diameter and 1.63 cm. (0.64 in.) internal diameter, 274 cm. (9 feet) long, mounted in the shaft $s\ s$, which is driven by the motor shaft $S\ S$ through the velocity-reducing gear-wheels H, h. When adjusted in position, the wire $W W'$ stood parallel to the shafts $s\ s$, $S\ S$. The radius L of the test-wire from the axis of the shaft $s\ s$ was adjustable be-

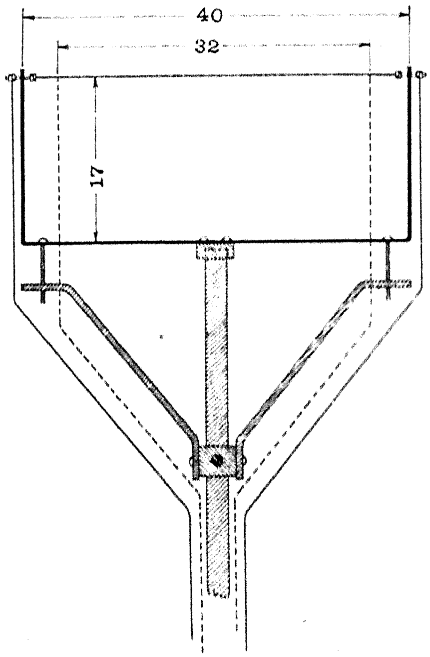

FIG. 9.—Details of fork. Dimensions in centimetres.

tween certain limits. The usual radii selected were about 118 cm. (46.5 in.) and 180 cm. (70.8 in.), respectively. This change in radius not only enabled high and low velocities to be more easily attained; but also enabled the same velocity to be secured with two radii, as a check upon the observations. A suitable counterpoise C had to be used in each case.

The motor M was a $\frac{1}{2}$-h.p., 115-volt, 4-pole machine, running at 1200 rev. per min. Its speed of rotation was controlled, in the usual way, by resistances in its field and armature circuits, not shown in the illustrations. The velocity-reducing gear

ratio of 18/120, enabled the fork-shaft to make 180 rev. per min., when the motor-shaft made 1200 rev. per min. The four slip-rings were of brass, each 4 cm. (1.57 in.) in diameter, and 1.5 cm. (0.58 in.) wide. The copper gauze brushes resting on them were 3.8 cm. (1.5 in.) long, 0.95 cm. ($\frac{3}{8}$ in.) wide and 0.67 cm. (0.26 in.) thick. The brushes were soldered to their respective connecting wires, and were spring-pressed into contact with their slip-rings. As an extra precaution, two brushes were used on each of the voltage slip-rings G_3 G_4.

The voltmeter V and ammeter A^1 were standard portable instruments, specially calibrated for these tests. The frame on the end of the fork supporting the test wire is represented in detail in Fig. 9. It was made of brass strip 1.9 cm. ($\frac{3}{4}$ in.) broad and 0.32 cm. ($\frac{1}{8}$ in.) thick, so arranged as to disturb the air in the immediate neighborhood of the test-wire as little as possible during the motion.

An illustration of the general apparatus is given in Fig. 13, reproduced from a photograph.

Sample series of measurements. In making a series of tests on a test wire, the temperature elevation of the wire was selected in advance, and the resistance of the wire for that temperature elevation was computed by equation (1) from its resistance at the room temperature. A slide-rule was then set to this resistance, in such a manner that for any current strength through the wire, the corresponding potential difference required by Ohm's law could be read off directly. The fork radius L, Fig. 8, was then set, and the motor M brought into action. The speed of the motor-shaft was then adjusted to a suitable value. All speeds except the lowest were measured by means of an electromagnetically driven stroboscopic fork*, and through the slits in its vibrating shutters, a target K, mounted on the motor-shaft, was observed. The speed could thus be measured, with all desired accuracy, within from three to fifteen seconds after bringing the fork to the eye. The current was then increased until the correct current-voltage ratio was obtained, the speed again observed, and all three values recorded.

Each series comprised two different fork radii (L, Fig. 8), in such a manner that both radii were in use over a certain range of speed.

* The Measurement of Rotary Speeds of Dynamo Machines by the Stroboscopic Fork. By A. E. Kennelly and S. E. Whiting, TRANSACTIONS, American Institute Electrical Engineers, 1908, Vol. XXVII, p. 631.

Table IV contains a sample series of observations.

Column I gives the velocity of the wire, perpendicular to its length, through the air, or the "wind-velocity" using the formula:

$$v = 2\pi L N \qquad \text{cm. per sec.} \qquad (15)$$

TABLE IV

SAMPLE SERIES OF FORCED CONVECTION TEST

Run on wire of 2.038×10^{-2} cm. diameter.

Resistance held at 2.5×10^{8} absohms, giving 128.3 degrees cent. elevation.

Barometer 770.4 mm. @ 18.6 degrees cent.

Average room temperature, 21 degrees cent.

Assmann aspiration psychrometer gave readings for $t = 20.2$ degrees cent. and $t' = 12.6$ degrees cent.

Pressure 6.977 mm. corresponding to 0.905% or 9.05 l. of water-vapor per cubic meter of air.

I	II	III	IV	V	VI	VII	VIII
Wind velocity v cm./sec.	I Abs. amp.	I^2 Abs. amp.^{2}	$P = I^2 r_T$ Linear power Abwatts cm.	P_r Linear radiation Abwatts cm.	P_c Linear convection Abwatts cm.	P'_c/θ Linear convection per degree cent. elevation	$k = P'_c/(\theta\sqrt{v})$ Linear convection per degree cent. elevation and per $\sqrt{v}$
	$\times 10^{-1}$	$\times 10^{-1}$	$\times 10^{6}$		$\times 10^{6}$	$\times 10^{4}$	10^{3}
0	3.4	1.156	0.903	7.763×10^{4}	0.825	0.643	
114	5.6	3.136	2.450		2.372	1.849	1.731
205.7	6.2	3.844	3.003		2.926	2.380	1.591
330.5	6.8	4.624	3.613		3.535	2.755	1.515
522	7.5	5.625	4.394		4.316	3.364	1.472
395	7.05	4.970	3.884		3.807	2.967	1.493
561	7.5	5.625	4.394		4.316	3.364	1.420
708	8.0	6.400	5.000		4.923	3.837	1.442
880	8.5	7.225	5.645		5.567	4.340	1.463
1113	9.05	8.190	6.400		6.322	4.928	1.477
1321	9.42	8.874	6.931		6.853	5.342	1.470
775	8.3	6.889	5.382		5.305	4.135	1.485
1072	9.0	8.100	6.328		6.250	4.872	1.488
1383	9.55	9.120	7.127		7.049	5.494	1.477
1632	9.95	9.900	7.734		7.656	5.968	1.477
1862	10.35	10.712	8.368		8.291	6.463	1.498
0	0	1.225	0.957		0.879	0.685	

where L is the fork radius (Fig. 8) in cm., N is the speed of the fork-shaft in rev. per sec., and v is the wind-velocity in cm. per sec.

Column II gives the observed current, in absamperes through the wire, required to maintain its correct linear resistance in absohms per cm. at the working temperature T degrees cent.

Column III gives the current squared, expressed in absamperes squared. Column IV gives the linear dissipation in abwatts per cm. ($P = I^2 r_T$). Column V gives the linear radiation computed from the formula:

$$P_r = o\,\sigma\,(T_a^4 - t_a^4) \quad \text{abwatts per cm.} \qquad (16)$$

where o is the circumference of the wire in cm., σ is Stefan's coefficient, taken as 5×10^{-5} abwatts per sq. cm. and degree absolute. T_a is the absolute working temperature of the wire, and t_a the absolute temperature of the surrounding air. Column VI gives the linear convection $P_c' = P - P_r$, obtained by deducting the computed radiation from the linear dissipation. Column VII gives P_c'/θ or the linear convection divided by

TABLE V

PARTICULARS OF COPPER WIRES USED IN MEASUREMENTS OF FORCED CONVECTION

Wire No.	Diameter		Length between taps cm.	Cross-sectional area cm.^{2}	Linear surface cm.^{2}	Linear resistance @20 degrees cent. absohms/cm.	Linear resistance @0 degrees cent. absohms/cm.	Resistivity @ 0 degrees cent. absohm-cm.
	Cm.	Inches						
1	0.01008	0.004	32.5	0.7971×10^{-4}	3.167×10^{-2}	2.18×10^7	2.011×10^7	1603
2	0.0159	0.0063	31.6	1.985 "	4.995 "	0.8703 "	0.8028 "	1594
3	0.02038	0.008	32.0	3.262 "	6.402 "	0.5219 "	0.4815 "	1600

the inferred temperature elevation of the test-wire. The last column gives $P_c'/(\theta\sqrt{v})$ or the quotients obtained by dividing P_c'/θ by the square root of the wind velocity. It will be observed that except for the lowest wind velocity of 114 cm. per sec. the entries in the last column are seen to be substantially constant near 1.497×10^3.

Copper wires. The accompanying table presents the dimensions of the copper wires employed in the measurements of forced convection.

Correction for linear radiation. It will be observed that in the case presented in Table IV, the computed linear radiation P_r' is only 3% of the linear dissipation P at the lowest recorded speed—114 cm. per sec.—and is only about 1% of the linear dissipation at the highest recorded speed—1862 cm. per sec.

This is a fair sample of the relation between the linear radiation and dissipation for all of the wires used. It shows that whereas the error due to incorrect estimation of σ, which might enter into the results for free convection, would be small but appreciable, the corresponding error in the results for forced convection would be practically quite inappreciable.

Results of measurements of forced convection. Out of the 16 series of measurements at different dates on the three test wires, all of which are in good agreement, 10 series are plotted on the logarithmically scaled chart shown in Fig. 10. The ordinates are in current units of amperes, or of 10^{-1} absamperes, (2 to 20 amperes) and the abscissas in units of wind-velocity of 10^2 cm. per sec., (200 to 2000 cm. per sec.). It will be observed that, except at the lowest velocities, the observations conform closely to straight lines whose inclinations with the abscissa axis are all 14°, or $\tan^{-1} 0.25$; so that the observations indicate a relation

$$I_\theta = K_\theta \sqrt[4]{v} \qquad \text{absamperes} \qquad (17)$$

in which K_θ is a constant for each wire and temperature elevation θ. The diameter of the wire is indicated by the type of line in Fig. 10, and the temperature elevation of the series in degrees cent. is marked on each line.

In Fig. 11 the results for the same ten series are plotted as linear convections in ordinate units of 10^6 abwatts per cm., against wind-velocity in abscissa units of 10^2 cm. per second. The lines are here inclined at an inclination of $26\frac{1}{2}° = \tan^{-1} 0.5$, and the results indicate a relation for linear convection:

$$P_c' = K_\theta' \sqrt{v} \qquad \text{abwatts per cm.} \qquad (18)$$

where K_θ' is a constant for each size of wire and temperature elevation θ.

Equations (17) and (18) cease to hold below wind velocities of 200 cm. per sec., apparently for the reason that these equations require zero linear convection at zero wind velocity, or stationary air. But free convection continues under the latter condition; so that these equations fail at low velocities by ignoring the influence of free convection.

The following table summarizes the results for the ten series of observations presented graphically in Figs. 10 and 11.

Column III gives the temperature elevation in degrees cent.
" IV " " linear convection from Fig. 11 for v = 900 cm./sec.
" V " $K_\theta' = P_c'/\sqrt{900} = P_c'/30$ abwatts per cm and cm./sec.
" VI " $k = K'/\theta$ abwatts per cm. for unit speed and 1 degree cent. elevation.

It appears, therefore, that for wire No. 1 the linear convection is:

$$P_c' = 884\,\theta\sqrt{v} \qquad \text{abwatts per cm.} \qquad (19)$$

TABLE VI

I Test wire No.	II Diameter. cm.	III Temperature elevation. θ deg. cent.	IV Linear convection P'_c for v = 900. abwatts/cm.	V K'_θ for v=900. abwatts/cm	VI k abwatts per cm. and deg. cent. elevation	VII Mean k
1	0.01008	106.05	2.81×10^6	0.9366×10^5	883	884
		179.	4.63 "	1.547 "	864	
		251.8	6.83 "	2.277 "	904	
2	0.0159	117.4	4.16 "	1.387 "	1181	1212
		211.1	7.74 "	2.58 "	1222	
		305.	11.28 "	3.76 "	1233	
3	0.02038	51.	2.225 "	0.7416 "	1454	1475
		128.3	5.76 "	1.92 "	1497	
		205.5	9.08 "	3.027 "	1473	
		282.8	12.52 "	4.173 "	1476	

for wire No. 2:

$$= 1212\,\theta\sqrt{v} \qquad \text{abwatts per cm.} \qquad (20)$$

and for wire No. 3:

$$= 1475\,\theta\sqrt{v} \qquad \text{abwatts per cm.} \qquad (21)$$

Consequently, between the limits of $v = 200$ and $v = 2000$ cm. per sec. (4.5 and 45 miles per hour), and up to temperature elevations of 300 degrees cent., the linear forced convection is expressed by

$$P_c' = k\,\theta\sqrt{v} \qquad \text{abwatts per cm.} \qquad (22)$$

where k increases nearly in direct proportion to the diameter. A first approximation to the forced linear convection for a small wire of diameter d cm. would be:

$$P_c' = 76{,}000\, d\, \theta \sqrt{v} \qquad \text{abwatts per cm.} \qquad (23)$$

If we attempt a second approximation, which will more nearly satisfy equations (19) (20) and (21), we obtain:

$$P_c' = (300+58{,}000\, d)\, \theta \sqrt{v} \qquad \text{abwatts per cm.} \qquad (24)$$

Influence of water vapor in the air upon forced linear convection. On the occasion of each series of observations, a measurement of the amount of water-vapor in the air of the room was made by means of an Assmann psychometer, in which a fan driven by clock-work forces air over the wet and dry bulbs of a pair of thermometers placed side by side. The amount of water-vapor varied between 5.8 and 13.8 liters per cubic metre, on different days.

One or two measurements were taken with the air almost completely saturated with moisture after boiling water in the room for some hours. The object was to ascertain whether the presence of the water-vapor in excess appreciably altered the linear forced convection. It was found that starting with a clean test wire 0.0159 cm. in diameter, worked at 70 degrees cent. elevation in normally dry air, at a barometer reading of 778 mm., the series obtained was

$$P_c' = 1255\, \theta \sqrt{v} \qquad \text{abwatts per cm.} \qquad (25)$$

The next day, with moisture-saturated air and a barometer reading of 772 mm., the series obtained was about

$$P_c' = 1210\, \theta \sqrt{v} \qquad \text{abwatts per cm.} \qquad (26)$$

At first sight the reduction of about 4% seemed attributable to the excess of water-vapor; but it was found that the test wire had become decidedly blackened and superficially oxidized by being driven in the moist air. On repeating the series on the day following, with the same wire in normally dry air, at a barometer reading of 765 mm., the series obtained was:

$$P_c' = 1210\, \theta \sqrt{v} \qquad \text{abwatts per cm.} \qquad (27)$$

or the same as in the saturated air. Consequently, the effect of moisture in the air was relatively small, and was masked by the influence of changes in surface condition of the test wire or changes in the barometric pressure. The observations have not yet even served to determine whether the linear forced convection is increased or diminished by the presence of water-vapor.

Correction for free convection at low wind velocities. If formulas (23) and (24) are applied to the case of the test wire at rest and subject only to free convection, using the value of k obtained with higher wind velocities, we obtain a certain fictitious value of v which may be called the virtual wind-velocity of free convection. It is a matter of interest to examine this virtual velocity in the different series of observations. They are presented in the following table:

TABLE VII

VIRTUAL WIND VELOCITIES OF FREE CONVECTION FOR DIFFERENT TEST WIRES

I Diameter of test wire cm.	II Temperature elevation θ degree cent.	III Constant k abwatts/cm. per degree cent. and unit velocity	IV Linear free-convection $v = 0$ abwatts per cm. and degree cent.	V Virtual wind velocity v' cm. per sec.	VI Mean v' cm. per sec.
0.01008	106.05	883	4640	27.5	
	"		4790	29.3	
	"		4438	25.4	
	179	864	3968	21.1	27.5
	"		4794	30.7	
	251.8	904	4739	27.5	
			5042	31.0	
0.0159	117.4	1181	5269	20.0	
	"	"	5420	21.0	
	117.4	"	5395	20.8	
	"	"	5777	23.8	
	211.1	1222	6072	24.7	22.9
	"	"	5850	22.9	
	"	"	5850	22.9	
	305	1233	6016	23.7	
	"	"	6660	29.1	
0.02038	51	1454	5515	14.4	
	"	"	6354	19.1	
	128.3	1497	6434	18.5	
	"	"	6855	21.0	
	205.5	1473	6818	21.5	20.7
	"	"	7196	23.8	
	282.8	1476	7018	22.6	
	"	"	7378	25.0	

Column II gives the temperature elevation of the test, column III the values of k deduced for that test in Table VI. The linear convection per degree cent. elevation with the wire at rest is

given in Column IV. This is linear free convection; but, treating the convection as though it were due to wind velocity, we obtain the values of the virtual wind velocity given in Column V, with the mean values for each wire in Column VI. It appears that the virtual wind velocity is independent of the temperature elevation and that it is about 23 cm. per second. It is independent of the size of wire to a first approximation; but is slightly lowered by increasing the wire diameter.

If we increase all the actual wind velocities by 25 cm. per second, so as to correct for free convection, we obtain an empirical formula which conforms satisfactorily with the observations not only between the speed limits of 200 and 2000 cm. per sec., but also down to zero speed. We obtain from (23):

$$P_c' = 76{,}000\, d\, \theta \sqrt{v + 25} \qquad \text{alwatts per cm.} \qquad (28)$$

or taking formula (24) as more nearly representing the observations, we have:

$$P_c' = (300 + 58{,}000\, d)\, \theta \sqrt{v + 25} \qquad \text{alwatts per cm.} \qquad (29)$$

Possible application to anemometry. The law indicated by the results above stated, that the linear forced convection from a thin copper wire is proportional to the temperature elevation and to the square root of the wind velocity, after allowing a small correction for free convection, may be capable of application to the measurement of wind-velocities. It would probably be necessary to maintain a constant surface condition for the test wire by gilding or platinizing it, in order to keep the wire from oxidizing. A short vertical test wire carrying, say, constant-current strength under 110 volts, and with a steadying resistance of German-silver in its circuit, to absorb all but 6 volts, would fall in temperature and in resistance with an increase of the horizontal component of the wind. A recording voltmeter, connected to pressure taps near the ends of the test-wire, would leave a record that would be interpretable in wind-velocity. The instrument would have the maximum sensitiveness at low wind velocities and the sensitiveness would diminish as the wind velocity increased. Or, the test wire might be worked at constant voltage of, say, 6 volts from a storage battery, and a recording ammeter in circuit with the wire would leave a record interpretable in wind velocity.

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Summary of experimental results. From the measurements above described on the dissipation of heat from thin copper wires (diameters 0.1 to 0.7 mm., or 4 to 27 mils) the following facts may be stated for conditions of free convection in a tank:

At constant standard air-pressure of 1 atmosphere, the linear convection was proportional to the temperature elevation of the wire, and was in the neighborhood of 6,000 abwatts (6×10^{-4} watt) per linear cm. per degree cent. and, to a first approxima-

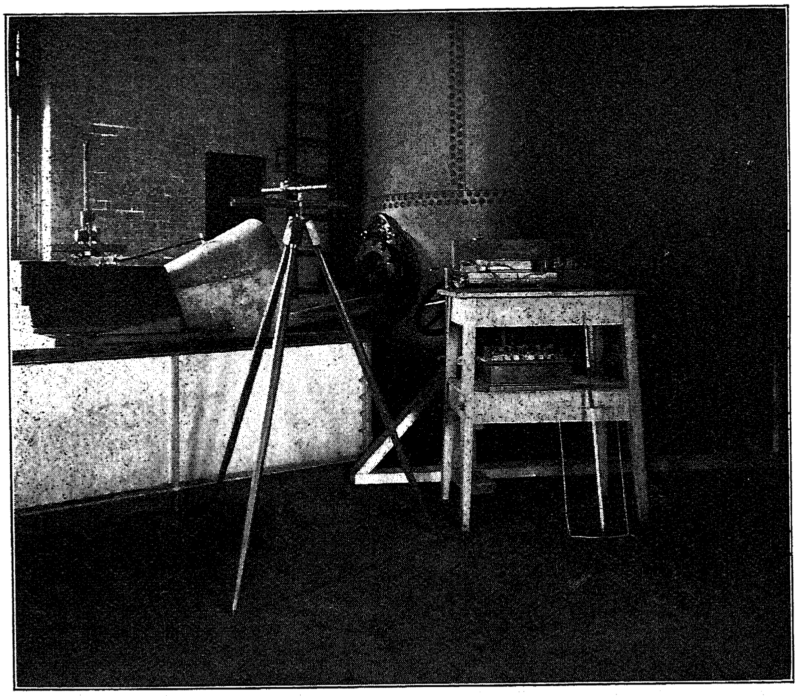

FIG. 12.—Apparatus in free convection tests.

tion, independent of the size of wire, although increasing slowly with the diameter. The results do not yet warrant a decision as to the precise increase with diameter, but it represented about $4{,}000 + 64{,}000\ d$ abwatts per cm., where d is the diameter in cm.

Under varying atmospheric pressure, the linear convection increased with the pressure, substantially according to an exponential law, or to a straight line on logarithm paper, the exponent being the same for the same wire at different temperature elevations but with different wires increasing slowly

with the diameter of the wire, or, to a first approximation, as the cube root of the diameter.

The current strength required to maintain a wire at constant temperature under free convection in varying atmospheric pressure, followed an exponent of the pressure, or a straight line on logarithm paper, the exponent being the same for different temperature elevations (up to 180 degrees cent.) of the same wire, but increasing slowly with the diameter of the wire.

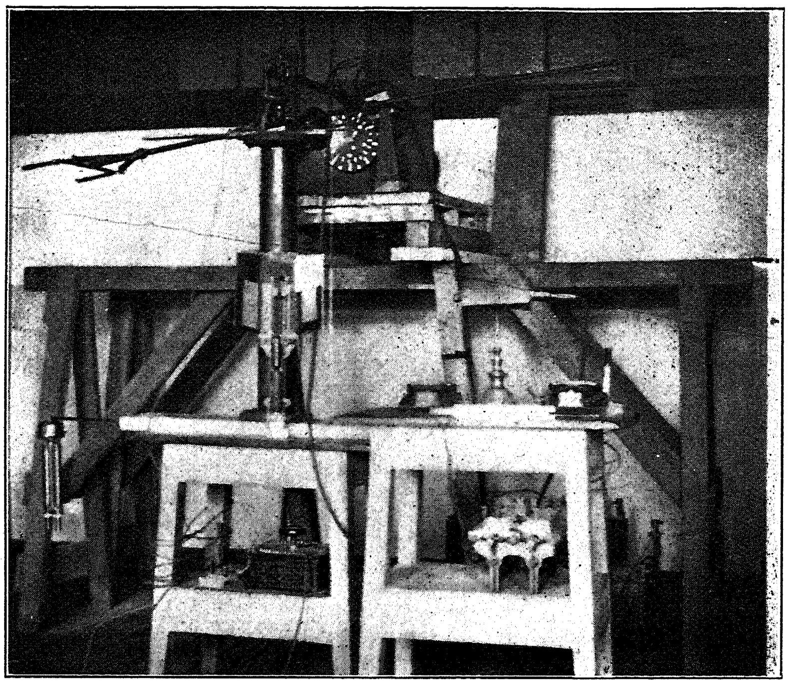

FIG. 13.—Apparatus in forced convection tests.

For conditions of forced convection, or wind, on a thin wire, the following observations may be stated:

The linear convection was proportional to the temperature elevation of the wire at the same wind velocity.

The linear convection, at any single wind velocity between 200 and 2000 cm. per second, increased with the size of wire and, as a first approximation, in direct proportion to the wire diameter.

Under varying wind velocities, within the range of 200 to

2000 cm. per second, the linear convection increased as the square root of the wind-velocity.

At wind-velocities below 200 cm. per second, the linear convection was greater than that predicted by the last preceding rule, apparently owing to the presence of free convection in air not moved by an external force.

The free convection from the wires at rest corresponded to a forced convection at a virtual wind-velocity of about 25 cm. per second. To a first approximation this virtual wind velocity of free convection was constant for the different temperature elevations, and also for the different sizes of thin wire tried; but it diminished slightly as the diameter of the wire increased.

The correction for radiation on a wire of 0.2 mm. diameter in a wind velocity of 500 cm. per second was less than 2 per cent.

The linear convection from a wire under a wind velocity of 2000 cm. per second was nine times that when the wire was at rest.

If a virtual wind velocity of 25 cm. per second, for free convection, be added to all the actual wind velocities in forced convection tests, the linear convections conform satisfactorily to the law of square roots of wind velocity from 2000 cm. per sec. down to 0 cm. per sec.

A thin layer of oxide on the wire slightly diminished the forced convection.

List of Symbols Employed

A_θ, B_θ = constants of linear free convection, abwatts and absamperes per cm. at 1 megabar.

a, b, c = constants of linear free convection, abwatts per cm. per degree cent. at 1 megabar.

d = diameter of test-wire, cm.

$G\,G'$ = resistances of differential galvanometer coils, ohms or absohms.

I_θ = current strength in test-wire at $\theta°$ cent, temperature elevation, absamperes.

$K_\theta\,K_\theta'$ = constants of linear forced convection, absamperes and abwatts per cm. at 1 cm. per sec.

k = constant of linear forced convection, abwatts per cm. per deg. cent., at 1 cm. per sec.

N = rotary speed of fork-shaft, rev. per sec.

m, n = exponents of the atmospheric pressure, numerics.

o = circumference of a test wire, cm. = linear surface of a test wire, sq. cm./cm.

p = atmospheric pressure, megabars.

R = resistance of test wire, ohms or absohms.

R' = Constant comparison resistance of German silver, ohms or absohms.

R_0 = Resistance of a test wire at 0° cent., absohms.

R_T = Resistance of a test-wire at T° cent., absohms.

r = Linear resistance of a test-wire, absohms.

r_T = Linear resistance of a test-wire at T degrees cent., absohms.

ρ = adjustable resistance in galvanometer circuit, ohms, or absohms.

σ = Stefan's coefficient, abwatts per sq. cm. per degree absolute.

$P = I_\theta^2 r$ = linear dissipation of power in test-wire, abwatts per cm.

P_c = linear free convection from test-wire, abwatts per cm.

P_c' = linear forced convection from test-wire, abwatts per cm.

P_r = linear radiation from test-wire, abwatts per cm.

T_a = Temperature, in degrees absolute, of test-wire.

T = Temperature, in degrees cent., of test-wire.

t_a = Temperature, in degrees absolute, of surrounding bodies.

t = Temperature, in degrees cent., of surrounding bodies.

$\theta = T - t$ = Temperature elevation, degrees cent., of hot wire.

t' = Temperature in degrees cent. of wet bulb thermometer.

v = velocity of wind, cm. per second.

v' = virtual wind-velocity of free convection, cm. per second.

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DISCUSSION ON "THE CONVECTION OF HEAT FROM SMALL COPPER WIRES." FRONTENAC, N. Y., JUNE 28, 1909

V. Karapetoff: Dr. Kennelly uses the absolute electromagnetic system of units, and besides he uses the abbreviation "abs." for absolute. I have always found this system of notation to be very convenient, as well as the notation "*abstat*" for absolute electrostatic units. But while fully appreciating the convenience of the use of the absolute system in some cases, I would like to ask Dr. Kennelly the reason for using this system in this paper? For instance, referring to Table I, or to any other Table in the paper, it will be seen that the current values could just as well have been expressed in ordinary amperes, because they come very distinctly within the range of amperes. In the same way Dr. Kennelly measures resistance in absohms, or absolute ohms, then multiplies by 10^{-9}, and finally gets ordinary ohms.

I would also like to ask the reason for introducing such unfamiliar units as megabars for pressures, when they could have been expressed in atmospheres, or, to be more consistent, in metric atmospheres (one metric atmosphere is one kilogram pressure per square centimeter)? I see that power could not be expressed conveniently in watts in this case, but I do not see why abwatts or absolute watts could not be employed, or, still better, milliwatts or microwatts, which every electrical engineer would understand.

Dr. Kennelly calls formulas (8) and (9) *exponential.* I think it more correct to call them *parabolic.* What is known as an exponential expression is of the form $y = Ce^{\mp x}$; there are several phenomena in physics and in engineering that follow this exponential law. The expression used in the paper is of the form $y = Cx^n$, and it would be more consistent to call it a parabolic law. However, this is a minor point, provided the distinction be clearly understood.

Formula (28) is rather a common form of a law that is met with in engineering; that is to say, where one variable varies as a certain power of another variable, and then there is a correction term, in this case = 25 (correction for free radiation). If it were not for the correction term, the coefficients of the formula could be easily evaluated from the results of experiments, using logarithmic paper. I would ask Dr. Kennelly if there is any method by means of which formulas (28) and (29); that is, those containing a correction term, could also be handled by means of logarithmic paper; or do we have to go back to the laborious method of least squares?

Charles P. Steinmetz: I have given considerable attention to the study of a rational reduction of empirical phenomena and have also found the feature mentioned by Professor Karapetoff, the existence of a constant term, a great annoyance when using logarithms to derive a parabolic law. But I have found

that with a little experience the method can still be used. If the logarithmic line is curved slightly, concave or convex, it signifies a positive or negative constant term. Subtracting or adding a constant term to the function, and replotting, produces a second logarithmic curve. If this has the same curvature, it indicates that the constant term is too small; if it has the opposite curvature, by interpolating between the two logarithmic curves there results a value of the constant term which gives minimum curvature. Where necessary, by a second interpolation, a still closer approximation can be obtained, thereby evaluating a parabolic function with constant terms. This appears to me the simplest method. If any one knows of a simpler method I shall be glad to hear of it, because the quick evaluation of empirical curves is frequently of great importance.

Chas. F. Scott: I would reinforce what has been said about the nomenclature in this paper. The facts set forth are engineering facts, which should be known and appreciated and understood by ordinary engineers as well as by scientists. I am inclined to think that some people who may want to use this paper would be apt to interpret "abs" as meaning absurd. Dr. Kennelly himself, in presenting the paper, in order that those present may have some conception of what he is talking about, says in one place "that is approximately equal to one atmosphere." The term "megabar" conveys very little information to the average engineer, and other strange names are equally perplexing.

Appearances to the contrary, the paper is not abstruse, nor does it deal with the mysterious. It deals with phenomena which are matters of ordinary experience and for which there is an adequate, everyday language. Without questioning the excellence of the absolute system of units, I fail to see its utility in the present case. Even if it were conceded useful to call an ampere or a watt by some other name, why should we mystify the barometric pressure by some unfamiliar term that means nothing to most of us? So far as I see there is no fundamental relation in these tests requiring absolute units to express the watts of the atmospheric pressure—the latter is simply recorded as a condition under which the tests are made and its value does not enter into any formula any more than does the serial number of the test.

Another point, if I understand correctly; Dr. Kennelly points out in the main test that the curves showing the relation between convection and wind velocity cannot be drawn down to zero; they would tend to show zero as convection of heat at zero speed although there is, of course, a considerable loss. Now the speed that is really involved is not the absolute speed of the wire, but the relative speed between the wire and the air. When the wire is stationary, there really is some breeze created by the natural rise of air passing the wire. The natural draft caused by the rise of the hot air when the wire is actually

stationary in regard to surrounding objects, may result in an actual velocity with respect to the air, which possibly accounts for the apparent discrepancy. Possibly we could get the zero point; that is, the zero of relative motion between wire and air, by raising the wire at a definite speed so that the wire is continuously surrounded by the same air.

Paul M. Lincoln: I would like to express my appreciation of the data which Dr. Kennelly has presented in this paper. The engineer who is designing electrical apparatus is confronted at once with the questions: 1. How much heat can be radiated from the surface of this apparatus with a given temperature? and 2. How is the law of radiation changed by forced ventilation? The data which Dr. Kennelly has presented here give the best answer to those questions that I know of.

In this paper Dr. Kennelly gives data as to the rate at which heat is dissipated from bare metallic surfaces. The designing engineer is particularly interested in the rate at which insulated surfaces will dissipate heat. A study of the changes, if any, which will be introduced into the values determined by Dr. Kennelly by covering bare surfaces with the ordinary types of insulation would be of great assistance to the designing engineer. I hope that he will give us the result of such study at some future meeting.

A. E. Kennelly: In regard to the matter of "abs" and "abwatts", an apology is offered for the names, which are slang, but I offer no apology for using the C. G. S. system. Our reason for using the C. G. S. system in this case is that we have to deal with certain fundamental physical effects. This is a physical paper, rather than an electrical engineering paper. This report is only a first step towards electrical engineering.

It is true that we actually used ammeters, worked with amperes, and put them back into absamperes, in the manner Professor Karapetoff pointed out. We thought it simpler to keep to one and the same system of units, and not to oscillate between practical and absolute units. We may have been wrong in taking the absolute system as our choice, but having once adopted it, I think we were right in adhering to it.

I think Professor Karapetoff is right as to the term "parabolic." We should have said parabolic instead of exponential. We should have been glad to adopt that amendment in terminology.

In regard to what Mr. Scott has pointed out, the reason we chose the *megabar* is because it is the C. G. S. unit of pressure, one megadyne per square centimeter. If we are going to use C. G. S. units at all, we must use them throughout in order to be consistent. I do not think we should have gained in simplicity of statement if we had tried to define an atmosphere with equal precision in the ordinary system.

The remarks of Mr. Scott in regard to the matter of the virtual velocity of free convection are very pertinent and seem likely

to be correct, but we did not dare to say that. We do not know that the velocity of the air past the wire was twenty-five centimeters per second. The formula which we give to include the virtual velocity of free convection is an empirical formula, but we hope it may be possible to demonstrate that the formula is rational, as Mr. Scott suggests.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

ALTERNATOR FOR ONE HUNDRED THOUSAND CYCLES

BY E. F. W. ALEXANDERSON

Before entering into a description of the form of generator dealt with in this paper, it may be of interest to give a short review of the history of the high-frequency alternator. Heretofore the 10,000-cycle machine has represented the highest frequency in commercial use. Such a machine was designed by Steinmetz in 1900 and a number of them are now in service. A similar machine for 10,000 cycles was described by Lamme in a paper before the Institute in 1904.* Reference may be made particularly to a paper by Dudell read before the Physical Society of London, giving a synopsis of his work with this kind of apparatus and describing the experiments made by him.

In 1904 when the author began to develop for Professor Fessenden, an alternator which should have a frequency of about 100,000 cycles and an output sufficient for commercial work in wireless telegraphy, not much confidence was placed in the possibility of constructing such a machine. The doubts expressed in the discussion of Dudell's paper as to the practical application of such machines to wireless telegraph work seemed quite just, in view of the fact that the highest frequency that had been produced with any appreciable output was 10,000 to 15,000 cycles, while Dudell's machine, which had been worked up to 120,000 cycles, had an output of a fraction of a watt.

Since then three machines of the design described in the following pages, and referred to in several papers by Professor Fessenden, have been placed in service. In this paper there

* TRANSACTIONS A.I.E.E., 1904 Vol. XXIII. p. 417.

will be described a perfected form of the same general type, rated at 2 kw. and 100,000 cycles.

Early types. The stages of development which led to the present type of machine were substantially the following:

In view of the special precaution which had to be taken to reduce the core loss in the alternator built for 10,000 cycles, it was natural in the first attempts at building a machine for 100,000 cycles to try to eliminate the iron in the armature.

The first machine built was a model with a revolving armature of the Ferranti type, 3 in. in diameter. The experiment was encouraging, inasmuch as it demonstrated the possibilty of generating an appreciable amount of power at 100,000 cycles, but the machine could never be brought up to a corresponding speed on account of mechanical difficulties. The armature had 200 conductors consisting of 100 U-shaped phosphor-bronze wires anchored in insulated segments clamped in the rotating body. As long as the machine was run at no load, it acted comparatively well. The centrifugal force and air pressure on both sides of the wires seemed to keep them away from the stationary pole faces; but as soon as the field was applied the magnetic action on the current in the conductors was sufficient to distort the wires so that they would strike the poles and break.

From this experience it was apparent that a successful machine must have a stationary field winding as well as a stationary armature and that the armature should not contain any iron. The outcome of these conditions was a type of machine illustrated in Fig. 1. The armature is made of wood and the two rotating discs form a complete magnetic circuit which is energized by a stationary field coil suspended inside the armature between the discs. In connection with this machine a number of mechanical problems were to be solved, which will be referred to when describing the latest form of the alternator. The chief reason why this machine could not be considered as the final development was the fact that the rotating discs could not be made symmetrical. Owing to this fact the centrifugal force at high speed changed the shape of the discs in such a way that the air-gap between the field and the armature was increased; the result being that the voltage of the generator did not increase in proportion to the speed, but reached a maximum at about 60,000 cycles and decreased at higher speed.

Type of construction. Fig. 2 shows the construction arrived at in a machine recently completed. As there was a question

whether it was practicable to use an iron armature at a frequency of 100,000 cycles, the machine was built with two kinds of armatures for trial, one of iron and one of the winding placed in wood. The iron proved to be preferable, giving a higher output and a better mechanical structure. The success of the iron

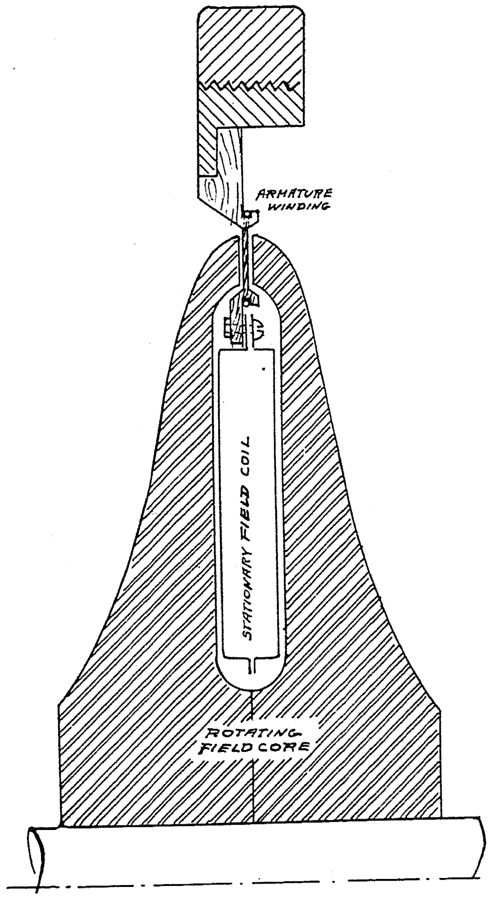


Fig. 1.

armature is largely due to the fact that the volume of iron subject to alternating flux is extremely small. The main part of the laminated armature carries a constant flux, while the pulsating flux occurs only in the teeth that separate the armature conductors.

The alternator shows in every way the same characteristics

as a generator of moderate frequency. The following are the constructional data of the machine: the armature has 600 slots with one conductor per slot of 0.016 copper wire, triple-silk covered, and varnished. The wire is wound in a continuous wave up and down in the successive slots. The rotating field has 300 projections, which, as the machine is of the inductor type, correspond to 600 poles for a machine of the ordinary alternate-pole type.

It appears from the electrical characteristics that the output

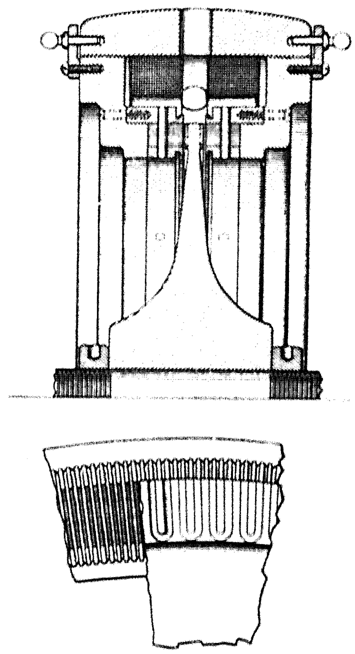


FIG. 2.

of the generator depends largely upon the air-gap clearance, because the voltage of the machine is nearly inversely proportional to the air-gap. The characteristic curves have been given at an air-gap of 0.015 in. but the output of the generator can be doubled if a smaller air-gap be used; in fact the machine has been operated with an air-gap as small as 0.004 in., though such a clearance would not be practicable for continuous operation. In view of this condition it is desirable to regulate the air-gap so as to offset any wear in the bearings, and to select the gap which is suitable for the work to be done. If the machine

is to be operated continuously without attention, a comparatively large clearance is necessary, while for a special experiment in which higher power is required a small gap may be used.

The sensitiveness of the air-gap regulation is one of the reasons why, instead of using the usual drum winding construction of the armature, the winding was applied on a radial face, as shown in Fig. 2. The armature is mounted on a frame that is threaded into the stationary field frame, and the air-gap can be set exactly by screwing the armature tight against the disc and then moving it back a distance corresponding to the desired clearance. For this purpose the stationary frame is provided with a scale, so that rotating the armature on its thread one division corresponds to one thousandth of an inch change of the air-gap.

Flexible shaft. At a speed of 20,000 rev. per min., it would not be practical to use a rigid shaft as in ordinary machines. It was therefore necessary to adopt the principle employed in the deLaval steam turbine, that of a flexible shaft which allows the disc to revolve round its exact mass center, thus avoiding any strain on the bearings due to centrifugal force. In a turbine of this type designed for a speed of 20,000 rev. per min., the shaft is very thin, about the size of a lead pencil, but not very long. A shaft of such proportions would not be suitable for the alternator in which the exactness of the air-gap is essential and the disc is subject to the magnetic attraction between field and armature. The shaft of the alternator must have not only considerable stiffness, in order to keep the disc in an accurate position, but also sufficient flexibility to allow the rotating disc to adjust itself to its own mass center. To meet these requirements the shaft is designed like a stiff spring with uniform strength so as to obtain maximum work out of the material without exceeding the elastic limit in any part. It has a diameter of 1.25 in. in the middle and 0.625 in. at the ends, the length between the centers of bearings being 28 in.

When the machine is brought up to speed, severe vibration occurs at two distinct critical speeds, one at 1700 rev. per min. and the other at 9000 rev. per min. The vibration increases gradually when the machine approaches the critical speeds. When the vibration has reached its maximum, it ceases suddenly and the machine runs smoothly and with surprisingly little noise, indicating that the disc has fallen into step with itself,

so to speak, and is rotating around its own mass-center. The critical speed, which can be predetermined approximately from the design of the shaft and the disc, occurs when the natural vibration of the shaft with the disc coincides with the revolutional speed.

The first point of resonance is the one at which the shaft vibrates like a string, as shown in Fig. 3. The spring force of

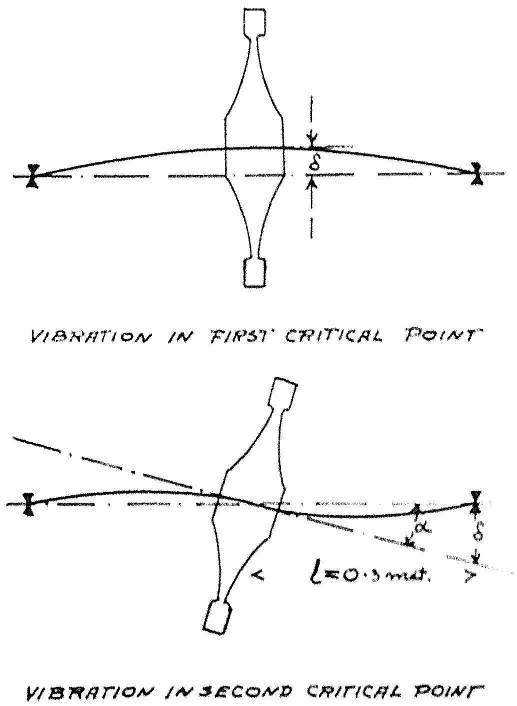


FIG. 3.

the shaft tending to hold the disc in its original position is proportional to the deflection from the same position.

Let the deflection $= \delta$
The spring force $= P$
Hence $P = k\,\delta$

in which k is a constant corresponding to the stiffness of the shaft. The constant k can be determined by calculating the deflection of the shaft from its dimensions. The result of such a calculation gives $\delta = 0.0024\ P$ cm. or 0.000024 meters in which

P is expressed in kilograms. The weight of the disc is 13.6 kg. and the mass $= m = \frac{13.6}{9.8}$.

The equation for resonance in the first critical point is derived by assuming that the shaft swings as a string with a weight in the middle. In order to define the condition for resonance, it is assumed that the disc is brought out of center by an arbitrary amount δ. The condition for mechanical resonance occurs if the centrifugal force of the mass of the disc rotating at a radius δ is equal to the centripetal force created by the deflection of the shaft to the amount δ from its normal shape.

Centrifugal force $= m\,\delta\,\omega^2$

Where ω is the angular velocity.

The spring force of the shaft has been designated P

$$m\,\delta\,\omega^2 = P = \frac{\delta}{0.000024}$$

$$\text{and introducing } m = \frac{13.6}{9.8}$$

$$\frac{13.6}{9.8}\,\delta\,\omega^2 = \frac{\delta}{0.000024}$$

$$\omega^2 = \frac{9.8}{13.6 \times 0.000024}$$

$$\omega = 173$$

Rev. per sec. $\frac{\omega}{2\pi} = 27.5.$

Rev. per min. 1650.

The second harmonic is the one at which the disc vibrates around an axis vertical to the shaft and the shaft assumes an S shape. The momentum exerted by the spring force on the disc is proportional to the angular deviation of the disc.

The relation is $P\,l = k\,\alpha$.

The constant k can be derived from the dimensions of the shaft. In this case

$$k = \frac{l^2}{0.000024} = \frac{0.3^2}{0.000024} = 3750$$

The inertia of the disc for rotation around an axis vertical to the shaft may be designated I. In this case

$I = 0.0052$ expressed in meters and kilograms.

The equation for mechanical resonance is

$$I \alpha \omega^2 = P l$$

in which ω is the angular velocity.

Substitute $P l = k \alpha = 3750 \alpha$

$$I \alpha \omega^2 = 3750 \alpha$$

$$\omega^2 = \frac{3750}{0.0052}$$

$$\omega = 850$$

$$\text{Rev. per sec.} = \frac{850}{2 \pi} = 136$$

Rev. per min. = 8200.

A number of mechanical features which have been the pri
problems in the development of this machine are descril
the following.

Method of drive. The first machine built was belt-c
After a good deal of experimenting with different kinds o
and pulleys, it was found feasible to run the machine at
rev. per min., corresponding to 75,000 cycles. However,
became apparent that this method of driving could not
cepted for practical use, so that the later machines have
equipped with gears. Although these gears are design
reduction of speed, they appear to work satisfactorily wh
action is reversed. The gear ratio is 10 to 1.

Revolving disc. The disc is designed so as to have u
stresses in all parts and consequently the greatest p
margin of safety. The disc is made of chrome-nickel
with an elastic limit of about 200,000 lb. per sq. in.
frequency of 100,000 cycles the speed is 20,000 revs. pe
The diameter of the disc being one foot, the peripheral
is 1,000 feet per second or 700 miles per hr.; in other
the disc would roll over to Europe in 4 hr. The cent
force of the material in the edge of the disc is 68,000 tir
own weight, but owing to the shape of the disc, which is de
for maximum strength, the stresses are only 30,000 l
sq. in., giving a factor of safety of 6.7 to 1.

Heating and air friction. If it be born in mind that th
has 300 teeth on each side spaced at a distance of 0.1
and that these teeth pass the face of the stationary arn

at a rate of 1000 ft. per sec. with a clearance of 0.015 in., it can be readily understood that the air friction in the gap must be considerable. It was found that the machine heated up excessively owing to air friction, but this has been overcome by filling the slots with non-magnetic metal so that the disc offers a perfectly smooth surface. Then another difficulty was encountered owing to centrifugal force acting on the fillers, each of which is subject to a centrifugal pull of 80 lb. During some of the first attempts, when the slots were simply filled with hard solder, it happened that some of the fillers broke loose. It was interesting to notice that an unbalancing centrifugal force of 80 lb., which would have been enough to bend the shaft at standstill, did not affect the machine, which, in every case, continued to run as if nothing had happened. In the final construction, in which the fillers are anchored so as to stand the strain, they consist of U-shaped phosphor-bronze

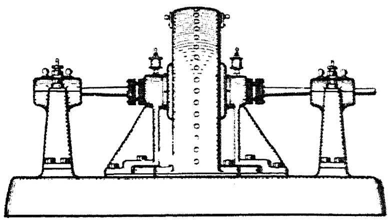


FIG. 4.

wires supported directly by the steel teeth so that the solder is needed only to fill the joints.

Bearings. The generator has two sets of bearings as shown in Fig. 4. The outside bearings support the weight of the revolving parts and are fed with a continuous stream of oil supplied by a pump. With this system of lubrication, the machine can run continuously at full speed with a temperature rise of the bearings of only 5 degrees above the temperature of the oil in the tanks. The necessity of the continuous oil feed is demonstrated by the fact that the temperature of the oil in the tank reaches 20 degrees above the surrounding air.

One of the functions of the middle bearings is to prevent excessive vibrations when the shaft passes the speeds of mechanical resonance. During normal operation, the shaft does not touch the middle bearings, which are bored out to give 1/64 in. clearance; at the critical speed, however, the shaft touches the bearings.

The end-thrust of the shaft is taken up partly on the middle bearings and partly on the end bearings. If the air gaps on both sides of the disc were not equal, the magnetic end thrust might be considerable, but with this arrangement of divided end thrust the adjustment seems to be to a certain extent

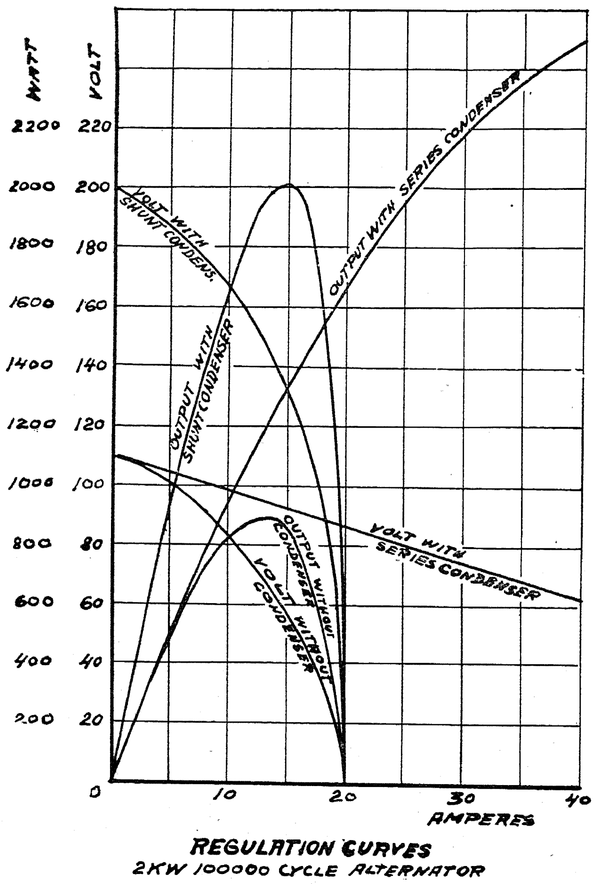


FIG. 5.

automatic. If the disc has a tendency to move over to one side due to a difference in size of the air-gaps, the corresponding middle bearing will heat up and the same side of the shaft will expand longitudinally, thereby relieving the middle bearing as well as correcting the air-gap.

Electrical characteristics. A generator of this kind is always

used in connection with a capacity, which is located either in the aerial, as in the case of wireless telegraphy, or in a condenser of suitable capacity used in order to obtain the proper tuning. The best method of showing the characteristics of the generator, without reference to any certain application, is to assume the load to be non-inductive and use a condenser of a fixed capacity as an attribute to the generator. The most

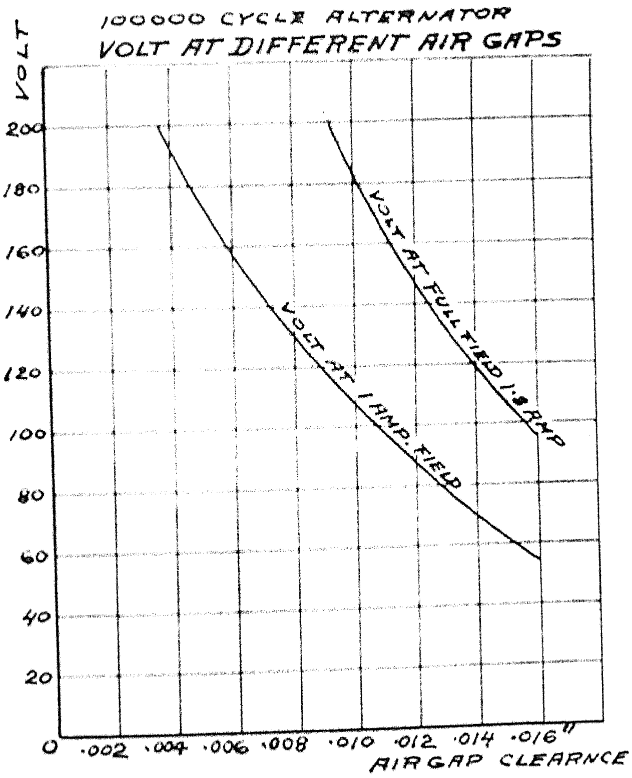


FIG. 6.

typical performance of the generator operating in conjunction with a condenser is the one in which the condenser has an impedance equal to the inductive impedance in the generator winding and is connected in series with the generator. This combination gives the generator a maximum output for any load. A condenser arranged in this way considerably increases the useful performance of the generator and should be regarded as a part of the generating outfit. The tests of the alternator

described here show that the reactance of the armature winding is 5.4 ohms at 100,000 cycles. The condenser which ought to be used in connection with the machine at the same frequency should have the same impedance expressed in ohms. If the capacity is expressed as usual in microfarads, it should be

$$C = \frac{1}{5.4 \times 2\pi \times 100{,}000} \text{ farads}$$

$$= \frac{10}{5.4 \times 2\pi} = 0.3 \text{ microfarads}$$

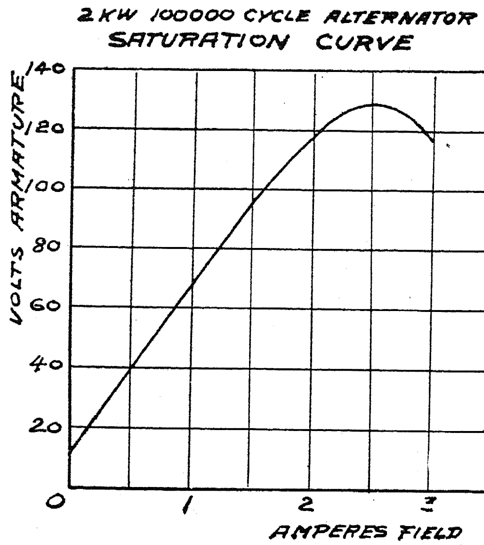


Fig. 7.

Regulation curves of the alternator are shown in Fig. 5. From this diagram it appears that the combination of alternator and series condenser gives the same terminal voltage at any load as would a generator which had the same electromotive force and the same ohmic resistance but no inductive drop. In other words, the condenser completely offsets the reactance of the winding, and the regulation is determined entirely by the ohmic resistance.

Tests, as well as practical use of these machines for wireless telegraph work, have shown that the commercial output of

the alternator is limited by heating of the armature winding. In building these alternators for practical purposes, the need for a commercial rating of the machines has already been realized. The basis for rating which has been adopted for these machines

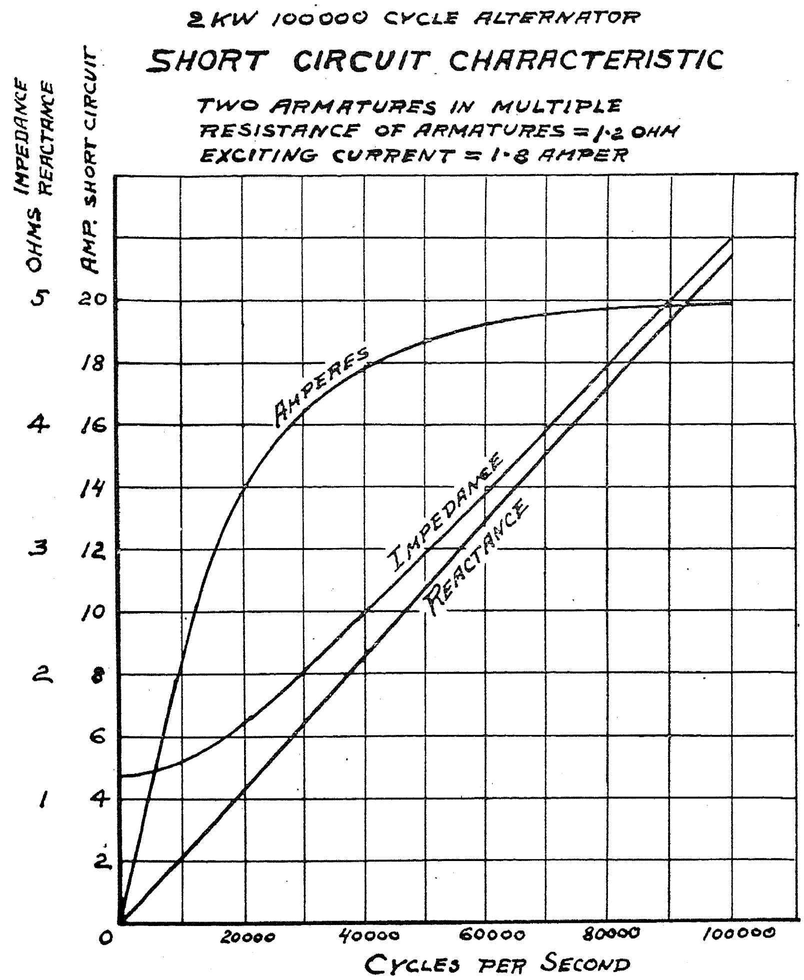


Fig. 8.

is illustrated by the rating of the 2-kw., 100,000-cycle alternator.

The characteristics of the machine as given by test are the following:

No load voltage, 110.

Ohmic resistance, 1.2.

Short-circuit current, 20 amperes.

Continuous capacity at 45 degrees rise, 30 amperes.

From the above data, it is apparent that the alternator cannot be operated up to its full thermal capacity without a condenser, and in selecting a condenser the capacity is regulated so as to compensate for the inductance of the winding as described. Under these conditions, the maximum continuous load of the alternator is 70 volts and 30 amperes or 2.1 kw. The commercial rating has therefore been made 2 kw.

It may be of interest to note that the product of the short-circuit current and the no-load voltage is 2.2 kilovolt-amperes, equal to the full load-capacity as determined by heating. However, the short-circuit current would not be a suitable basis for commercial rating; because the thermal rating depends entirely upon the ohmic resistance of the winding, while the short-circuit current is nearly independent of the resistance.

It would undoubtedly be of theoretical interest to be able to determine the properties of the iron core at the high frequencies dealt with in this generator. A test made at 60,000 cycles may be referred to for this purpose. The power required to run the whole set without excitation at the corresponding speed was 1340 watts and the additional power required to drive it at the same speed with full field was 330 watts. The only conclusion that can be drawn from this test is that the core loss is not higher than 330 watts. The main part of this loss is probably due to increased bearing friction by magnetic end thrust. By a more delicate laboratory test, it might be possible to separate the additional friction from the core loss, but the above is the only data that are available at present.

Regarding the possibilities of building high-frequency alternators of higher output, it may be mentioned that a machine for 50,000 cycles is being constructed of such dimensions that an output of 35 kw. can be expected. At the present state of the art, however, it is safer to build the machine first and determine its rating after it has been tested.

The question of whether machines can be designed for frequencies still higher than 100,000 cycles cannot be decided offhand. Considering, however, that the present generator has a large margin in the mechanical strength of the revolving disc as well as in the length of the air-gap, etc., it appears probable that still higher frequencies can be reached by direct generation with revolving machinery.

Discussion on "Alternator for One Hundred Thousand Cycles." Frontenac, N. Y., June 28, 1909

John B. Taylor: I wish to ask why, in Fig. 7, the voltage falls off with increasing field excitation?

J. C. Lincoln. I wish to ask why, in the design of a small machine of this sort, such a small diameter as one foot is selected for the diameter of the rotating part. The centrifugal strains would be less with a given peripheral speed if the revolving part were made 2 ft. or 18 in. in diameter. Also, larger pole space on the periphery would be obtained.

D. B. Rushmore. At one time the induction alternator was in general use in this country, especially when 133 cycles were in vogue. At 60 cycles the induction alternator easily held its own as a small high-speed machine, but became more expensive for other frequencies, and at 25 cycles could not compare with other machines. However, where high frequencies are desired it is necessary to use this form of machine.

The curve in Fig. 5 shows the increased output of the machine with the shunt condenser under somewhat the same conditions as exist in a transmission system. The critical speed is a matter of great interest and importance in many of the lines of mechanical work where, because of the type of prime mover, or to obtain higher efficiency at lower cost, the tendency is to use velocities limited only by the strength of material.

C. J. Fechheimer: I wish to ask Mr. Alexanderson if the critical speed is affected by the means of support with the flexible shaft used in this alternator; that is, will self-aligning or fixed bearings materially affect the critical speed?

I also wish to ask if careful tests were made of the tensile strength of the chrome-nickel steel. Mr. Alexanderson states that the elastic limit of the steel used is about 200,000 lb. per sq. in., whereas the chrome-nickel steel furnished by the steel mills has an elastic limit of about 110,000 lb. per sq. in.

A. E. Kennelly: It is interesting that a condenser should be considered as normally attached to the machine in order to obtain normal output; although this would be a dangerous precedent if it should be considered necessary in machines of larger proportions or lower frequencies, since rotary condensers might become necessary as part of the equipment of the machine.

I wish to ask Mr. Alexanderson whether he has made measurements of any kind, to show whether a physiological effect, or shock, can be felt from the machine at 100,000 cycles per second? We know that at the frequency of 2000 cycles per second a shock can be felt and that there is a certain high frequency at which a shock is not felt.

E. F. W. Alexanderson: Answering the question asked by Mr. Taylor: why the voltage of the generators decreased with increased field strength. The decrease of voltage is due to the nature of the inductor-alternator. The profile of the disc

looked at from the end is indicated in Fig. 1, showing one wire in each slot of the armature. The variation of field strength in the separate armature coils is produced by the varying air-gap when a tooth is opposite or when a gap is opposite to a coil. If the excitation is driven too high, the teeth saturate and the difference between maximum and minimum induction becomes less. That is the reason why a certain field excitation gives the maximum voltage.

It is, without doubt, true that a larger diameter would materially increase the capacity. A 1-ft. diameter was selected because in building the experimental machine the item of cost had to be considered. The machine now building, which is expected to have an output of 35 kw., is designed with a diameter of 3 ft.

The resonance of the shaft, so far as I have been able to observe, has nothing to do with the alignment of the bearings. The bearings are self-aligning. Whether it would be different with a non-lined bearing, I could not tell.

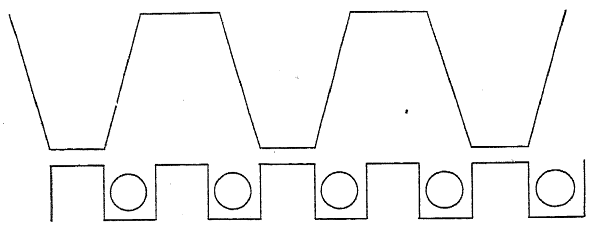


FIG. 1.

In making the disc in the lathe and in the milling machine, the opinion of those who worked on it is that it is an extremely tough material. It has, therefore, been, attempted and will probably work out successfully, to have the material cut before it has been tempered, because the high tensile strength of 200,000 lb. per sq. in. is not inherent in the material until it has been tempered by a special process. It is hoped it can be treated without changing the shape. The same process has been used generally in automobile gears.

Dr. Kennelly asked if it were possible to feel a 100,000-cycle shock. I can assure him that it is. I could not feel any difference between alternating current at 100,000 cycles and direct current. 100 to 150 volt, at the high frequency gives the same sensation as a direct current of the same voltage.

In developing the 100,000-cycle generator one practical point was discovered which may be of interest in regard to the discussion of Dr. Kennelly's paper, as to the radiation from wires. The wire placed in the slot has a diameter of 0.016 in. It has a triple silk covering and coating of varnish. One of these wires, when placed in the slot, carries 15 amperes at a rise of 25

degrees centigrade; that is, twice as much as it will carry in the open air. Naturally the iron, which is closely in touch with the coil, takes off more heat than the air. The measurements are made by resistance, and the rise of temperature of the wire takes only a few seconds. The current density in it is about 75,000 amperes per square inch, against 2000 to 3000 in the ordinary machine.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

METHOD OF TESTING TRANSFORMER CORE LOSSES, GIVING SINE-WAVE RESULTS ON COMMERCIAL CIRCUITS

BY L. W. CHUBB

It is well known that if the wave-shape or the frequency varies from the normal, the loss in a transformer core at normal voltage varies also. It is obvious, however, that the normal loss, that is, the loss with the sine-wave shape and the normal frequency of, say, 60 cycles, can be obtained under the abnormal conditions of wave-shape and frequency by either raising or lowering the voltage. What is needed, then, is an indicator which will enable the voltage to be so adjusted that the loss in a transfomer core as measured upon an ordinary wattmeter will be the same as it would be at normal voltage on a circuit having a sine-wave shape of normal frequency.

Suppose such an instrument or indicator were so made that when connected to an alternating-current circuit, it would indicate the effective value of a sine-wave voltage of a certain normal frequency, such as 60 cycles, which would cause the same watt loss in a transformer core as the circuit to which it is connected. For core-loss tests it would be necessary simply to adjust the voltage of the circuit so that the special instrument would indicate the normal voltage, say, 110 volts, although an ordinary voltmeter might read several volts higher or, lower, and take a wattmeter reading of the input to the transformer under test. This reading then would be the same as would be found if the measurement were made at 110 volts on a 60-cycle circuit having a sine wave of voltage. Such an instrument, termed an iron-loss voltmeter, is described in this paper, which takes up in some detail the practical importance of the subject

of transformer core-loss tests, the nature and magnitude of errors, and the methods proposed for correcting them.

To the central-station operator or user of distributing or power transformers, the iron loss of his transformer is of the greatest importance. But of all tests on electrical apparatus one of the most difficult to check and agree upon is the measurement of the core loss of a shunt transformer. Reports of tests for the same transformer taken at different times or in different places will seldom agree within three or four per cent, and often differ as much as ten per cent. The difference is sometimes due to an actual change in the transformer, such as aging of the iron core or a change in the mechanical make-up, or physical stresses of the core due to extreme temperature changes or vibrations during transportation.

Changes in testing conditions and errors of testing methods, however, are more often responsible for the differences and errors in core-loss results. Frequency, temperature and wave-shape variations; wattmeters which are inaccurate on low power-factor tests; voltage control by inductance and resistance; connecting voltmeter and wattmeter pressure leads on the line side of the wattmeter; and paralleling windings which are of wrong ratio, are all frequent sources of error. Of all these the wave-shape difficulties are the most serious, are almost always present, and have been the most difficult for which to make corrections in the usual commercial tests.

The voltage wave-shapes of alternating-current circuits differ greatly. The difference or variation from the sine may be inherent in the alternator, or distortions may be caused by abnormal conditions of load and voltage drops in the circuits. Such voltage distortions are generally caused by having high inductance and resistance in the circuit such as the alternator armature, the coils of measuring instruments, and the rheostats and choke coils which are often used to regulate the voltage. A peaked wave of voltage will cause a lower core loss, and a flat wave a higher loss than a sine wave of the same effective voltage.

On account of this variation with wave-shape, it is desirable that the core-loss tests and data for transformers be expressed in terms of some certain wave-shape, as well as frequency and voltage. Guarantees of core losses are generally based on a sine wave, but we sometimes find them stated in terms of an "average commercial wave." The wave-shapes of commercial circuits may vary between wide limits, and in the majority of

cases will be found to be peaked, especially at the terminals of a transformer under test.

The Standardization Rules for a sine wave allow a deviation from the equivalent sine of ten per cent of the maximum ordinate of the equivalent sine wave. If instead of a true sine wave, this nominal sine is to be used as a basis for core-loss guarantees, the conditions of test and results obtained will still be indefinite. Figs. 1 and 2 show two waves which meet sine-wave specifications, but a transformer tested for core loss on them will differ

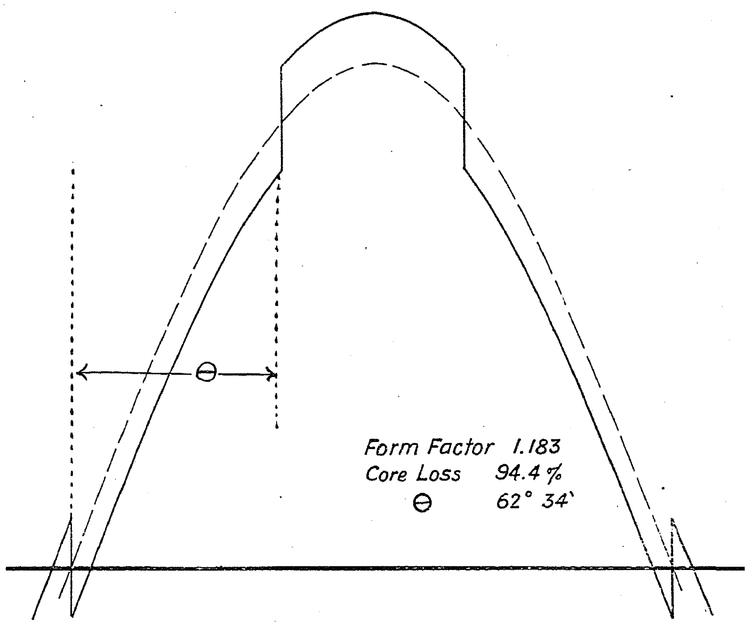


FIG. 1.

from 7.8 per cent below to 5.8 per cent above the loss which would be obtained on the equivalent sine. These are, of course, theoretical waves, but they show the limit of core-loss error for an average transformer, caused by waves which meet sine-wave specifications.

On account of the accuracy required for core-loss tests, the difficulty of obtaining a good sine wave for test, and the difficulty of correcting results to a sine value by the methods which are now in vogue, it seems very desirable that a method of test be available which will remove the wave-shape difficulty.

It is the purpose of this paper to present a method of testing which will give sine-wave values of transformer core losses on very distorted testing waves, without the aid of an oscillograph, contactor, synchronous commutator, or even exact knowledge of the wave-form or frequency of the circuit.

The iron loss of a transformer consists of the eddy-current loss and the hysteresis loss which are present in about the same relative proportions in transformers designed for the same frequency. The eddy-current loss in a given transformer is dependent upon the root mean square of the instantaneous

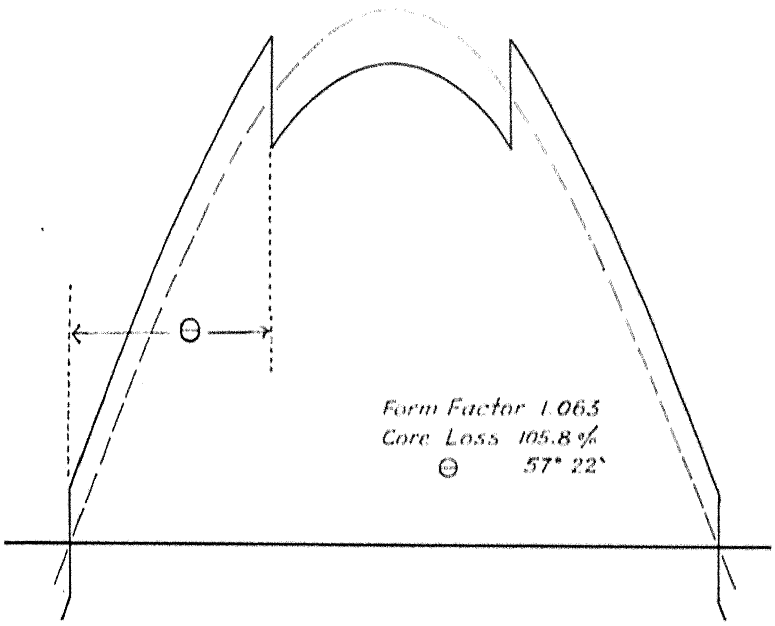


FIG. 2.

values of the induced voltage, while the hysteresis loss is dependent upon the maximum magnetic induction in the iron, which in turn is dependent upon the average of the instantaneous values of the induced or counter voltage. This dependence of eddy loss upon root-mean-square volts and of hysteresis upon average volts, is practically independent of wave-shape variation, except in cases when the voltage curve passes through zero more than twice per cycle; or the flux distribution is varied; or the impedance to high-frequency components of the eddy loss varies greatly.

It is obvious that if the usual alternating-current voltmeter is used to adjust voltage in core-loss tests and the wave-shape of voltage is not a sine, then the eddy loss in the transformer will be correct; that is, the sine value and the hysteresis loss will be low or high, depending upon whether the form-factor of voltage is respectively above or below 1.11. If the average voltage reading, taken from a voltmeter indicating the average, not the root-mean-square, voltage, or a synchronous rectifying commutator is used for the adjustment of voltage, the hysteresis loss will be correct and the eddy loss will be high or low to agree with the high or low form-factor of voltage.

The total loss as measured in either of the foregoing methods is therefore not a sine-wave result unless the wave of impressed voltage has a form-factor of 1.11. To obtain a sine-wave result of total loss on a distorted wave, either of the following is necessary:

Determine the form-factor and make the proper corrections.

Adjust the testing voltage by an instrument whose readings are such a function of both the average and effective voltage that when it reads the normal voltage of the transformer, the error in the eddy loss will be equal and opposite to the error in the hysteresis loss, thus giving a correct total or core loss.

Suppose a 110-volt, 60-cycle transformer is to be tested and that the result of the core loss is required in terms of a sine wave of impressed voltage. The testing circuit available has a wave-form of unknown shape, and there are no convenient means of finding the form-factor of the wave. Suppose another very small transformer to be on hand with the same ratio of eddy loss to hysteresis loss at 110 volts, 60 cycles, as the first transformer, the sine-wave core loss of the second transformer being 20 watts at 110 volts, 60 cycles. This small transformer and a wattmeter may be used in determining the proper voltage at which to test the unknown core loss in the larger transformer, on the given circuit. The voltage of the circuit should be adjusted until the energy input to the small transformer core is 20 watts, as indicated by the wattmeter. The core loss in the larger transformer is then read on a separate wattmeter, and the value thus obtained will be the same as would be measured at 110 volts on a 60-cycle sine wave of voltage. The above contains the essential elements of a method which will give sine-wave results of shunt-transformer core losses on any commercial alternating-current circuit.

The scheme is practically carried out by the use of a portable direct-reading instrument, known as an iron-loss voltmeter. This instrument indicates the effective or root-mean-square value of a sine wave of voltage which would cause the same iron loss in a transformer as the wave of voltage to which the instrument is connected will cause in the same transformer.

The iron-loss voltmeter consists essentially of a ring or core of laminated steel excited by a winding, and a wattmeter of suitable design on which to read the power input to the ring and all copper circuits in the instrument. The core and meter are

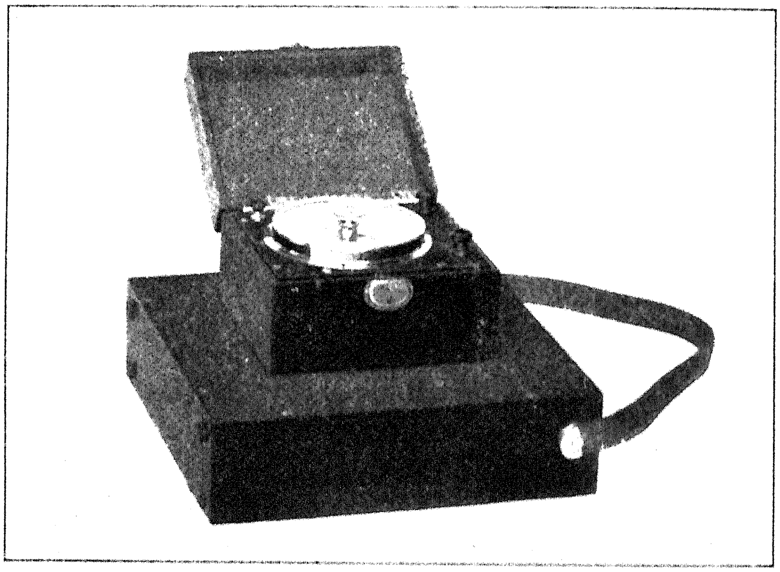


FIG. 3.

mounted in suitable wooden cases, the instrument appearing as in Fig. 3. All connections are made inside of the instrument so that there are but two terminal posts to which the line is connected, as is the case in standard alternating current voltmeters. Fig. 4 shows the arrangement of the circuits in the meter.

On the laminated core C is placed a winding W, the core member and winding being securely mounted in the lower case, and the two leads of the winding brought through the top and into the instrument case, which contains the meter movement.

A stationary current coil S is connected in series with the winding W and the terminals P P. The shunt circuit consists of the moving coil M, the non-inductive resistance R, and the compensating or adding coil C C, which is wound parallel to the series coil and of an equal number of turns. It can be readily seen that the deflection of the wattmeter movement will be caused by the total input to the instrument. This input is the hysteresis and eddy loss in the ring or magnetic rheostat, and the $I^2 R$ losses in all copper circuits of the instrument.

Of these losses the hysteresis in the magnetic rheostat is dependent upon the average value of the terminal voltage, and the eddy-current loss in the rheostat and the copper losses in the shunt circuit are dependent upon the effective value of the terminal voltage. The loss in the series circuit is a complex function of both the average and effective voltage, being pro-

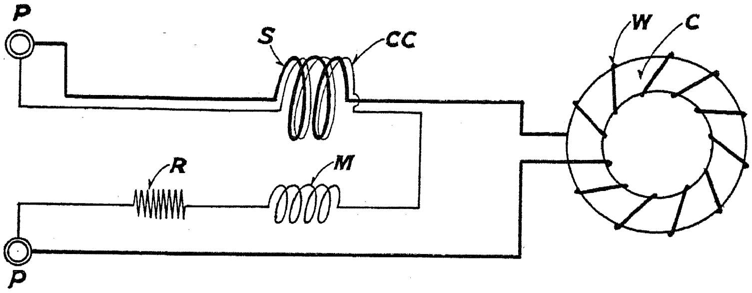


FIG. 4.—Internal connections of iron-loss voltmeter.

portional to the square of the exciting current of the ring. This series copper loss is so small that the exact law of its dependence on wave-shape is of no importance. The relative proportion of these component losses in the instrument is of importance.

Before calibrating the instrument for a certain frequency, the adjustment of the ratio of eddy loss in the ring plus shunt copper loss, to the total loss, is made by changing the non-inductive resistance in the shunt circuit and the turns on the ring. This ratio of $I^2 R$ losses to total loss is made to be 20 per cent at about six-tenths of full scale voltage. After this adjustment is made, the instrument is calibrated in parallel with an alternating-current voltmeter on the pure sine-voltage wave of the required frequency from a small smooth-core alternator. The scale is drawn in to agree with the alternating-current instrument.

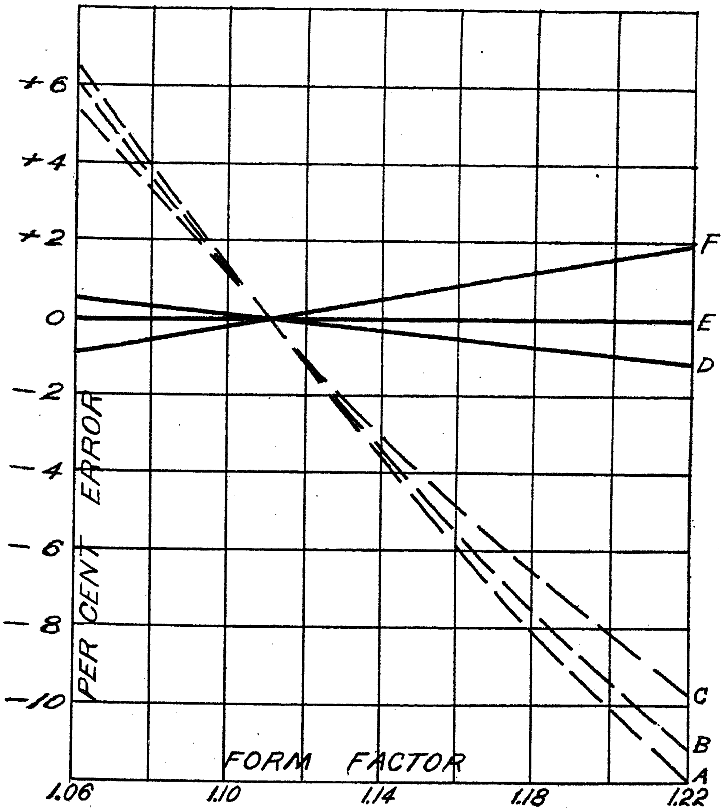
The iron-loss voltmeter, constructed and calibrated as described, may be used as a portable voltmeter in testing transformer core losses on any reasonably distorted wave of voltage. The test may be conducted without any knowledge of the wave-form. The voltage may be regulated through a considerable range by the aid of resistance or inductance, thus dispensing with the usual multi-voltage transformer or means of varying the field current of the alternator. The frequency need only approximate the normal value; and the final result of core loss as measured on a wattmeter will be the value which would be obtained on a sine wave of voltage at the normal frequency and voltage of the transformer.

By the Steinmetz formula the hysteresis loss in iron at constant frequency varies practically as the 1.6 power of the induction. This approximate exponent of 1.6 differs somewhat for different samples of sheet steel and it varies with changes of magnetic induction in the same sample of steel. If the hysteresis exponents of the iron in the voltmeter and the iron of the transformer are not equal, an error is introduced theoretically, but in no case of a commercial transformer or a commercial test is this difference of exponent great enough to introduce appreciable errors.

Another condition of a theoretically correct result is that the per cent of $I^2 R$ loss in the voltmeter and transformer shall agree at the testing voltage. Rather wide variations in this relation will, however, introduce errors which are very small in comparison to those obtained by using a root-mean-square voltmeter. No error is present on a sine wave, no matter what the relation of eddy and hysteresis may be in the transformer and instrument. When the wave-form is distorted the usual method of testing will introduce large errors and the iron-loss voltmeter method will be correct or introduce comparatively small errors. The curves of Fig. 5 show the per cent error that would be obtained when testing transformers of varying per cent eddy loss by the use of the root-mean-square voltmeter and an iron-loss voltmeter having 20 per cent eddy loss at the point of test.

Curves A, B and C show the errors which would be obtained by using the root-mean-square voltmeter in testing three transformers with 14 per cent, 20 per cent and 30 per cent eddy loss respectively on wave-shapes which have form-factors ranging from 1.06 to 1.22. Curves D, E and F show the corresponding errors which would be obtained on the three transformers when

ng the iron-loss voltmeter. Notice that the transformer th 20 per cent eddy loss can be tested without error on any ve-form by the new method, because the instrument assumed



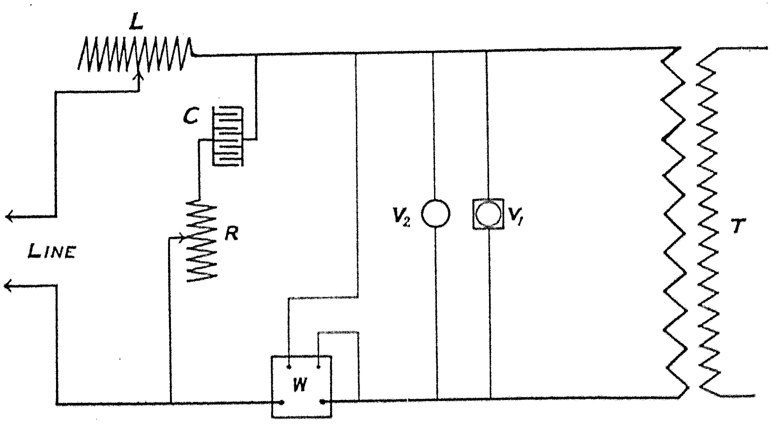
Curve	Transformer with	Tested with
A	14% eddy loss	R.m.s. voltmete.
B	20% " "	R.m.s. "
C	30% " "	R.m.s.
D	14% " "	Iron "
E	20% " "	Iron "
F	30% " "	Iron

. 5.—Curves showing effect of ratio of eddy and hysteresis loss on core loss tests by the two methods.

20 per cent copper loss. Sixty-cycle transformers of com-rcial design will be found to have about 14 per cent to 30 cent eddy-current loss and to average about 20 per cent eddy

loss at normal voltage. Curve *B* then shows the usual errors by the old method. The errors by the new method will seldom be more than one per cent and, even with the worst proportions of eddy and hysteresis loss and worst wave-shapes, will not exceed two per cent.

The foregoing method gives all that should ever be required for commercial accuracy, but as a point of interest a method of form-factor correction is presented which will eliminate even the small errors caused by unequal ratios of component losses already discussed. This method will give a theoretically correct result of core loss for laboratory tests or when great



T—Transformer under test, *W*—Wattmeter, V_1—Iron-loss voltmeter, *V*—Root-mean-square voltmeter, *R*—Variable resistance, *L*—Variable inductance, *C*—Aluminum electrolytic cell.

Fig. 6.—Connections for correcting voltage form-factor.

accuracy is required. The form-factor of the voltage wave is first corrected to 1.11 and the loss is then read.

As shown in Fig. 6, an inductance *L* is placed in series with the line and an aluminum electrolytic cell *C* is connected across the line and between the inductance *L* and the wattmeter *W*. The inductance causes a peak in the wave, and the aluminum cell, which is working above its critical voltage, short-circuits or flattens out the peaks until the form-factor is 1.11. This correction or adjustment of form-factor is made by varying the resistance *R* and the inductance *L*. When the readings of the root-mean-square voltmeter and the iron-loss voltmeter agree, the form-factor is 1.11 and the core-loss reading is taken. This

method does not necessarily give a sine wave for the test but it gives a wave having the same form-factor, and has been found to be very accurate.

Some specimen tests are shown herewith for the sake of clearness and to give an idea of the accuracy and commercial value of the method. Table I shows comparative tests on two 3000-kw., 60-cycle transformers by the usual method and by the iron-loss voltmeter method. Voltage was taken from a 115-volt tap in the low-tension winding. It will be noted that such large units distorted the wave-form enough to make serious errors. Each transformer was tested alone by each method and the sum of the separate tests compared with a test made with both in parallel. Assuming the separate tests by the new method and their sum to be correct, it will be noted that the test with both in parallel by the new method checks within

TABLE I.

Test No.	Transformers tested	Core loss in kilowatts: Iron-loss voltmeter method	Core loss in kilowatts: Root-mean-square voltmeter method
1	*A*, alone	17.7	16.6
2	*B*, alone	17.8	17.0
	Sum of separate tests	35.5	33.6
3	Both tested in parallel	35.1	31.5

1.1 per cent, which is within the error of observation. Also note that by the root-mean-square voltmeter method, *A* alone gives a result 6.2 per cent low; *B* alone, 4.5 per cent low, and the parallel test does not agree with the sum of the separate tests and is over 11 per cent below the correct sine value.

Table 2 shows a similar comparative test on three smaller transformers, made with an alternator having a good sine wave at no load. The results of the last column show the errors of the root-mean-square voltmeter method under conditions, which were supposed to be very good. They also show how consistent the results of the new method are. Tests numbered 5 and 10 were made with some inductance in series with the line. It will be seen by the voltmeter readings of test 5 that the wave was quite distorted, and that the result by the new method was practically unaffected, while the result by the root-mean-square

TABLE II

IRON-LOSS VOLTMETER METHOD

No.	Transformer	Volts: Root mean square	Volts: Iron-loss meter	Total watts	Instrument losses: $VM + WM$	Instrument losses: Iron loss Vm	Instrument losses: Total	Core loss (watts)
1	*A*...........	111.2	110.0	193.6	15.9	27.7	43.6	150.0
2	*B*...........	111.2	110.0	189.9	15.9	27.7	43.6	146.3
3	*C*...........	111.2	110.0	189.9	15.9	27.7	43.6	146.3
	Sum							442.6
4	*A*, *B*, & *C* tested in parallel	112.6	110.0	475.5	9.9	27.7	37.6	437.9
5*	*A*, *B* & *C* tested in parallel.....	122.3	110.0	478.0	11.7	27.7	39.4	438.6

ROOT-MEAN-SQUARE VOLTMETER METHOD

No.	Transformer	Volts: Root mean square	Volts: Iron-loss meter	Total watts	Instrument losses: $VM + WM$	Instrument losses: Iron loss Vm	Instrument losses: Total	Core loss (watts)
6	*A*...........	110.0		162.3	15.5		15.5	146.8
7	*B*...........	110.0		158.3	15.5		15.5	142.8
8	*C*...........	110.0		158.3	15.5		15.5	142.8
	Sum							432.4
9	*A*, *B* & *C* tested in parallel....	110.0		430.5	9.5		9.5	421.0
10*	*A*, *B* & *C* tested in parallel....	110.0		387.5	9.5		9.5	378.0

* Voltage wave purposely peaked with inductance.

TABLE III

IRON-LOSS VOLTMETER METHOD

No.	Volts: Root-mean-square meter	Volts: Iron-loss meter	Core loss	Remarks
1	118.5	110.0	185.2	Alternator #1 light load..........Fig. 7
2	115.0	110.0	184.6	Alternator #1 loaded..................
3	115.5	110.0	185.6	Alternator #2 light load..........Fig. 8
4	123.4	110.0	186.2	Alternator #2 loaded............Fig. 9

ROOT-MEAN-SQUARE VOLTMETER METHOD

No.	Volts: Root-mean-square meter	Volts: Iron-loss meter	Core loss	Remarks
5	110.0		165.1	Alternator #1 light load..........Fig. 7
6	110.0		171.1	Alternator #1 loaded..................
7	110.0		170.4	Alternator #2 light load..........Fig. 8
8	110.0		157.7	Alternator #2 loaded............Fig. 9

method was lowered an additional 10 per cent. The additional columns show the intermediate steps used to work out the core loss from the observed readings of volts and total watts input. The wattmeter had no compensating coil to correct for its shunt power and, since the voltmeter and pressure leads of the wattmeter were connected to the terminals of the transformer, corrections had to be made for the instrument loss of the voltmeter and the shunt circuit of the wattmeter. The loss in the iron-loss voltmeter is known from a calibration curve to be 27.7 watts when the instrument reads 110 volts, but to correct for the shunt power in the wattmeter we must use the formula $\frac{E^2}{R}$, where E is the root-mean-square voltage. This makes it clear why the root-mean-square voltmeter reading is also taken.

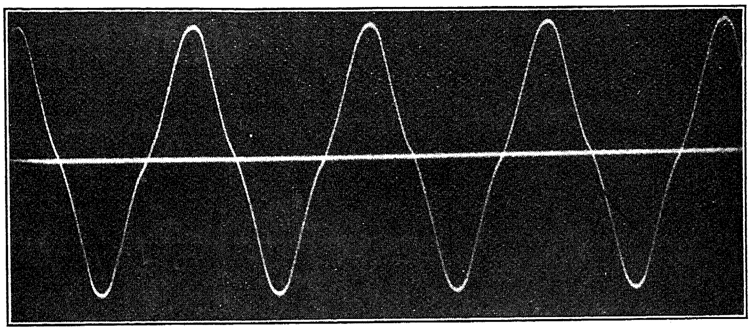


FIG. 7.

In this table also it may be seen that the sum of the separate tests by the root-mean-square voltmeter method does not check with the sum as tested in parallel, and that the errors of the tests by this method are from 2.1 per cent to 13.8 per cent low.

Table 3 shows some tests made on two different alternators at light load and during heavy low-power-factor load. Both the alternators in these tests have a peaked wave on no load. Alternator No. 1 is a large, and No. 2 is a small, machine.

Oscillograms of Figs. 7, 8, and 9 show the voltage wave at the terminals of the transformer under the different conditions of test in Table 3. This table shows the results by the new method to agree within less than nine-tenths of one per cent, while the results by the root-mean-square voltmeter method are from 7.7 per cent to 14.9 per cent low and differ among themselves over 8 per cent.

Table 4 shows the effect of errors of frequency on the core losses measured by the two methods. The results of core-loss tests by the new method are practically independent of frequency errors in testing voltage. This means that the core loss

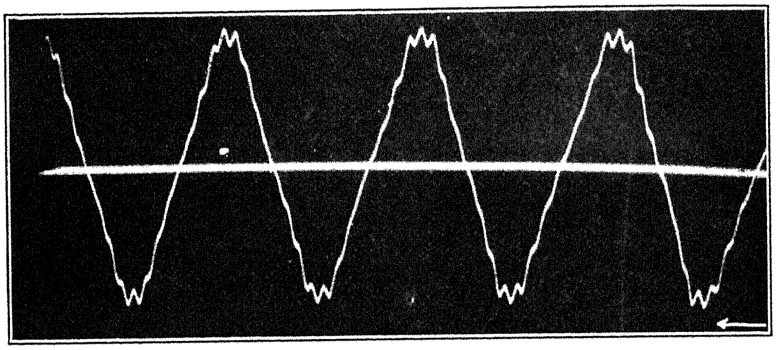


Fig. 8

at normal frequency and sine voltage can be obtained on a circuit with both frequency and wave-shape errors. It also means that transformer core losses, on a distributing line where the wave-shape is distorted and the frequency unknown, can be

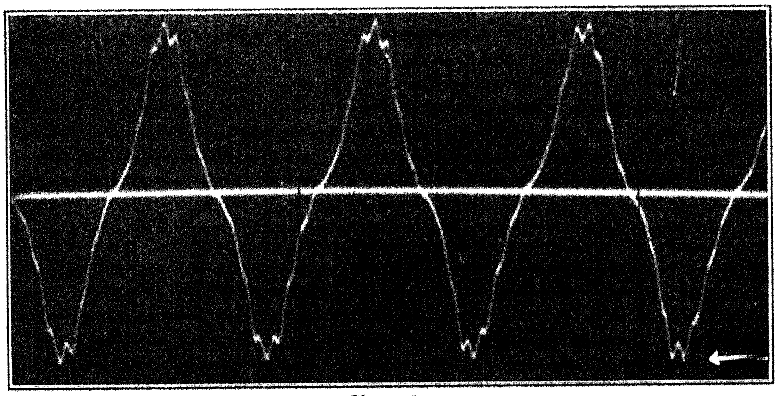


Fig. 9.

tested about as accurately as in a laboratory or at the source of power where the frequency can be accurately adjusted and the voltage wave-shape kept constant by adjusting voltage with the alternator field or a multi-tap transformer.

The tests of Table 4 were made on an alternator with good

wave-form, the voltage being adjusted by means of an auto-transformer. The conditions of wave-shape voltage were supposed to be very good and yet it will be noted that the error at 60 cycles by the root-mean-square voltmeter method is as great, owing to wave-form distortions, as the error at 53 cycles by the method.

In closing, it is only fair to speak of the disadvantages of the iron-loss voltmeter and the new method of test. The instrument is necessarily heavier than the ordinary alternating-current voltmeter. The internal losses of the instrument are high and in testing very small transformers the correction for instrument losses is as large as the core loss of the transformer. For high-voltage tests the usual voltage transformer cannot be used on account of the relatively large current taken by the instrument and its poor power-factor. A voltage transformer to be used with the iron-loss voltmeter must be especially designed with heavier copper and low reactance. The instrument can be made direct reading for only one frequency, and, when used on other frequencies, must be used with a calibration curve. These disadvantages are not of great importance as there are generally low-voltage taps on high-voltage transformers, to which the meter can be connected, and there are but few standard frequencies in terms of which tests will be required.

TABLE IV

Iron-loss voltmeter method				Root-mean-square voltmeter method			
Frequency		Core loss		Frequency		Core loss	
Cycles	% error	Watts	% error	Cycles	% error	Watts	% error
68.1	+13.5	201.1	+1.6				
66.4	+10.7	201.0	+1.5	67.1	+11.8	183.1	-7.5
64.5	+7.5	199.3	+0.7	63.3	+5.5	188.9	-4.6
63.7	+6.2	199.5	+0.8	62.1	+3.5	191.6	-3.2
60.0	Normal	198.0	0.0	60.0	Normal	195.6	1.2
59.5	-0.8	197.8	-0.1	57.0	-5.0	202.7	+2.4
59.0	-1.7	197.7	-0.2	54.2	-10.0	208.7	+5.4
56.8	-5.3	196.8	-0.6	52.3	-12.8	211.3	+6.7
55.2	-8.0	197.1	-0.5	50.5	-15.8	218.6	+10.4
53.0	-11.7	195.8	-1.1				
50.5	-15.8	195.1	-1.5				

DISCUSSION ON "METHOD OF TESTING TRANSFORMER CORE LOSSES GIVING SINE-WAVE RESULTS ON COMMERCIAL CIRCUITS." FRONTENAC, N. Y., JUNE 28, 1909

Frederick Bedell: The author is to be thanked for perfecting as an instrument that which, no doubt, more than one of us has already used as a method. We have used a calibrated transformer, whose losses under certain conditions have been accurately determined, for comparison with the transformer under test. Mr. Chubb's instrument will certainly prove most valuable if its calibration can be depended upon. Where one has a testing laboratory at his disposal he can readily check up such an instrument from time to time. When, however, one is remote from such testing facilities and has used an instrument of this kind for a number of years, a question arises as to whether it can be depended upon and will not be affected by time and temperature. I should like to hear from the author in regard to this.

Charles P. Steinmetz: In Mr. Chubb's paper a definition is given for the form-factor which I consider rather unfortunate. It is as follows: the ratio of the effective to the mean value. In this sense this term is frequently used, but it gives to the sine-wave a value of 1.11. Since in all electrical engineering we consider the sine-wave as the standard, it certainly is very unfortunate to define form-factor so that for the standard wave it does not give unity. Form-factor should be defined so as to give unity for the sine-wave. That is, *form-factor is the ratio of the effective to the mean value of the alternating wave, divided by the corresponding ratio of the sine-wave.* Hereby the sine-wave has form-factor 1, the form-factor of any other wave would directly represent the deviation from the standard wave, and the 1.6 power of the form-factor would be the correction factor of the hysteresis loss.

M. G. Lloyd: It seems to me that the instrument described ought to have a wide use, especially among smaller manufacturers and central stations where there are no facilities for getting sine-waves. I know of one case, for instance, which comes up in connection with some government work. The Navy Department in its permanent specifications for transformers at present requires the manufacturer who bids on its proposals to have the means of using and determining the sine-wave and control of the frequency. That prevents a great many of the smaller manufacturers from bidding on government contracts. An instrument like this would enable one to make tests without these facilities. If admitted by the Navy Department—I am told it is considering the use of such an instrument—it would be possible for a great many companies without testing facilities to participate in bids on such contracts. The difficulty of getting a sine-wave, as I have found by experience, is not at all small. In my paper on the "Testing of Transformer

Steel" I cite an instance of a generator which, to look at the oscillogram only, one would suppose gave a sine-wave. It had a very smooth and even curve and appeared to be perfect in all respects. An actual measurement, however, showed that the form-factor was considerably off. I take it to be a fact that it is not at all easy to get a machine that gives a form-factor sufficiently near to the sine-wave.

As regards the general use of these instruments, it is evident that the errors are only eliminated when the eddy currents bear a definite proportion to the hysteresis. Of course the errors in the instrument increase but slowly with variation of that ratio; but it occurs to me, in using it with widely different kinds of material, such as silicon-steel and the ordinary steel, that the same instrument perhaps could not be calibrated to read very well on both classes of material. While silicon-steel is in almost general use for certain classes of transformers, still, I understand, on low frequency many manufacturers still use the old variety of steel. Perhaps this is one thing that limits the use of the instrument.

As Mr. Chubb pointed out, I suppose the instrument could be supplied with different coils and calibrated to use on different frequencies, such as 25 and 60 cycles, which would make it of general application.

It seems unfortunate, too, that the instrument cannot be produced in a lighter form. Its excessive weight would hardly permit it to be classed with portable instruments, as it would not be convenient to carry such an instrument around.

L. T. Robinson: I wish to emphasize what Mr. Lloyd has just said about this instrument. It seems to me that it is a very good and very useful device, but is it not true that its application is limited to certain cases—to a line of transformers built, for instance, by certain designers for which they have prepared a suitable instrument? If such an instrument were trusted universally, would there not be found in commercial practice variations enough in the relation between eddy-current losses and hysteresis losses in transformers, and also variations enough in the form-factor of the waves, to make the results misleading?

Personally, I have used a method that amounts to about the same thing, but has not been worked up into combined instrument form. To a certain extent it is satisfactory but beyond that point it can not be trusted. I would prefer to see accurate work attempted in some other way; that is, by some instrument that would measure directly the form-factor based on which the correction would be applied to the transformer under test, either by separating the core loss of this transformer into its components or by using the somewhat definite knowledge that one would have of the constructive data.

Frederick Bedell: With reference to the definition of form-factor, I agree with Dr. Steinmetz that the present use is not the most logical. Form-factor as first used, following the lead of

Dr. Roessler in Germany, was defined as the ratio of the average to the effective value, which is the reciprocal of the value used in this paper. The form-factor of a square-topped wave is then unity, and of any other wave is less than unity, being 0.9 for a sine-wave. This is in conformity with efficiency and power-factor, which have maximum values of 100 or unity, and proceed from these values down to zero. For a number of years form-factor was used by different writers in each of these two ways. Although the definition of Roessler has historical precedence, the definition of form-factor used in several papers presented to us at this meeting seems to be now nearly universal, and it is probably now too late to revert to the original definition or to make any other change.

Chas. F. Scott: This instrument is essentially a practical instrument. The scientific principles which have been explained show that under a fairly wide variation of the very many variables which enter into the problem, it is substantially correct. To get a true idea of the value of the instrument, we must consider what must be done if this instrument were not available. Mr. Chubb has summarized the conditions which ordinarily apply, not only on the circuit of the operating company, but on the circuit of the manufacturing company in the testing laboratory. In order to get an accurate knowledge of the conditions of wave-form, it is necessary to make very elaborate, extended, and cumbersome measurements. It has been the custom at some factories to specify that a particular generator shall be used for supplying the current for transformer measurements and that the measurements are to be made on the machine when it is not loaded, it being known that the particular machine gives approximately a sine-wave. It recently happened that in a certain case transformers that were to be subjected to a certain test were measured with this new instrument, but it was found out afterwards that the generator, instead of being empty, as specified, was really carrying a load, and probably had a distorted wave form. In the days of the old method of measurement, with an ordinary voltmeter, there would be an error of probably twelve to fifteen per cent. The transformers, which had already been sent to the shipping room, were taken back and remeasured, and it was found that the measurements, under the corrected conditions, the specified conditions of the test, showed practically the same results as those formerly taken. Those who are the most familiar with the difficulties involved in the measurement of transformers, either in a large factory or at a remote station, will probably be the first to appreciate a method which is as good as this, although it may not be correct to the highest degree of refinement.

Charles P. Steinmetz: This instrument is extremely useful. We must realize, however, that in the usual case of transformer testing, where only a small amount of power is required, and therefore the efficiency of supplying the power is immaterial, we can get an almost absolutely perfect sine-wave from a cir-

cuit of any kind of wave shape, in the following manner. (Fig. 1). Suppose we have at D a supply of alternating voltage of known pressure and frequency, but of any kind of wave shape, no matter how distorted, and insert two inductances of equal reactance, x. into the circuit, and shunt a condenser, C, across the circuit, adjusting the condenser for the same reactance, x, as the inductances, so that if there is a short-circuit at B, there will be a minimum current from D. Then, connecting a resistance r across the circuit, and tapping off this resistance, a part of it or all, at S, any apparatus connected across S, if the current taken by it is small compared to the current in the resistance r, receives a perfect sine-wave. There is a case of resonance in this combination which eliminates everything but the fundamental sine-wave. This arrangement may be useful, at least for calibrating such a core-loss voltmeter; otherwise, after a year or so it may be doubtful whether the meter is correct or not.

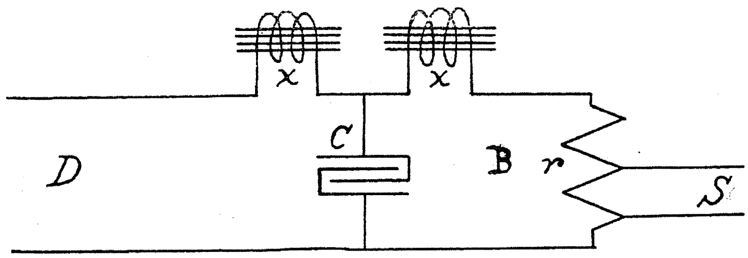


Fig. 1.

L. W. Chubb: Answering Dr. Bedell as to the reliability of calibration; we have used the instruments since March, 1908, and have had remarkably good results. Constancy of calibration was obtained by making the steel core of the instrument of laminated steel rings of non-aging material, without air-gaps and by thoroughly impregnating this core in a hard gum. Repeated calibration has shown the instruments to be reliable.

There is great need of such a method and instrument, as very large transformers cannot be even approximately tested by the usual method on the small generators that must be used for testing purposes. The new designs of transformer with silicon-steel cores, and their peaked exciting currents, will invariably distort the voltage wave of the testing generator, and unless the core-loss-voltmeter method, or the more complicated correction method, is used, the core-loss test is practically worthless.

Dr. Steinmetz suggests that the definition of form-factor be changed so as to give unity for the form-factor of a sine-wave. This would be a great convenience. The present method of having 1.11 as the sine form-factor introduces fractional coefficients in the usual formulas.

Dr. Lloyd speaks of the present government specifications

for equipping and method of testing transformers, prohibiting the small producers from taking contracts. Such has been the case, but satisfactory specifications can certainly be written requiring the use of the core-loss voltmeter, which will allow the small producer to compete, and which will greatly simplify the tests of the larger companies. Table III and the oscillograms in this paper show results of tests made by the Navy Department with a view of adopting the method.

In regard to different metals which may be used in the transformer core. It makes practically no difference whether the transformer under test is built with sheet-iron, open-hearth, or silicon-steel. The same instrument can be used and the result of core loss will in all cases vary from the true sine-wave value much less than if the root-mean-square voltmeter is used. The curves of Fig. 5, show the errors introduced by different ratios of eddy and hysteresis losses. A general statement, with no figures, was made in regard to the effect of steels having different exponents of hysteresis. Suppose we take an example of an extreme case to show the effect of the variation of hysteretic exponent: Assume a form-factor of voltage of 1.20; hysteret c exponent of instrument core 1.6; of transformer 2.0; sine-wave core loss of transformer 80 watts hysteresis, 20 watts eddy loss, 100 watts total; sine-wave loss in instrument 16 watts hysteresis, 4 watts eddy current, 20 watts total. On the peaked wave the voltage must be raised 6 per cent to give the required voltmeter indication and input of 20 watts to the instrument. At this point the input to the transformer will be hysteresis 76.910 watts, eddy current 22.472 watts, total 99.382 watts. Indeed, it will be seen that the component hysteresis and eddy-current losses are very much in error; but one is practically as much low as the other is high, so that the sum is only 0.6 per cent in error. In this case we have considered the hysteretic exponents in instrument and transformer differing as much as will be found in service, and the error of less than 1 per cent is commercially allowable.

In regard to the portability of the instrument, it is certainly heavy as now made, but a much lighter instrument is being designed which will be quite portable.

Mr. Robinson expressed the opinion that it can be used only in certain cases. The experience we have had with the instrument shows clearly that such is not the case. Any transformer of a certain frequency which is of commercial design may be tested with any core-loss voltmeter, adjusted for that frequency, and in all cases, it may be said that the core-loss tests will be more accurate than with the usual voltmeter.

L. T. Robinson: I agree substantially with everything that has been said, that is, for ordinary conditions, but I do think the instrument should be used with considerable caution, because there are variations in form-factor on systems that might sometimes make it misleading to use an instrument like the one described.

L. W. Chubb: You mean that the purchaser of a transformer does not want to know the core loss on a form-factor of 1.11, but that he is interested in what the loss will be on his line.

The transformer which has the lowest loss on a sine wave will have the lowest loss on the customer's wave. Specifications and tests should all be based on the same wave, and then guarantees and test results will be directly comparable. The manufacturers do not know the form-factor of the customer's line. If they did it would not be practicable to work out special performance guarantees for the customer's circuit. It is far better to have a standard wave shape on which to base the performance, and require that the tests be made in terms of this standard wave, or sine wave. Such tests can be made with the iron-loss voltmeter on any commercial circuit.

The next question referred to the proportion of eddy current and hysteresis loss varying in the instrument. The percentage of hysteresis does vary at different scale readings, but within a readable range the ratio is sufficiently constant to give accurate results.

It was also suggested to use an instrument to find the form-factor and make corrections in the observed core-loss result. It should be remembered that if this is done it is necessary to make several elaborate tests and calculations, while with the iron-loss-voltmeter it is only necessary to adjust the voltage until the meter reads the normal value, and then read the core loss directly on a separate wattmeter.

Dr. Bedell suggests having the square type wave or the unity form-factor wave as standard, and then comparing other waves to it; in fact, treating the matter as in the case of power-factor. The unity power-factor is the most desirable and convenient reference value for the cosine of the angle of lag or lead, while the unity form-factor is a limiting as well as a maximum value, and a condition which is never acquired in the usual alternating-current circuits.

Dr. Steinmetz speaks of using a harmonic filter, as shown in his diagram, for obtaining a sine wave for testing transformers or calibrating the iron-loss voltmeter. The scheme will give a sine wave at *S* at no load, or when the current drawn at this point is very small in proportion to the current through the resistance. However, a transformer exciting current is a distorted low-power-factor current with a decided peak, when the values of voltage are low or going through zero. On account of this distorted and low-power-factor current there will be out-of-phase resistance drops in the resistances shown, which will peak the voltages at *S*.

The scheme outlined by Dr. Steinmetz could be used in calibrating meters taking very small current, but I believe the iron-loss voltmeter takes too much current to be successfully calibrated by this method.

In reply to the question concerning the form-factor correction scheme, I will say that oscillograms were not taken. However, voltage drops and currents were observed at the cell and at other points of the circuit. We found experimentally that the relative readings of the root-mean-square and iron-loss voltmeters, connected to the terminals of the transformer, could be easily and quickly adjusted by means of the inductance L and resistance R, thus making the form-factor correspond to a sine wave, a peaked or a flat wave.

Paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

THE TESTING OF TRANSFORMER STEEL

BY M. G. LLOYD AND J. V. S. FISHER

Summary. The object of this paper is to discuss the conditions which should be realized in the measurement of energy losses in sheet-iron and steel subjected to alternating magnetization, with a description of a modification of the Epstein method and apparatus. This method is believed better to satisfy the desired conditions and to give an accuracy of one per cent with the use of less than two kilograms (4.4 lb.) of material.

Results are given showing a wide range in the quality of material in general use, quality having slight connection with price. Several foreign specimens of ordinary steel and silicon-steel are included for comparison with the American product. Silicon-steels contain from 3 to 4 per cent silicon, the quality not depending upon the exact percentage of silicon.

By making measurements at two frequencies, the eddy-current and hysteresis losses have been separated and a study made of the variation of each with flux density. The values of the hysteretic constant and the total loss in watts per pound at 60 cycles and 10,000 gausses are tabulated.

The effects of artificial aging are shown to depend upon the flux density selected for test, the hysteresis increasing more for 5000 than for 10000 gausses. Tests should therefore be made at the density to be used. The aging is usually negligible in silicon-steels.

Many methods have been employed for the testing of sheet-iron and steel for energy losses when subjected to alternating magnetization, but all that have so far been employed have been

lacking in some desirable qualifications. The plotting of a hysteresis loop from readings of magnetizing force and the resulting magnetic induction, and the measurement of the area of this loop, is a slow and tedious process and gives no indication of the eddy-current losses. Methods depending upon the relative motion of the specimen and a magnet necessitate an air-gap in the magnetic circuit, with a resulting induction in the specimen which is far from uniform. Moreover, if a permanent magnet be used, as in the Ewing and the Blondel apparatus, one is restricted in the values of the flux density which may be used, and the apparatus is suitable only for comparative measurements at one flux density.

Consequently, methods of testing with alternating currents have come to be regarded as the only satisfactory way of making these measurements, and present efforts are directed to securing the best conditions for this form of test. In the wattmeter method, the electrical energy which is supplied to maintain the alternating magnetization is measured with a wattmeter, while the maximum magnetic induction produced in the specimen is determined by a voltage measurement at the terminals of the magnetizing winding or at the terminals of a secondary winding placed around the same core of test material.

The specimen may be employed in three forms: (1) It may be in the form of straight strips placed in contact with a yoke, thus forming a closed circuit of ferro-magnetic material. (2) The straight strips may be used without any yoke. (3) The specimen may be arranged to form a closed magnetic circuit in itself.

The second form gives a distribution of flux which is far from uniform, and is therefore objectionable. The first form gives a more uniform flux, but it is necessary to distinguish between the energy supplied to the specimen and that supplied to the yoke. This can be done satisfactorily only by knowing the constants of the yoke, and only then by having the distribution of flux uniform, a condition difficult to secure. Consequently, for accurate measurements the third form is the most reliable, although for factory use the first or second may prove more convenient where accuracy can be sacrificed for other considerations.

Assuming a closed magnetic circuit of the material to be tested by the wattmeter method, the following conditions should be realized as far as possible:

1. The flux should be uniformly distributed over the cross-section of the specimen, and should be the same at every section; this requires that there should be no leakage of flux through the air.

2. A definite form of wave of magnetic flux should be used, or in other words, a definite form of wave of secondary electromotive force, since the form-factor of this wave enters into the computation of maximum flux density from the observed effective voltage.

3. The material used should be cut in a form such that only a small part of it is contiguous to a cut edge, since it is well known that all methods of cutting have a hardening effect upon the material bordering upon the cut. This means that the strip, whether straight or in ring form, should not be too narrow. This condition may be dispensed with if all specimens are annealed under definite conditions after cutting to size and prior to testing.

4. The amount of material required should not be greater than is necessary to get a fair average value.

5. The corrections to be made in the readings of the instruments to obtain final values should be as few and as small as possible.

Two general forms of magnetic circuit are available. The material may be stamped into rings or the circuit may be built up from straight strips. Leakage is most effectually avoided by using rings. With this form of specimen, however, it is impossible to satisfy simultaneously conditions 1 and 3, unless rings of very great diameter are employed, and in the latter case there is a very great waste of material. The non-uniformity of flux existing in rings of small diameter, even when uniformly wound, and the errors resulting therefrom, have been discussed elsewhere.[1] The use of rings is thus restricted to cases where the material is annealed after stamping, and the radial width of the ring should be very small in comparison with its diameter. When rings are employed, the labor of winding each specimen separately with a magnetizing coil may be obviated by the use of the apparatus of Esterline[2] or Möllinger.[3]

To meet condition 2 it is sufficient to know or measure the

1. M. G. Lloyd, Bulletin of Bureau of Standards, 5, p. 435; 1909. *Electrical World* 52, p. 1395; 1908.

2. J. W. Esterline, Proc. Am. Soc. Testing Materials 3, p. 288; 1903.

3. J. A. Möllinger, Elektrot. Zs. 22, p. 379; 1901.

form-factor of the secondary voltage.[4] By making runs at two frequencies it is then possible to separate the eddy-current and hysteresis losses, and, if desired, to compute the eddy-current loss for a standard wave-form.[5]

It is far preferable, however, to work throughout with a sine wave, when a generator is available which will fulfill this condition. There are three things which may prevent the realization of this condition. In the first place the machine may not generate a sinusoidal electromotive force. In fact, it may be stated as a general proposition that no generator gives a perfect sine wave. The only question is as to the magnitude of the harmonics present, and whether these are negligible. It cannot be assumed that these are negligible simply because the machine was designed to give a sine wave, or because a rough oscillogram does not indicate definite distortion. The only way to be certain is to take an accurate curve from the machine and analyze it by measurement of the ordinates.

As an example of the necessity of this, we cite an instance occurring at the Bureau of Standards. Here the tests are usually made with a generator whose electromotive force wave contains a third harmonic whose amplitude is 0.6 per cent of the amplitude of the fundamental, and none of the higher harmonics are present to such an extent as 2 per cent. The form-factor is almost exactly that for a sine wave. One day a specimen, which had already been tested with this generator, was tested with a second generator supposed to give a sine wave whose oscillogram appeared smooth and inoffensive. The losses appeared more than 4 per cent lower than by the previous test. This led to a closer examination of the wave given by the generator, the wave being traced by the Rosa apparatus,[6] analyzed and found to contain nearly 7 per cent of the third harmonic, sufficient to account for the observed difference in losses.[7]

A second cause of distorted wave-form is to be found in armature reaction, which may alter the electromotive force of a loaded generator when the curve on no load is sinusoidal. For this reason, it is best in testing to use a machine so large that it is only slightly loaded by the test current.

4. An apparatus for measuring form-factor is described by Lloyd and Fisher, Bull. Bur. Stds. 4, p. 469; 1908.

5. See M. G. Lloyd, Bulletin Bureau of Standards 5, p. 381; 1909

6. See E. B. Rosa, Phys. Rev. 6, p. 17; 1898. TRANS. A. I. E. E. 1898.

7. See M. G. Lloyd, Bulletin Bureau of Standards. 4, p. 484; 1908.

A third cause of distorted wave is to be found in the drop of potential due to the impedance of the circuit. The generator electromotive force is made up of two parts, one of which is balanced by the electromotive force induced by the changing flux, while the other produces current. If the flux be sinusoidal, the electromotive force induced by it is also sinusoidal. But owing to the fact that the permeability of the material varies with the magnetic induction, the magnetizing current cannot be sinusoidal. The component of the electromotive force producing current has the same form of wave as the current, and it also cannot be sinusoidal. The total electromotive force of the generator then, to produce sinusoidal flux, is made up of one component which is sinusoidal and one which is not, and therefore is not sinusoidal. Its shape must vary with varying conditions, and hence it would be useless to attempt to secure such a form of generator electromotive force. To approximate sinusoidal flux it is necessary to have a sinusoidal electromotive force at the generator and to make the component which sends current (equal to the product of current and resistance) negligible in comparison with the component which balances the electromotive force induced in the apparatus. It is desirable then to keep both resistance and current low in the magnetizing circuit. If this ohmic drop of potential can be made negligible, then the wave-form of flux will differ from a sine curve only by a negligible amount.

The resistance in the magnetizing circuit consists of the armature, leads, magnetizing coil, measuring instruments and perhaps of windings of transformers used to step up or down to the proper voltage. It should not include a regulating rheostat, but the current should be controlled through the generator field. Each of these items should be made as low as possible, and thus appears another reason for choosing a generator of capacity large in comparison to the load to be placed upon it. The magnetizing current may be kept low by having a magnetic circuit of low reluctance. Air-gaps should be avoided, and any joints in the magnetic circuit be made as good as possible.

With the same magnetizing current, the induced electromotive force is proportional to the cross-section of test material; consequently the greater the quantity of material used, the less the distortion of the wave. With a definite cross-section and windings, the induced electromotive force is proportional

to the maximum flux density, but the magnetizing current is not proportional to the flux density, owing to varying permeability. The greater the permeability, the larger the ratio of flux to magnetizing current. The distortion will consequently be less if the iron be magnetized in the region of maximum permeability, and the distortion is sure to become appreciable if the flux density be carried too high, and may become appreciable at very low flux densities, even when it is negligible through the range of working flux densities used industrially.

In the method of Epstein,[8] which has been adopted as standard in Germany,[9] these conditions are fairly well met. Ten kilograms of sheet are cut into strips 50 by 3 cm. and assembled into four bundles, over which solenoids are slipped. The four bundles are arranged in the form of a square, having butt joints at the corners, where the magnetic material is separated by a sheet of thick paper. This interruption to the magnetic circuit tends to make the flux more uniform across the section of test material, but it also makes the leakage greater, and the flux less equal at different sections. Thus the flux at the center of one bundle may exceed the flux near one end by as much as 8 per cent. This air-gap also increases the reluctance of the magnetic circuit, consequently increasing the magnetizing current, and a large quantity of test material must be used. Condition 4 is here sacrificed for the better attainment of conditions 1, 2, and 3, and yet conditions 1 and 3 are not very well satisfied.

A modification of this method has been developed at the Bureau of Standards differing from the foregoing principally in the arrangement of the test material. A smaller quantity of material in wider strips may be used, while at the same time a greater uniformity of flux is secured. The amount of material used is from 1.5 to 2 kg. (less than 4 lbs.), and an accuracy of one per cent is attained.

Description of the Apparatus

The specimen to be tested is cut into strips 25.4 by 5 cm. (10 by 2 inches). These are assembled into four bundles, in each of which adjacent strips are separated by strips of press board of equal width and thickness, but 2 cm. shorter. Each bundle is wrapped with friction tape and is inserted in a solenoid.

8. I. Epstein, Elektrotechnische Zeitschrift 21, p. 303; 1900.
9. Elektrot. Zs. 24, pp. 657, 684; 1903.

The four bundles are then arranged in a square so that the plan view shows the edges of the strips, as in Fig. 1. The solenoids are wound upon fiber frames 22.7 cm. long, and have inside dimensions 5 cm. by 1 cm. At the corners of the square, short pieces of test material are bent at right angles and interleaved between the strips of adjacent bundles, as shown in the figure. There are as many of these corner pieces as there are test pieces,

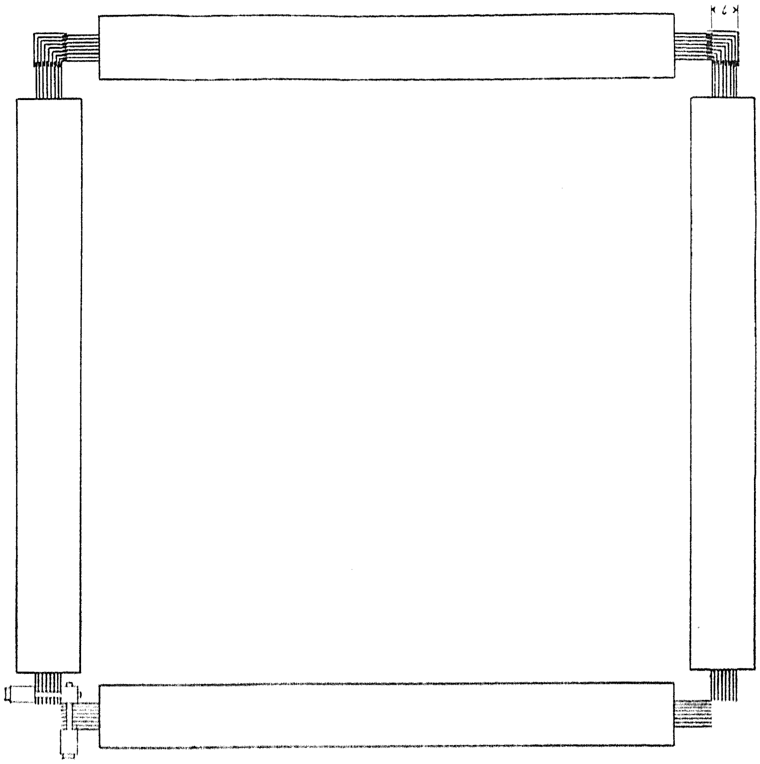


FIG. 1.—Plan of apparatus, showing corner pieces in place at two corners and clamp in place at one corner.

and they are graduated in length so as to give a uniform lap of about 2 mm. A special clamp, shown in Fig. 2, is tightened over these laps, so as to give a good magnetic joint.

Each solenoid has in its first layer two windings of double-silk-covered No. 20 wire, each consisting of 45 turns. Over these are wound 250 turns of No. 14 copper wire, also double-silk-covered, to form a magnetizing coil. The four solenoids are connected in series, making a total of 1000 magnetizing

turns and two secondaries of 180 turns each. One of these secondaries is connected to a voltmeter for determining the magnetic flux. This instrument is a deflecting mirror dynamometer, giving a sufficient deflection with 0.001 ampere. The other secondary is connected to the moving-coil circuit of a watt-dynamometer of the same type as the voltmeter. The magnetizing current traverses the field coils of this wattmeter, whose deflections are a measure of the power supplied to the core and the secondary coils. The copper loss in the primary is thus eliminated from the power measurement,[10] as is evident from the following considerations.

Let N_1 = primary turns.
N_2 = secondary turns.
Φ = flux threading both primary and secondary.
i = primary or magnetizing current.
e = electromotive force applied to primary.
L = self-inductance of primary due to any flux not included in Φ.

Then

$$e - N_1 \frac{d\,\Phi}{d\,t} - L \frac{d\,i}{d\,t} = r i$$

The instantaneous power expended is

$$e\,i = r\,i^2 + N_1\,i \frac{d\,\Phi}{d\,t} + L\,i \frac{di}{d\,t}$$

The integral of this expression, extended over a complete cycle, will give the net power. The term $r\,i^2$ represents the primary copper loss. The term $L\,i\,\frac{d\,i}{d\,t}$ when integrated over a complete cycle, is equal to zero. The term $N_1\,i\,\frac{d\,\Phi}{d\,t}$ will not integrate to zero when there is either hysteresis in the core or secondary currents (including eddy currents) are flowing, since in either case Φ is not in phase with i. This term represents the power expended in the core and in the secondary circuits. $N_2\,i\,\frac{d\,\Phi}{d\,t}$ is proportional to this, and its integral value represents

10. Due to C. P. Steinmetz, TRANS. A.I.E.E., Vol. IX, p. 624; 1892.

the reading of the wattmeter when connected as in this apparatus. Hence the wattmeter reading, when multiplied by $\frac{N_1}{N_2}$, gives the power expended in the core and in the secondary circuits. Error will arise only when there is flux threading the core and linked with the primary, which is not linked with the secondary. This is avoided by winding the secondary under the primary, and making the two coextensive in length. The energy in each secondary is obtained by squaring the secondary voltage and dividing by the resistance of its circuit. By using a low number of turns in the secondaries and sensitive instruments, these corrections are kept very small and are accurately known.

The voltmeter and the wattmeter each have variable multipliers, whose resistance is adjusted to give a suitable deflection in each case. The accuracy of reading is usually better than 0.1 per cent, and is higher than the conditions require.

The frequency is determined by a Hartmann and Braun frequency meter, which has been calibrated by the use of a chronograph, or where greatest accuracy is required the chronograph is read directly.[11]

When it is desired to measure the exciting current, an ammeter can be introduced into this circuit, but the exciting current for ordinary inductions is so low that it is difficult to secure an ammeter of sufficiently low resistance. The only type of portable instrument which has served this purpose is the Duddell thermo-ammeter. This can be obtained with a range of 0.5 ampere and a resistance of 0.2 ohm.

The use of corner pieces bent at right angles caused at first some apprehension as to its effect upon the results. It is known that bending, like any other mechanical treatment, will change the magnetic properties of the steel. If this affected enough of the steel seriously to alter the average value, it would condemn the method. Experiments which were directed to the determination of this effect showed that it was of no importance. The corner pieces are bent sharply in a machine which distorts the material to only a very short distance from the angle. The distorted material has its constants considerably changed without doubt, but as the corner pieces constitute only about 5 per cent of the

11. This use of the chronograph, with specimen record, is described by M. G. Lloyd, Bulletin Bureau of Standards, 5, p. 388; 1909.

entire circuit, and as only a part, say 30 per cent, of this material is altered by a fraction, say 20 per cent, of its initial value, an error of much less than 1 per cent is made by regarding this material as unaffected by the bending, and 1 per cent is the limit of accuracy claimed for the method.

To examine experimentally the effect of bending, a measurement was made in the usual way; the strips were then removed and each was bent at right angles and then bent straight at a point close to the first bend, so that the length was only slightly altered. A new measurement showed that the losses had increased by 1 per cent. A single bend would of course affect them by much less than this. Another experiment consisted in making measurements upon annealed specimens, and others not annealed, each with corner pieces of its own material. The corner pieces not annealed were then used with annealed test pieces, and after making the proper corrections and allowances for loss in the corner pieces, as described later, the values for the test pieces were found in agreement with those previously obtained with annealed corner pieces. As the annealed material is very much more sensitive to mechanical treatment than that not annealed, the effect must have caused different results in the two cases if it were operative to more than a negligible degree. A third method of checking this point was tried, and led to the same conclusions, but the foregoing are considered sufficient evidence.

On account of the lapping of the corner pieces over the ends of the test pieces, the flux density is low in this part of the material, necessitating a correction in the results. The amount of lap is determined by the relative weights of corner and test pieces, as compared with the relative lengths of the two parts of the circuit. When the corner pieces are of the same material as the test pieces, it is assumed that the flux density is halved in the portion of material which laps, and the energy loss is consequently only one-third normal.

B = nominal, or average, maximum flux density.
B_1 = maximum flux density at ends of test pieces.
M = mass of test pieces.
m = mass of corner pieces.
l = dimension shown in Fig. 1.
W = measured loss.

$\frac{m}{M}$ = proportional increase in mass of magnetic circuit, due to corner pieces.

$\frac{l}{25.4}$ = proportional increase in length of magnetic circuit, due to corner pieces.

$\frac{m}{M} - \frac{l}{25.4} = c$ = mass of corner pieces which lap, expressed in terms of mass of test pieces.

$2c$ = total material (lapped and lapping steel) in which flux density is $\frac{B_1}{2}$

$$2cW\left[1-\left(\frac{B_1}{2B}\right)^x\right] = 2cWk = \text{correction to } W \text{ for lap,}$$

where x expresses the law of variation of loss with B.

k varies slightly with the conditions. If $B_1 = B$ (no leakage)

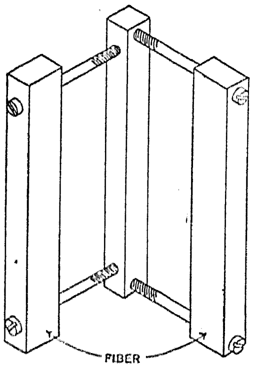


FIG. 2.

and $x = 1.6$, $k = 0.67$. For $x = 2.0$ this becomes 0.70. For 4 per cent leakage, $x = 1.6$, $k = 0.68$. With sufficient accuracy for the purpose k may be taken as 0.70 throughout, so that the correction becomes $1.4\ cW$ and the loss per unit mass is

$$\frac{W}{M+m}(1+1.4\,c) \text{ or } \frac{W}{(M+m)\,(1-1.4\,c)} \text{ with sufficient ac-}$$

curacy. The latter form is the most useful in practice, since a number of observations at different flux densities are usually made upon a single specimen, and the correction may be made once for all to the mass. The quantity $(M+m)\ (1-1.4\,c)$ may be called the "effective mass."

At first, a set of corner pieces was made for each set of test pieces of the same material, but this has been found unnecessary.

Corner pieces of approximately the same quality and thickness may be used with satisfactory and reliable results, provided the constants of the material are known. The Bureau of Standards has now accumulated a sufficient variety of corner pieces so that it is seldom necessary to make a new set, unless unusual precautions are to be taken. When using corner pieces of different material from the test pieces, it is necessary to compute the loss in the entire corner pieces, and then determine an "effective mass" resulting from the lap reducing the flux in the test pieces, Since the thickness of corner pieces may be different from that of the test pieces, it is necessary to consider this fact, and the flux at the lap may be considered to divide evenly between the two, or in proportion to their thickness. As the results do not differ materially, we assume that in each lapped part the flux density is half the value in the rest of the material.

Let

t = thickness of test pieces.

t_1 = thickness of corner pieces.

w = loss per unit mass in corner pieces.

and other quantities as before. We neglect the leakage, which is small. c must now be computed by using for M the mass M_1 of test pieces of the same material as the corner pieces. The loss in the corner pieces, if there were no lap, would be $w\,m\left(\frac{t}{t_1}\right)^x$.

Considering the effect of lap the loss is $w\ (m - 0.7\,c\,M_1)\left(\frac{t}{t_1}\right)^x = W_c$. The correction to be applied to the loss in test pieces due to lap is $0.7\,c\,(W - W_c)$ and the loss per unit mass is $\frac{W - W_c}{M\,(1 - 0.7\,c)}$.

If the corner pieces are of the same thickness as the test pieces, the loss in them becomes $w\ (m - 0.7\,c\,M_1)$, and this expression will usually give the correction closely enough. Here again the quantity $(m - 0.7\,c\,M_1)$ can be determined once for an entire set of measurements.

If the flux at the lap be assumed to divide between the two in proportion to thickness, we have for the loss in the corner pieces

$$w\left[\ (m - c\,M_1)\left(\frac{t}{t_1}\right)^x + c\,M_1\left(\frac{t}{t+t_1}\right)^x\right]$$

and the correction for lap in the test pieces is

$$c\,(W - W_c)\ \left[1 - \left(\frac{t}{t+t_1}\right)^x\right]$$

The leakage with this arrangement of test material, that is, the difference between flux at the ends of test piece and at the middle, is usually not greater than one per cent. Since the greatest length of a line of induction is 8 per cent larger than the least, there is a probability of the same difference in flux density between the outer and inner sheets. As the average flux density is measured, this cannot make an error greater than a small fraction of one per cent.[12] Inequality in the four arms has an equally slight effect. Upon removing one sheet in 12 from opposite sides, the loss per unit mass was not appreciably altered.

Observations and Results

The procedure in making a test is as follows: The material is cut into strips of the given dimensions by the use of a sharp machine shear with nearly parallel jaws. The number of strips is determined by the thickness, and for gauge No. 29 amounts to 48. The strips are then weighed, bundled and mounted in the solenoids. The effective voltage corresponding to any given flux density and frequency is computed from the following relation.

$$E = 4\,f\,N\,n\,\Phi\,10^{-8} = \frac{4.44 \times 180 \times 10^{-8}\,B\,n\,M}{101.6\,\rho}$$

where n = frequency, Φ = total flux, f = form-factor of the secondary electromotive force, and ρ = density. In the work here reported ρ has been taken as 7.77 grams per cubic centimeter, but it is proposed hereafter to determine the density for each specimen.

The dynamometer voltmeter is calibrated for one voltage as determined above, and when taking observations for watt loss, the generator voltage is adjusted until the same deflection is obtained. For other frequencies and flux densities, the resistance in the voltmeter circuit is altered until it is proportional to the product $B\,n$, so that the same deflection is always used. For the lower values of this resistance, the slight correction due to the

12. See M. G. Lloyd, Bulletin Bureau of Standards, 5, p. 435; 1909.

inductance of the instrument is also made. In computing the power supplied to the voltmeter circuit, it may then be remembered that this energy is also proportional to $B\,n$, since the same current is used throughout. A similar series of resistances is usually used in the potential circuit of the wattmeter, so that the power consumed here is also readily computed. The deflection, however, will increase as the product $B\,n$ increases, but will usually remain within working limits of permissible deflections. These limits are so chosen that within them the deflections are proportional to the watts. The multipliers for the potential circuit are so chosen that for one of them the actual watts corresponding to a given deflection, when multiplied

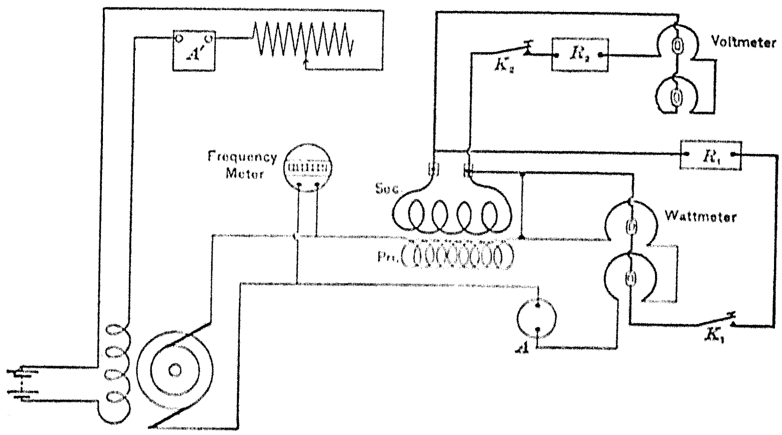


FIG. 3.—Diagram of connections.

by $\frac{N_1}{N_2} = \frac{1000}{180}$, are numerically equal to the deflection. The wattmeter is then direct-reading for this range for the values of the watts required, and for other ranges a simple factor determined by the value of $B\,n$ gives the desired result.

The generator used gives an electromotive force wave which is sufficiently close to the sinusoidal, and the form-factor of the secondary electromotive force has been determined and found sufficiently near to that assumed through the working ranges of flux density at the frequencies used, which usually are 60 and 30 cycles, the latter being chosen because it makes the separation of hysteresis and eddy-current losses easy to compute. The generator is driven by an electric motor whose

field circuit contains a rheostat in the laboratory, permitting adjustment of speed for a definite frequency. The motor is supplied with power from a storage battery so that a steady speed may be maintained. The field circuit of the generator is connected through another rheostat in the laboratory, which permits adjustment of the generator voltage to give the flux density desired. No rheostat is used in the magnetizing circuit. When necessary, a transformer of ample capacity is used to step up or down to the voltage required for the test.

The electrical connections are shown in Fig. 3, where for simplicity a single secondary is represented, which may indeed be used in practice. When the generator voltage has been adjusted to give the proper reading on the voltmeter connected to

TABLE I.

SPECIMEN K

TEST PIECES = 1327 GRAMS. CORNER PIECES = 80.2 GRAMS. $c = 0.029$.
EFFECTIVE MASS = 1350 GRAMS

Cycles	Flux density	Watt-meter deflection	Watts	Instrument losses	Iron losses	Joules per cycle	Ergs per gram per cycle	Hysteresis	Eddy currents
60	10000	21.38	4.276	0.079	4.197	0.0699	518	394	124
30	10000	18.87	1.887	0.0395	1.848	0.0616	456	394	62
60	5000	13.54	1.354	0.0395	1.314	0.0219	162	130	32
30	5000	12.20	0.610	0.0197	0.590	0.0197	146	130	16

the secondary circuit, the wattmeter is read. These settings are repeated twice. An adjustment is then made for a different flux density and readings are taken as before. When the exciting current is desired, an ammeter is included in the magnetizing circuit, and its indications noted. Whenever a change is made from a higher to a lower flux, the current is reduced gradually to the lower value in order to demagnetize the material. This descending set of observations is usually made at a frequency of 30 cycles. Whenever the magnetizing circuit has been broken, it is closed through a considerable resistance, which is continuously reduced to zero in order to prevent a large first surge and consequent high magnetization, which would require subsequent demagnetization.

Table I gives a specimen set of observations where the corner

pieces were of the same material, while in Table II they were of different material.

The total loss is separated into two components, due respectively to hysteresis and eddy currents, as follows, using the Steinmetz equation,

$$W = \eta\, n\, B^x + \zeta\, n^2\, B^y$$

where the symbols have the same significance as before, η and ζ being constants of the material. By taking observations at two frequencies, n_1 and n_2 we have

$$\frac{W_1}{n_1} = \eta\, B^x + \zeta\, n_1\, B^y = a + b\, n_1$$

TABLE II

SPECIMEN H_1

TEST PIECES = 1304 GRAMS. CORNER PIECES OF SPECIMEN K = 80.2 GRAMS. c = 0.029. EFFECTIVE MASS = 1278 GRAMS

Cycles	Flux density	Watt-meter deflection	Watts	Instrument losses	Iron losses	Joules per cycle	Corner pieces	Test pieces	Ergs per gram per cycle
60	10000	21.55	4.310	0.076	4.234	0.0706	0.0028	0.0678	531
30	10000	18.95	1.895	0.038	1.857	0.0619	0.0024	0.0595	465
60	5000	13.74	1.374	0.038	1.336	0.0223	0.0009	0.0214	168
30	5000	12.34	0.617	0.019	0.598	0.0199	0.0008	0.0191	150

$$\frac{W_2}{n_2} = \eta\, B^x + \zeta\, n_2\, B^y = a + b\, n_2$$

where a is the hysteresis loss per cycle and $b\,n$ the eddy-current loss per cycle.

$$a = \frac{\frac{W_2}{n_2}\, n_1 - \frac{W_1}{n_1}\, n_2}{n_1 - n_2}$$

$$b = \frac{\frac{W_1}{n_1} - \frac{W_2}{n_2}}{n_1 - n_2}$$

If $n_1 = 2\,n_2$ this computation is greatly simplified; for then

$$b\,n_2 = \frac{W_1}{n_1} - \frac{W_2}{n_2}$$

$$b\,n_1 = 2\,b\,n_2$$

$$a = 2\,\frac{W_2}{n_2} - \frac{W_1}{n_1} = \frac{W_2}{n_2} - b\,n_2$$

While the Steinmetz equation, and consequently this separation, is not accurately in accordance with the facts, the errors

TABLE III
VARIATION OF EXPONENTS WITH FLUX DENSITY
SPECIMEN P_1

Cycles	B	Loss per cycle	Hysteresis	Eddy currents	Exponents: Hysteresis	Exponents: Eddy currents
60	12000	842.7	445.7	397.0		
30	12000	644.2	445.7	198.5		
					2.12	2.03
60	10000	576.8	302.8	274.0		
30	10000	439.8	302.8	137.0		
					1.75	1.97
60	8000	381.4	204.8	176.6		
30	8000	293.1	204.8	88.3		
					1.68	1.83
60	6000	230.9	126.5	104.4		
30	6000	178.7	126.5	52.2		
					1.53	1.97
60	4000	114.9	67.9	47.0		
30	4000	91.4	67.9	23.5		
					1.51	2.03
60	2000	35.4	23.8	11.6		
30	2000	29.6	23.8	5.8		

are very small in thin sheets. The exponents x and y can be determined by observations at different flux densities, and these have been computed for a number of specimens, as shown in Tables III to VII. Where the eddy-current loss is small, as in silicon-steel, the values of y are subject to greater error.

While these exponents do not exhibit any definite and constant value, it will be noticed that the hysteresis exponent does not differ much from 1.6 for flux densities between 5000 and 10000 gausses, while for densities exceeding 10,000 it is in the neighborhood of 2, with a definite tendency upward. The

exponents for eddy-current loss vary rather widely from 2 in some instances, but with specimen *C* in Table VII, where the eddy-current loss is greater and the results consequently more accurate, the values come close to 2.

In order to test the proportionality between eddy-current loss and frequency, some runs were made with a second generator which gave 180 cycles at normal speed and 90 cycles at half speed. The wave-form of this generator is not so pure as that of the one used for the lower frequencies, but the loss could be altered by less than two per cent at most from this cause, and probably not over one per cent. The error from this cause

TABLE IV
VARIATION OF EXPONENTS WITH FLUX DENSITY
SPECIMEN *K*

Cycles	Flux density	Loss per cycle	Hysteresis	Eddy currents	Exponents: Hysteresis	Exponents: Eddy currents
60	12500	820	638	182		
30	12500	729	638	91	2.16	1.7
60	10000	518	394	124		
30	10000	456	394	62	1.71	1.9
60	7500	313	241	72		
30	7500	277	241	36	1.53	1.9
60	5000	162.2	129.2	33		
30	5000	145.7	129.2	16.5	1.47	2.0
60	2500	54.6	46.6	8.0		
30	2500	50.6	46.6	4.0		

would be less at 180 cycles than at 90 cycles. The measured loss, however, is a great deal lower at 180 cycles than is computed from the results at 30 and 60 cycles. This is illustrated by the experiments exhibited in Table VIII, where the hysteresis and eddy-current losses are separated by use of the readings at 30 and 60 cycles. The hysteresis is assumed constant, and the resulting eddy-current loss does not increase as rapidly as the frequency. The eddy-current loss falls off more rapidly, the thicker the specimen. The effect is greatly accentuated in specimen *A*, which consists of sheets 1.6 mm. thick, and was tested by using butt joints at the corners of the apparatus. The results of specimen *P* are plotted in the curve of Fig. 4. The

TABLE V
VARIATION OF EXPONENTS WITH FLUX DENSITY
SPECIMEN W_1

Cycles	Flux density	Loss per cycle	Hysteresis	Eddy currents	Exponents Hysteresis	Exponents Eddy currents
60	12500	379	309	70.		
30	12500	344	309	35.	2.04	2.0
60	10000	241	196	45.		
30	10000	218.5	196	22.5	1.68	1.9
60	7500	146.8	120.8	26.0		
30	7500	133.8	120.8	13.0	1.60	2.0
60	5000	74.7	63.1	11.6		
30	5000	68.9	63.1	5.8	1.59	2.2
60	2500	23.6	21.0	2.6		
30	2500	22.3	21.0	1.3		

TABLE VI
VARIATION OF EXPONENTS WITH FLUX DENSITY
SPECIMEN R

Cycles	Flux density	Loss per cycle	Hysteresis	Eddy currents	Exponents Hysteresis	Exponents Eddy currents
60	14000	606	524	82		
30	14000	565	524	41	1.88	1.6
60	12000	456	392	64		
30	12000	424	392	32	1.71	2.1
60	10000	331	287	44		
30	10000	309	287	22	1.68	1.7
60	8000	227	197	30		
30	8000	212	197	15	1.58	2.2
60	6000	141	125	16		
30	6000	133	125	8	1.65	1.65
60	4000	72.2	64	8.2		
30	4000	68.1	64	4.1	1.69	1.8
60	2000	22.1	19.7	2.4		
30	2000	20.9	19.7	1.2		

intercept of this curve upon the vertical axis represents the hysteresis loss. A dotted straight line has been drawn through the points for 30 and 60 cycles. We cannot be far wrong in assuming that the curve coincides with this at low frequencies, and consequently the method used for the separation of hysteresis and eddy-current losses is justified for thin sheets.

The falling off of the eddy-current loss at higher frequencies is explained by the consideration that the magnetizing force of the eddy currents reduces the flux in the center and crowds it towards the surfaces of the specimen; the short eddy-current

TABLE VII
VARIATION OF EXPONENTS WITH FLUX DENSITY
SPECIMEN *C*

Cycles	Flux density	Joules per cycle	Hysteresis	Eddy currents	Exponents	
					Hysteresis	Eddy currents
60	13000	0.3554	0.2872	0.0682		
30	13000	0.3213	0.2872	0.0341		
60	11000	0.2611	0.2119	0.0492	1.82	1.96
30	11000	0.2365	0.2119	0.0246		
60	9000	0.1868	0.1540	0.0328	1.59	2.00
30	9000	0.1704	0.1540	0.0164		
60	7000	0.1247	0.1047	0.0200	1.53	1.98
30	7000	0.1147	0.1047	0.0100		
60	5000	0.0741	0.0637	0.0104	1.48	1.93
30	5000	0.0689	0.0637	0.0052		
60	3000	0.0345	0.0307	0.0038	1.43	1.96
30	3000	0.0326	0.0307	0.0019		

paths enclose a smaller flux, while the longest still enclose the same; hence the average electromotive force of the eddy circuits does not increase as fast as the frequency.

Some experiments were made with thick copper sheets in a solenoid. On account of the low flux densities secured, small quantities of energy had to be measured and the accuracy was not high, but the results indicated that the eddy-current loss did not increase as rapidly as n^2, but somewhat more rapidly than B^2. The latter result is, however, somewhat doubtful.

By taking readings upon an ammeter in the magnetizing circuit, it is possible to compute the wattless component of

exciting current, and a curve of such values is plotted in Fig. 5 in relation to the flux density. Such a curve is just as valuable to the designer, or perhaps more valuable, than a magnetization curve obtained by the ballistic method. A similar curve for silicon-steel is plotted in the same figure. A ballistic curve which has been obtained by Dr. C. W. Burrows for a sample of silicon-steel from the same source, but from a different lot, is also shown but to a different scale. The low magnetizing current required by silicon-steel at low flux densities makes it par-

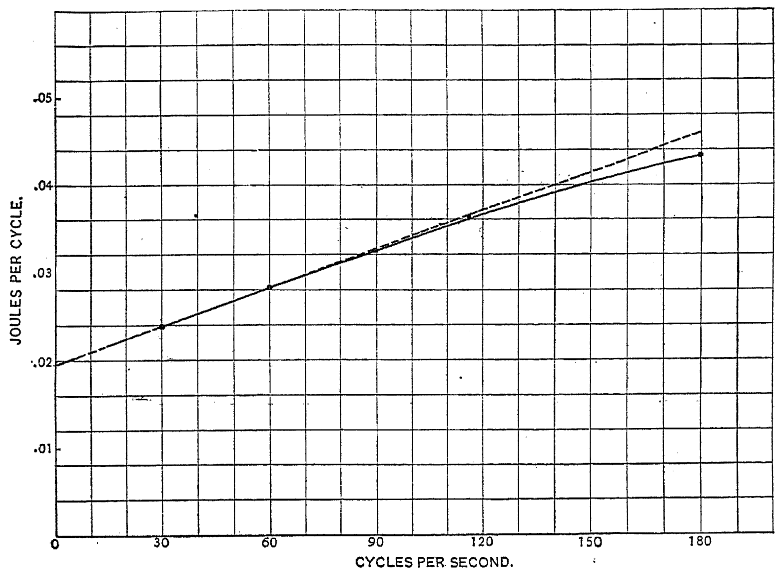


FIG. 4.—Transformer sheet No. 27 gauge at 5000 gausses.

ticularly suitable for current transformers which must have close regulation, as when used with measuring instruments.

That the hysteresis loss is larger when the steel is magnetized normal to the direction of rolling, than when magnetized parallel to the direction of rolling, is shown by Table IX and the curves of Fig. 6. The eddy-current loss in the two cases is practically the same, but the hysteresis is 5 to 10 per cent higher for magnetization normal to the direction of rolling. This specimen is of basic open-hearth steel, but the fact is typical of steel from all sources, though the magnitude of the effect is very variable. Table X shows a similar test upon steel from another source.

The authors have made tests of sheet-iron and steel from a great variety of sources, and have been surprised at the great range in quality of the material in general use by electrical manufacturers, the quality usually having no close relation to the price. Table XI shows some of the results obtained, all the materials having been secured from electrical manufacturers,

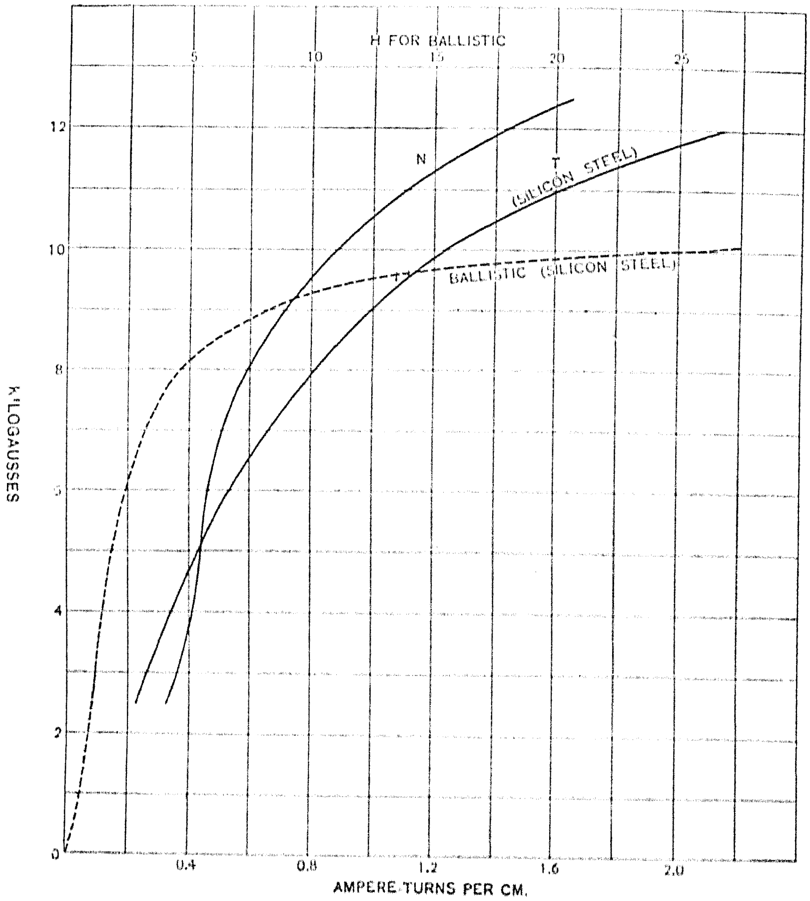


Fig. 5.—Magnetization curves.

or from iron mills and dealers supplying the electrical trade. Some foreign samples have been included in the table for comparison. The values of η have been computed from the relation $\frac{W}{n} = \eta B^{1.6}$; where $\frac{W}{n}$ is the hysteresis loss in ergs per cubic centimeter per cycle at 10,000 gausses.

The adjustments can be made and the readings taken so quickly after the circuit is closed, that the specimen becomes heated appreciably only when it is of poor quality or when extremely high frequencies or flux densities are used. The results consequently apply to room temperature. This varies from time to time through several degrees, but it has not been thought necessary to give the temperature in each case. The hysteresis

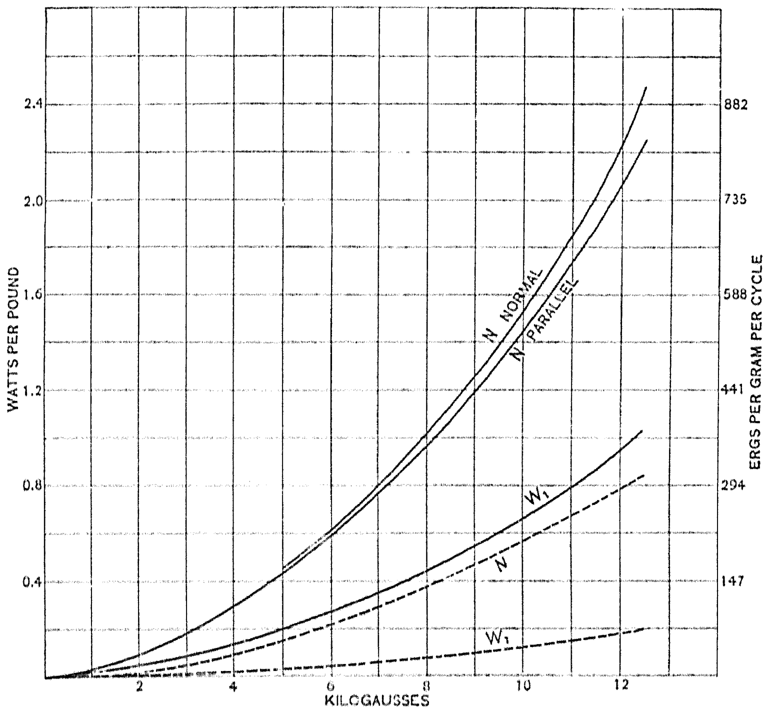


FIG. 6.—Relation between flux density and losses at 60 cycles when magnetized parallel and normal to direction of rolling. Solid lines represent total loss, dotted lines eddy-current loss, difference is due to hysteresis. N = ordinary steel; W_1 = silicon-steel.

varies only slowly with the temperature, and the eddy-current loss, which varies more rapidly, is the smaller part of the total, especially with silicon-steel.

Specimens P and P_1 should perhaps be classed as silicon-steels, although their silicon is not in the proportion which is typical of the alloy. Moreover, they are not put upon the market as an alloy steel, but are sold at about the price of

TABLE VIII

VARIATION OF EDDY-CURRENT LOSS WITH FREQUENCY

SPECIMEN C. $B = 5000$ GAUSSES. THICKNESS = 0.0422 cm.

Cycles	Joules per cycle	Hysteresis	Eddy currents observed	Eddy currents computed	Difference per cent
30	0.06844	0.06221	0.00623		
60	0.07467	0.06221	0.01246		
90	0.08074	0.06221	0.01853	0.01869	0.9
180	0.09626	0.06221	0.03405	0.03738	9.8

SPECIMEN P. $B = 5000$ GAUSSES. THICKNESS = 0.0437 cm.

Cycles	Joules per cycle	Hysteresis	Eddy currents observed	Eddy currents computed	Difference per cent
30	0.02385	0.01944	0.00441		
60	0.02826	0.01944	0.00882		
90	0.03249	0.01944	0.01305	0.01323	1.5
180	0.04329	0.01944	0.02385	0.02646	11.0

SPECIMEN A_1. $B = 3000$ GAUSSES. THICKNESS = 0.16 cm.

Cycles	Joules per cycle	Hysteresis	Eddy currents observed	Eddy currents computed	Difference per cent
30	0.0781	0.0510	0.0271		
60	0.1052	0.0510	0.0542		
90	0.1263	0.0510	0.0753	0.0813	7.4
180	0.1765	0.0510	0.1255	0.1626	22.8

TABLE IX

SPECIMEN N

MAGNETIZED PARALLEL TO DIRECTION OF ROLLING

Cycles	Flux density	Ergs per gram per cycle	Hysteresis	Eddy currents
60	10000	531	321	210
30	10000	426	321	105
60	7500	318	196	122
30	7500	257	196	61
60	5000	161	105	56
30	5000	133	105	28

MAGNETIZED NORMAL TO DIRECTION OF ROLLING

Cycles	Flux density	Ergs per gram per cycle	Hysteresis	Eddy currents
60	10000	562	352	210
30	10000	457	352	105
60	7500	332	208	124
30	7500	270	208	62
60	5000	167	110	57
30	5000	138.5	110	28.5

ordinary steel; hence they are classified as such. None of the samples analyzed showed more than the slightest trace of vanadium. Specimen Q contained 0.3 per cent of aluminum. For the chemical analyses we are indebted to Dr. H. C. P. Weber and Mr. J. R. Cain, of the Bureau of Standards.

Artificial aging has been practiced upon a number of the specimens, and consists in baking in an oven whose temperature is kept between 90 degrees and 100 degrees cent. The baking is interrupted only for the purpose of taking observations, which is done after the specimen has cooled to the room temperature.

TABLE X

SPECIMEN P_1

MAGNETIZED PARALLEL TO DIRECTION OF ROLLING

Cycles	Flux density	Ergs per gram per cycle	Hysteresis	Eddy currents
60	10000	576	302	274
30	10000	439	302	137
60	5000	168	96	72
30	5000	132	96	36

MAGNETIZED NORMAL TO DIRECTION OF ROLLING

Cycles	Flux density	Ergs per gram per cycle	Hysteresis	Eddy currents
60	10000	608	336	272
30	10000	472	336	136
60	5000	175	103	72
30	5000	139	103	36

The results are shown in Table XII, where the per cent change in total loss is given after various periods of aging. The time given is only approximate. The silicon-steels are almost entirely free from aging, but all the other specimens tested aged considerably, except L_1, and the test upon this was not continued for a very long period.

Caution must be exercised in applying the results of such tests, as one may be easily misled by them. Thus the hysteresis and eddy-current losses may be differently affected and, moreover, the hysteresis at different flux densities may be differently

TABLE XI

Designation	Thickness	Ergs per gram per cycle							
		10000 gausses				5000 gausses			
		60 cycles	30 cycles	Hysteresis	Eddy currents at 60 ~	60 cycles	30 cycles	Hysteresis	Eddy currents at 60 ~
Not annealed	cm								
A	0.0399	1785	1692	1599	186	608	585	562	46
B	0.0326	1290	1223	1156	134	420	402	384	36
C	0.0422	1274	1153	1032	242	426	391	356	70
D	0.0381	1193	1101	1009	184	401	377	353	48
Annealed									
E	0.0476	971	853	735	236	304	275	246	58
F	0.0280	766	716	666	100	247	233.5	220	27
G	0.0394	773	668	563	210	247	220	193	54
*H	0.0307	558	485	412	146	177.5	158	138.5	39
*H_1	0.0277	531	465	399	132	168	150	132	36
J	0.0318	543	442	341	202	166.5	139	111.5	55
*K	0.0282	518	456	394	124	162	146	130	32
*K_1	0.0280	541	479	417	124	170	152	134	36
†L	0.0346	565	473	381	184	175	150	125	50
†L_1	0.0366	615	516	417	198	192	165	138	54
B_1	0.0338	554	454	354	200	173	144.5	116	57
M	0.0335	550	461	372	178	173	150	127	46
N	0.0340	531	426	321	210	161	133	105	56
N_1	0.0312	523	435	347	176	162.5	137.5	112.5	50
P	0.0437	518	426	334	184	157	132	107	50
P_1	0.0470	576	439	302	274	168	132	96	72
Silicon steels									
†Q	0.0361	357	330	303	54	113	105.5	98	15
†Q_1	0.0366	390	360	330	60	124	117	110	14
R	0.0315	330	309	288	42	104	98.5	93	11
S	0.0452	350	314	278	72	108	99	90	18
T	0.0338	310	280	250	60	96	87	78	18
U	0.0346	312	291	270	42	98	92	86	12
U_1	0.0325	322	300	278	44	101	94	87	14
*V	0.0310	298.5	275	251.5	47	92	85.5	79	13
*V_1	0.0297	303	280	257	46	93	87	81	12
*W	0.0305	240	218.5	197	43	74.7	68.5	62.3	12.4
*W_1	0.0311	241	218.5	196	45	74.7	68.9	63.1	11.6
X	0.0430	265	232.5	200	65	80.8	72.5	64.2	16.6

* German.
† English.

TABLE XI.—Continued.

Designation	x	y	η	Per cent silicon	Watts per pound at 60 cycles and 10000 gausses: Eddy-current loss for gauge No. 29 see note	Hysteresis	Total
Not annealed							
A	1.51	2.02	0.0049		0.41	4.35	**4.76**
B	1.59	1.89	0.00358		0.44	3.14	**3.58**
C	1.51	1.79	0.00319		0.47	2.81	**3.28**
D	1.52	1.94	0.00312		0.44	2.74	**3.18**
Annealed							
E	1.58	2.02	0.00227		0.36	2.00	**2.36**
F	1.60	1.88	0.00206		0.44	1.81	**2.25**
G	1.54	1.96	0.00174		0.47	1.53	**2.00**
*H	1.58	1.90	0.00127		0.54	1.12	**1.66**
*H_1	1.60	1.87	0.00123		0.60	1.08	**1.68**
J	1.62	1.88	0.00105	0.0	0.70	0.93	**1.63**
*K	1.61	1.90	0.00122		0.54	1.07	**1.61**
*K_1	1.62	1.82	0.00129	0.4	0.55	1.13	**1.68**
†L	1.61	1.88	0.00118		0.535	1.035	**1.57**
†L_1	1.60	1.87	0.00129		0.515	1.135	**1.65**
B_1	1.61	1.81	0.00110	0.0	0.61	0.96	**1.57**
M	1.55	1.95	0.00115		0.55	1.01	**1.56**
N	1.62	1.90	0.00099		0.63	0.87	**1.50**
N_1	1.63	1.81	0.00107		0.64	0.94	**1.58**
P	1.64	1.88	0.00103	1.3	0.34	0.91	**1.25**
P_1	1.66	1.92	0.00094	0.7	0.43	0.82	**1.25**
Silicon-steels							
†Q	1.63		0.00094	3.1	0.14	0.825	**0.965**
†Q_1	1.58		0.00102		0.16	0.90	**1.06**
R	1.64		0.00089	3.4	0.15	0.78	**0.93**
S	1.63		0.00086	3.5	0.12	0.755	**0.875**
T	1.68		0.00077	2.8	0.18	0.68	**0.86**
U	1.66		0.00084		0.12	0.735	**0.855**
U_1	1.68		0.00086	3.9	0.145	0.755	**0.90**
*V	1.68		0.00078		0.17	0.685	**0.855**
*V_1	1.67		0.00080	3.8	0.18	0.70	**0.88**
*W	1.67		0.00061	3.4	0.16	0.535	**0.695**
*W_1	1.64		0.00061		0.16	0.535	**0.695**
X	1.65		0.00062	3.2	0.12	0.545	**0.665**

NOTE.—In order to make a fair comparison the eddy-current loss has been computed fo a thickness of 0.0357 cm. (gauge No. 29), assuming the loss proportional to the square of the thickness.

altered. In the present cases, measurements were also made at 5,000 gausses and at 30 cycles. Separation of the losses after aging indicates which component is active. It is found that the hysteresis is nearly always responsible for the increase, although the eddy-current loss may be either increased or diminished.

The decrease in eddy-current loss may sometimes mask the increase in hysteresis. Thus specimen Q_1 immediately exhibited a decrease of 14 per cent in eddy-current loss, amounting to 2 per cent of the total loss. After 250 hours the hysteresis increased one per cent at 10,000 gausses, and 2 per cent at 5,000 gausses, and yet the total loss still showed a decrease. None of the other silicon-steels showed any change in eddy currents, the slight changes in total loss being entirely due to increased hysteresis.

TABLE XII

PER CENT INCREASE IN TOTAL LOSS AT 60 CYCLES AND 10000 GAUSSES FOR DIFFERENT PERIODS OF AGING

Specimen	Time in oven					
	100 hr.	250 hr.	500 hr.	750 hr.	1000 hr.	2000 hr.
G	25	58	67	67	68	67
J	1	5	10	12	15	17
K_1		2	6	9	11	
L_1	−2	−1				
P_1		2	3	5		
Q_1	−2	−1				
R	0	0	0			
T		1	2	4		
U_1		0	0	0		
V_1		−1	0	0	1	
W		−2	−1	0	0	
X		0	0	3		

In the specimens G and J the eddy currents at first increased, but in the second thousand hours showed a marked decrease, which was sufficient to balance the steady increase in hysteresis. The result is a nearly stationary total loss. Specimen L_1 showed a slight initial decrease in hysteresis, and a later increase which was masked by decreasing eddy loss.

Specimen P_1 is a good example of the fact that the loss at different flux densities may be differently affected. The hysteresis throughout rose more rapidly at 5,000 than at 10,000 gausses, the increases amounting after 750 hours to 22 and 16 per cent respectively. This means that the law of variation of hysteresis with flux density has been changed. The exponent of B to which hysteresis is proportional has been diminished from $x = 1.66$ to $x = 1.60$. In the meantime the eddy currents have decreased, so that the increase in total loss at 10,000 gausses appears as only 5 per cent. Meanwhile, the total loss at 30 cycles and 5,000 gausses has increased 12.5 per cent. It is thus evident that an aging test should be conducted with measurements at the same flux density as that at which the material is to be used; otherwise the results may not apply to working conditions.

Discussion on "The Testing of Transformer Steel." Frontenac, N. Y., June 28, 1909

L. T. Robinson: As the subject of iron testing has been of great interest to me for a number of years I am prompted by Dr. Lloyd's paper to ask what sort of method is this that he presents? Is it to be a standard method? Is it to be a workshop method for ordinary use, or will it necessitate the sending of samples of iron to a testing laboratory? I do not know whether there is a similarity between the methods used or not, but I believe that all users of iron have methods for testing it that have proved satisfactory. By means of one method, at any rate, with which I am quite familiar we can make an apparently satisfactory test at a labor cost of something less then 5 cents per sample.

With the advent of German activity in iron, etc., it becomes necessary to know what the results obtained in Germany mean, and we must, therefore, interpret our results by means of the Epstein apparatus. From my point of view I think it exceedingly unfortunate at this time, so long after the present methods have become well established and familiar to those most interested, that we have to consider the adoption, or the use, or the interpretation of results obtained by still another method.

Will Dr. Lloyd explain why the Epstein method is not perfectly satisfactory as a commercial standard method, and why some modification of that method, which may mean an appreciably different result, should be introduced. I believe the objections to the lack of uniform distribution of the flux in the sample are fairly well met, by the introduction, in accordance with the German specifications, of the cardboard pieces in the corners.

As to his method being accurate within one per cent, I would say that we have always been accustomed to refer our tests to ballistic measurements and from them to judge whether or not any method is a good one. Now, a question arises, is it the intention to substitute Dr. Lloyd's method for the standard method of employing a ballistic galvanometer, or does the one per cent mean that ballistic measurements have been made on these identical samples, and that the results obtained by the two methods agree within one per cent?

It seems to me that before we adopt a new method it should be demonstrated on the basis of what we have always considered to be the standard method of test, that the old method is not good and that the new one is good. Or if the new method can not be so compared, something definite should be said to convince us that what we have always considered the absolute method, namely, suitable manipulation with ballistic galvanometer, is not correct and should not be used.

V. Karapetoff: Referring to the sketch, I see the advantage over the Epstein method of assembling the strips

on edge; but is not there a disadvantage in using separate corner pieces, making two joints instead of one joint as in the Epstein method? Could not a better joint be obtained by cutting the strips longer and bending them over the corner? This would do away with one of the joints, and, moreover, the joint would be made of the same material, instead of being made of a piece of iron that does not belong to the same sample.

I would also like to ask Dr. Lloyd with reference to Table XI, how he separated the hysteresis loss from the eddy-current losses? Did he assume any exponents beforehand, such as 2 for eddy currents and 1.6 for hysteresis, or did he determine the exponents experimentally from tests at different frequencies and different flux densities? As far as I am aware the values of the eddy-current loss and of the hysteresis loss are different, according to whether we assume the exponents beforehand, or determine them from the tests on the samples themselves.

C. E. Skinner: These tests have a degree of refinement rarely attempted in a commercial way, yet such refinement is necessary for the exact study of materials of this class.

After about two and one-half years' use of what is essentially the Epstein method we have found it to be the most satisfactory of any that we have tried. The insertion of bent corner pieces, as described in the paper, seems to me to be unnecessary in commercial work. The added accuracy of the corner-piece method when all necessary corrections are made, is unquestionable I think, but for commercial work the accuracy with the butt joints seems to be all that is really necessary, if inductions are kept relatively low. For inductions above 10,000 the joints may be lapped as in transformer work. We use a sample three or four times as large as that used by the authors of the paper, and the sample is of such width that material, if of satisfactory quality, can be used in commercial apparatus later, so that there is little or no actual loss of material due to the relatively large sample. While the strip is not as wide as that used by the authors we can detect little or no error from shearing provided the shear is kept sharp.

The accuracy claimed for the tests by the authors is within one per cent. I do not know the accuracy of our own method, but many checks lead me to believe it is within two per cent. On account of the ease and rapidity with which samples can be built and tested by our method I feel that it is to be preferred for commercial work where a large number of samples must be tested each day.

J. C. Lincoln: The first four samples *A*, *B*, *C*, and *D* have iron loss per pound given, and the annealed samples, which follow, have a very much decreased iron loss, sometimes as much as 50 or 60 per cent. I wish to inquire as to the method of annealing which gives this result.

Clayton H. Sharp: Is the wattmeter used in this test one of the ordinary commercial type, or is it something else? Is

any special provision made to maintain the temperature of the iron constant, or to determine the temperature of the iron, and how is it done?

Andrew Pinkerton: In testing iron as it is received we usually want the result of a large part of one or more whole sheets and not of a small sample, as the user of the material is interested in the results from whole sheets and not from small samples. That is my main objection to Dr. Lloyd's method.

Clayton H. Sharp: Regarding a remark of the latest speaker, I would say that Dr. Epstein once remarked to me that he had found that any test of iron made on a small sample had no value whatever.

E. E. F. Creighton: Some years ago we made tests of samples of iron cut from large sheets, and found a great difference in the result, depending on what part of the sheet the sample was cut from. The gases in the furnace have a great effect on the annealing of the iron. I have no doubt also that small differences in the temperature affect the results.

M. G. Lloyd: In regard to Mr. Robinson's question of the method itself and the comparison with other methods: the method that we have developed, it is true, is not quite as rapid as some of the workshop methods already in use, and it is not intended that it will replace them. For factory purposes it is often sufficient to use a method which is perhaps not so accurate, but which will serve when simply comparative results are desired, to determine the uniformity of material, or something of that kind.

Regarding the results obtained, however, I wish to say that our aim was to develop a method that would give as high an accuracy as possible; in other words, to get what we might call absolute values, constants that would apply to the material, and would not depend upon any peculiarities of the method; for that reason we wanted wide strips and uniform flux densities, and some of the other conditions mentioned. It is true that the method described is very similar to the Epstein method, and yet it differs from it materially. At the Reichsanstalt in Germany the Epstein method was compared with several others by what was considered an absolute method, and it was found that on the whole the Epstein method gave very satisfactory results for the total loss at 50 cycles and 10,000 gausses, but that the separate losses due to hysteresis and eddy currents individually were not so close, that the one came high and the other low by three per cent, and that they very nearly balanced up in the total loss. Consequently, in making separations of that kind, as is desirable for many purposes, it is not safe to use a method which is so much in error. It is true the separation cannot be made with nearly the accuracy of the individual observations, because the result at one frequency is often in error in one direction and in the case of a second frequency it may be in the opposite direction, and the errors multiply in making the separation.

It probably would be desirable for the Institute to adopt some definite method which could be universally used and accepted by manufacturers and users of steel, in order that when tests are made they shall be under the same definite conditions, and consequently not lead to any disagreement as to quality or interpretation of results. I do not see any harm in the meantime, however, in developing in practice any number of methods, in order to have the best possible selection when any attempt is made to adopt some definite method.

As regards the comparison with the ballistic method, I do not think we can expect any alternating-current method to give results which will be the same as those given by a ballistic method. If we consider the different methods of taking ballistic curves—such as that known as the step-by-step method, by which we get a ballistic reading for each slight increment in flux, and a ballistic method in which a large change is made in flux from the tip of the hysteresis loop to another point—it is found that the two ballistic methods will not agree between themselves; discordant results are obtained according to the process adopted in carrying out the measurements.

Similarly with alternating-current measurements, it is still an open question whether frequency affects the value of hysteresis, if everything else remains the same. Some investigations in the past have appeared to indicate large differences, as much as 30 per cent, in the value of hysteresis, due to different frequencies. When differences in conditions produce such results, and iron is put through a hysteresis loop with alternating current, it is not fair to take any ballistic measurements and say that because the alternating-current measurement did not agree with them, that the alternating-current measurement is in error. I do not think the ballistic measurement is any criterion of the accuracy of the measurements with alternating magnetization.

The error in our method has been determined simply through a consideration of the separate sources of error and the accuracy of the instruments, and by a comparison of measurements under slight variation of conditions. When I state the accuracy as one per cent, I consider that the results, before the separation of losses, are to be depended on, that is, the constants of the material are known to that degree of accuracy.

Professor Karapetoff's comments on the joints in the corners are partly answered in the paper itself, in the part which I did not take time to dwell upon. Ordinarily, the corner pieces are of the same material as the test pieces, but we have found it is not necessary that they should be. One can get sufficiently good results for all commercial purposes by using stock corner pieces, instead of making a set of corner pieces for each specimen, and for that reason, in practice, we use them very often in that way.

The objection to the second joint does not seem to be justified

by the results, because we find the difference in flux between the center and the end of the test piece to be very small indeed, and consequently any objection to the two joints at the corner is not substantiated.

The exponents have not been assumed in any case, as was intimated. The exponents are determined by obtaining values for hysteresis or eddy currents at two different flux densities, consequently derived from entirely independent measurements. The values of these two losses at any one flux density are determined by runs at two frequencies. Two observations at one flux density determine the separation of hysteresis and eddy-current losses. Then the values taken at two different densities are sufficient to determine the exponent which applies to these two particular flux densities.

In regard to the question on annealing, I may say that we have not done much of that. The specimens we have tested have in almost all cases been tested in the condition in which we received them, and the question of annealing was one of past history with us.

Dr. Sharp's question in regard to wattmeter and temperature conditions is answered in full in the paper.

As to the quantity of material, discussed by several, that is a point on which every person, I take it, must have his own opinion. Some think that the results of tests on samples are not of any value, and the test must be made on the entire material under question, and others seem to have found it satisfactory to use small test specimens. My own opinion is, that once a sufficient quantity is selected to represent a fair average of the material, if the method demands more beyond that, it is an undesirable feature of the method. The Epstein method, as usually employed, requires 10 kg. or 22 lb. of iron. What I take to be slightly better results are obtained with as small a quantity as 4 lb., and I think that is an advantage. Of course, if a person wants to test a whole sheet or larger quantities, it is easy to adapt the apparatus to the larger quantity. The question has been to find the apparatus which can be adapted to smaller quantities. With a method available for small quantities, one can always adapt it to one's own needs for large quantities.

I think I may be permitted at this time to call attention to the magnetization curves in Fig. 5, showing the wattless component of exciting current plotted against flux density for silicon-steel and ordinary steel. At low flux densities the silicon-steel is more highly permeable than the other, whereas at higher flux densities it is less so.

In regard to the effect of the higher magnetizing current, I wish to say one thing. It is of course undesirable for some reasons to allow the exciting current to run too high, and that has limited the design of transformers with this silicon-steel in one direction, that of pushing the flux density too high. It has been

very generally thought, however, that the regulation of the transformer would be affected by a high magnetizing current. I wish to speak of that, because it is an error which is commonly made. Most of the formulas which we see in print for the regulation of the transformer involve magnetizing current. As a matter of fact, magnetizing current has nothing whatever to do with regulation, and any objections to high magnetizing current in transformers are not along the line of inferior regulation. The reason for that is, there is a certain voltage drop in the transformer, due to magnetizing current, but it comes in both at no load and at full load, and it can be easily shown that it does not affect the regulation.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

A SKETCH OF THE THEORY OF THE ADJUSTABLE-SPEED, SINGLE-PHASE, SHUNT INDUCTION MOTOR

BY F. CREEDY

Up to the present time comparatively little attention has been devoted to single-phase motors having shunt characteristics, as they have been completely overshadowed by the series type, on account of its utility for traction purposes. Nevertheless, these shunt-wound machines have many advantageous features and should have a considerable field of utility. A certain amount of work has been done on them by Milch in this country and by Latour, Fynn and Punga in Europe, and considerable success has been attained in compensating the machine, that is, in raising the power-factor to unity.

The object of the present paper is to discuss the variation of the speed of these machines from synchronism. The usual impression, at any rate until recently, has been that the shunt type of machine must necessarily run at a speed in the neighborhood of synchronism. This is not true; theoretically, machines can be built to run at any speed, though in practice there are difficulties in running below synchronism.

First of all, it will be as well to describe briefly the general principles on which all single-phase shunt motors work. All practical machines of this class are developments of the machine shown in Fig. 1, usually known as the Atkinson commutator induction motor. This consists of a slotted stator or primary with a uniform air-gap all round, equipped with a single distributed single-phase winding, and a drum-wound rotor fitted with a commutator, as in a direct-current machine. On the commutator rest four brushes or groups of brushes (in a two-pole machine); the axis of one " pair " of brushes usually lies

parallel to that of the primary winding and that of the other perpendicular thereto, but this position is not essential. At standstill the primary coil merely induces a current on the $Y\ Y$ axis (Fig. 1) and hence there is no starting torque. But as the motor speeds up, a current is induced along the $X\ X$ axis which produces a flux along that axis. This current and the flux it produces are nearly in phase with the impressed electromotive force and hence in quadrature with the flux along $Y\ Y$. Therefore, the flux along $Y\ Y$ (the primary flux) can produce no torque in combination with the current along $X\ X$. In the neighborhood of synchronism, however, the current along $Y\ Y$, or the load current, is nearly in phase with the flux along

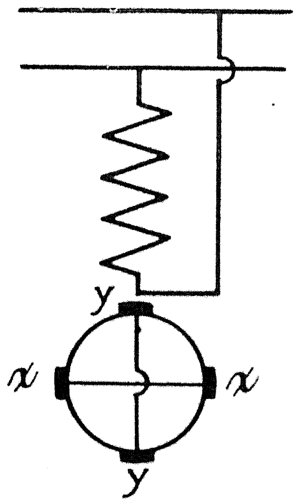


Fig. 1.

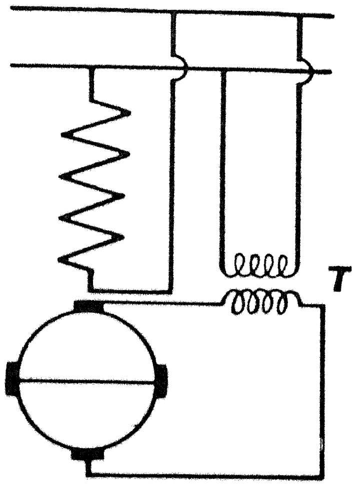


Fig. 2.

$X\ X$; hence there will be a torque due to these two in combination.

The flux along $X\ X$ may properly be regarded as the "field" flux of the motor, since it is the flux to which the torque is due. Similarly the current along $Y\ Y$ may be regarded as the armature current, since it has the same functions as this current in a direct-current machine, although, of course, it is induced from the stator, instead of being led directly in as in the continuous-current motor.

The electromotive force on the $X\ X$ axis is due to the movement of the rotor conductors through the primary flux, or flux

interlinking the primary circuit. It is, therefore, directly proportional to the speed. The flux along the $X X$ axis, which is determined by this electromotive force in the same manner that the primary flux is determined by the primary electromotive force, is therefore also directly proportional to the speed.

Such a motor reaches its limiting speed when the counter electromotive force, which may be represented by E_2, induced by the field flux (the flux along the axis $X X$) in the armature circuit along the axis $Y Y$ balances the electromotive force E induced therein by the primary flux; or, more exactly, when the vector difference, $E - E_2$, between these two is in quadrature with E_2, the counter electromotive force. This condition is clearly the same as that which limits the speed of the direct-current shunt-wound motor. Hence it would seem that the same methods which are available for varying the speed of the latter should be available here. There are two of these methods, (1) variable voltage control and (2) adjustable field excitation. To apply the first method to the single-phase induction motor we have merely to feed an extra voltage from the line into the $Y Y$ axis, say by means of a transformer as shown in Fig. 2, thereby increasing E. In order that the increased electromotive force E may be balanced by E_2 the motor must clearly run at a higher speed.

It might be thought at first sight that in order to increase E it is merely necessary to increase the primary flux, say by increasing the primary impressed electromotive force. This, however, is a fallacy, because the counter electromotive force E_2 is due to the rotation of the armature in the primary field and is therefore proportional to that field. Hence any variation of the primary field simply varies E and E_2 in the same proportion and the speed consequently remains the same. One may also remark that the secondary field flux becomes effectively equal to the primary flux at synchronous speed, and hence E_2 is effectively equal to E at that speed, which is practically the free running speed of the normal motor.

The above method of speed variation was first discovered by Mr. F. Punga, in 1905 and is probably on the whole one of the most satisfactory. It was subsequently reinvented by Mr. J. Perret, Mr. V. A. Fynn, the writer and others.

A second method, suggested by the writer, is to weaken the field along the $X X$ axis by inserting an inductance or a capacity in the $X X$ circuit, as shown in Fig. 3. An inductance tends to

raise the speed and a capacity to lower it. This method is precisely analogous to the speed variation of shunt-wound direct-current motors by field control, except that inductance or capacity must be used instead of resistance. The cause of this is that the circuit $X X$ is an almost purely inductive one, and consequently the magnetizing current which flows in it lags practically 90 degrees behind the electromotive force which produces it. It is not desired to alter the phase of the current, or of the flux it produces; hence in order to weaken it an inductance is inserted in the X X circuit, and to strengthen it, a

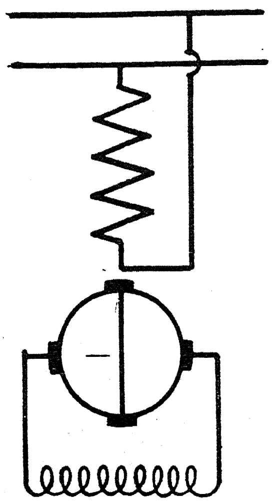


FIG. 3.

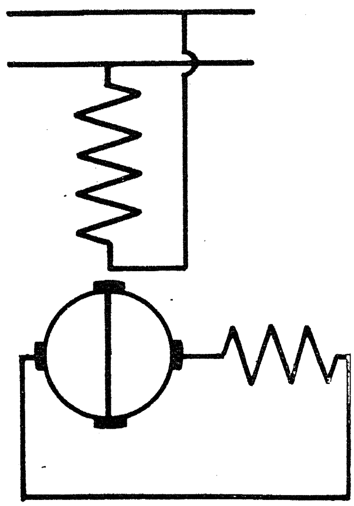


FIG. 4.

capacity. The use of resistance would tend to change the phase of the flux and impair the torque.

There is a third method which has no precise analogue in direct-current work. Clearly the flux necessary to balance the electromotive force E_1 induced in the $X X$ axis depends on the number of turns in the $X X$ circuit. Hence if a coil be put on the stator with its axis parallel to the $X X$ axis, and connected in series with the $X X$ brushes (see Fig. 4) it will either strengthen or weaken the flux along the X X axis, according as the magnetomotive force of this coil opposes or assists the rotor ampere-turns.

These three methods of speed variation are the only radically

distinct methods which may be applied to the single-phase induction motor pure and simple. They may, of course, be combined with each other, and with methods of phase compensation and of obtaining a starting torque, in an almost unlimited number of ways, as Messrs. Punga and Fynn have shown in their patent specifications dealing with the subject. Mr. Fynn has also invented a conductive type of machine in which the rotor circuit along the $Y\,Y$ axis is provided with a neutralizing coil, there being along the same axis a coil in shunt to the line, the sole purpose of which is to provide a flux capable of inducing an electromotive force on the $X\,X$ axis. He has also described a number of types in which the $X\,X$ axis is excited by means of a phase transformer, there being only one pair of brushes per pair of poles, but these types of machine do not come within the scope of the present discussion, which is confined to induction types.

Before deciding whether these methods of speed variation are really feasible and, if so, which is the most satisfactory, one must investigate the theory of the subject, as the introduction of the various voltages, etc., might so disturb the phase relations that some or all of the methods might be almost useless.

The clearest manner of describing the properties of an alternating-current motor is usually by means of the circle diagram. It is proposed, therefore, to describe the circle diagrams of the three types of motor described above, and to compare them with the well-known diagram of the ordinary single-phase induction motor.

It will be found that the deduction of these diagrams is quite complicated, as the properties of the motors seem to depend largely upon secondary reactions, which accordingly have to be taken fully into account. The most convenient way to calculate the position of the current circle is to determine three points upon it corresponding to three different speeds.

Discussing the motors in the order given above, the first thing we may remark is that if the electromotive force induced by the primary flux in the $Y\,Y$ axis be E volts and an electromotive force, E_8 be introduced, say, by a transformer, the increase of speed will *not* be in the ratio $\frac{E+E_8}{E}$, as it would be in a direct-current motor, because the field flux of the motor is directly proportional to the

speed instead of being constant. The change in speed therefore will be proportional to $\sqrt{\frac{E+E_3}{E}}$.

In the deduction of the circle diagram, the three most convenient points to take are (1) standstill, (2) synchronous speed and (3) the free running speed. The currents corresponding to these three speeds must be deduced

The deduction of the current at standstill presents no difficulty. If I_0 = magnetizing current of the machine or the current which the primary winding would take if the secondary were entirely open-circuited, and if I_s = standstill current, then

$$I_s = \frac{I_0}{\sigma},$$

σ being the dispersion coefficient.

The deduction of the other two currents is not so simple. Let E be the electromotive force induced by the primary flux on the $Y\ Y$ axis. Then on the $X\ X$ axis, at a speed k times synchronous speed, there will be an electromotive force k times as great and in quadrature with E; call this E_1. The electromotive force E_1 produces on the $X\ X$ axis a current I_2 proportional to itself. To this current the cross flux is due, and the electromotive force induced by this cross flux on the $X\ X$ axis or, in other words, the electromotive force of self induction due to this current, is represented in Fig. 5 as $I_2\ X$. This cross-flux produces, in its turn, owing to the revolution of the rotor, a back electromotive force on the $Y\ Y$ axis, which again will be k times as great as E_1. This opposes E and is the counter electromotive force of the motor. This is shown in the diagram (Fig. 5) as E_2. The resultant of E and E_2, shown as E_3, is the electromotive force which causes the current to flow through the local impedance of the $Y\ Y$ circuit. The object in drawing this diagram is to get at the electromotive force E_3. The magnitude of this electromotive force may be deduced more conveniently, however, by means of the simplified form of diagram due to Mr. Punga, shown in Fig. 6. First set out $O\ E$ as shown, and then cut off from E a length k times as great to represent E_1. This will be in quadrature with the true E_1 but will serve for the present purpose. Draw in $E_1\ A$ equal to $I_2\ R$ and $O\ A$ equal to $I_2\ X$, as before. Cut off from $O\ A$ a length k times as great to represent E_2. This vector

will have the correct phase relation, as may be seen by reference to Fig. 5, though it is shown for convenience with the opposite direction. Consequently, the electromotive force, E_3, is simply the vector difference between E and E_2. If the phase difference between E and E_2 were neglected, E_3 would be equal to $(1 - k^2) E$, since $E_1 = k E$ and $E_2 = k E_1 = k^2 E$.

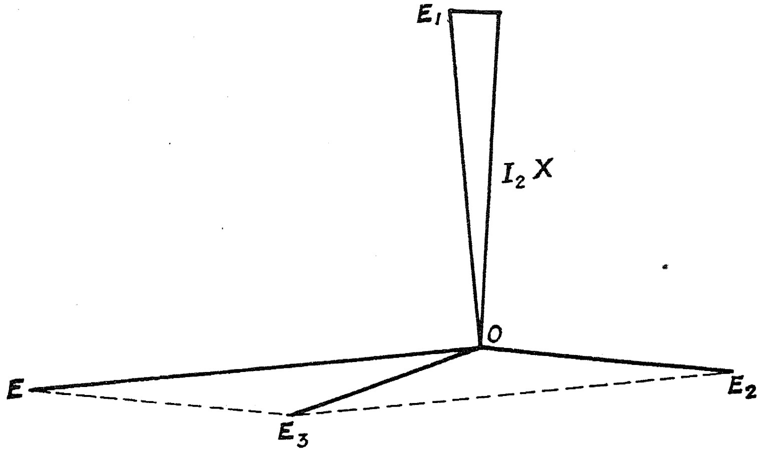


FIG. 5.

The action described above, however, is not the only one taking place. In addition to the main flux interlinking both primary and secondary along the $Y\ Y$ axis, there is the secondary leakage flux proportional to the current along $Y\ Y$, and interlinking the secondary only. Clearly all the arguments which apply to the

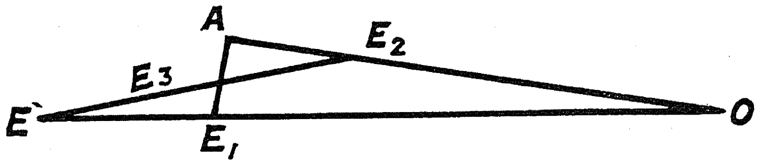


FIG. 6.

main flux must also apply to this secondary flux. Hence, a diagram may be drawn representing the effects of the leakage flux which will be precisely similar to the one of Fig. 6 Set out E_4, Fig. 7, to represent the electromotive force of self-induction along the $Y\ Y$ axis. Then the flux to which this is due will induce an electromotive force E_5 on the $X\ X$ axis k

times as great. Using the same convention as before, cut off from E_4 a length k times as great to represent this. The electromotive force, E_5, on the $X X$ axis will be balanced, as before, by a small supplementary current flowing on that axis, and producing a supplementary flux k times as great as that to which E_4 is due; $O A'$ is the electromotive force induced on the $X X$ axis by this flux. The same flux will induce on the $Y Y$ axis an electromotive force, E_6, k times as great as $O A'$.

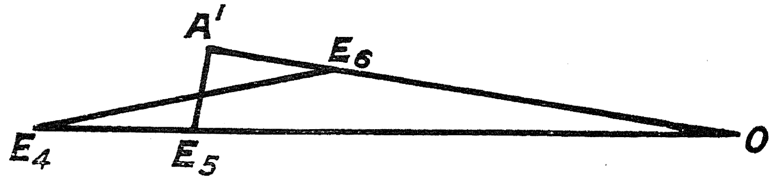


FIG. 7.

As before, we may say $E_5 = k E_4$ and $E_6 = k E_5 = k^2 E_4$, by neglecting the slight phase difference between E_4 and E_6. Then, since E_4 is the electromotive force of self-induction due to the current on the $Y Y$ axis, we may say:

In a single-phase induction motor running at a speed of k times synchronism, the electromotive force of self-induction due to a given current along the primary axis will have $(1 - k^2)$ times its value at standstill.

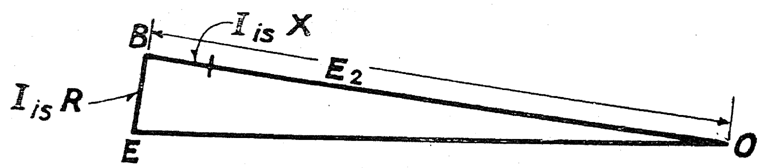


FIG. 8.

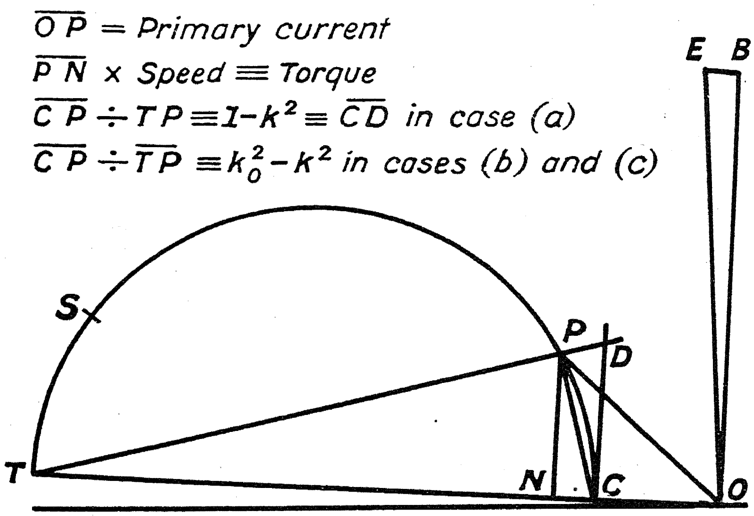
The slight phase difference above mentioned is strictly a second order effect and will be neglected in future.

In the polyphase induction motor it is well known that the rotor self-induction decreases with increasing speed, owing to the decrease in the frequency. The above investigation shows that a similar effect takes place in the single-phase motor.

With the aid of the foregoing diagrams it is not difficult to establish the circle diagram for the single-phase induction motor. Draw first the triangle of electromotive forces corresponding to

synchronous speed (Fig. 8). Here $\overline{OE}$ is, as before, the electromotive force induced by the primary flux and $\overline{OB}$ is equal to the counter electromotive force E_2 at synchronism.

The impedance of the circuit is $\sqrt{R^2+(1-k^2)^2X^2}$, as shown by the subsidiary diagram. At synchronism, then, when $k = 1$, the reactance vanishes. At any other speed, the current I_1 on the $Y\,Y$ axis may be resolved into two components, I_{1s}, the current at synchronous speed (equals $\frac{\overline{EB}}{R}$) and I_1', the remain-



Circle - Diagram of (a) the single phase induction motors; (b) the same with speed raised above synchronism by self induction, and (c) the same with speed raised by an auxiliary coil in the X X axis.

FIG. 9.

ing or load current; the first component may be considered present at all speeds. $\overline{OE}$ is also always present. Now $\overline{OE}+\overline{EB} = \overline{OB}$, and $\overline{OE_2} = \overline{OB}\times k^2$. Hence the resultant of $\overline{OE}$, $\overline{EB}$, and $\overline{OE_2}$ is $\overline{E_2B} = (1-k^2)\times\overline{OB}$, as may be easily seen.

Besides this we have $I_{1s}X\,(1-k^2)$ in quadrature with I_{1s}, that is, with $\overline{EB}$, and opposite in phase to $\overline{OB}$; this is a constant quantity at a given speed; $I_1'R$; $I_1'X\,(1-k^2)$.

The resultant electromotive force, then, which may be regarded as producing the flow of the current is $(\overline{OB}-I_{1s}X)\,(1-k^2)$ which may be called $E_7\,(1-k^2)$.

Hence,

$$I_1' = \frac{E_7\,(1-k^2)}{\sqrt{R^2+(1-k^2)^2\,X^2}} = \frac{E_7}{\sqrt{\left(\dfrac{R}{1-k^2}\right)^2+X^2}}$$

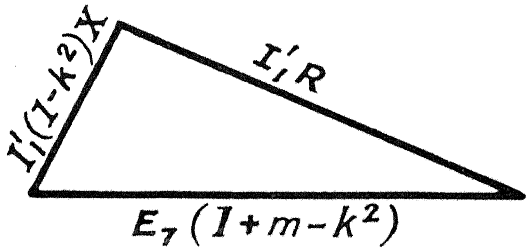


FIG. 10.

This is the equation for the current in a circuit of variable resistance and constant self-induction. It is well known that under such circumstances, if the electromotive force is constant, the current vector travels on a circle, and consequently it must do so here.

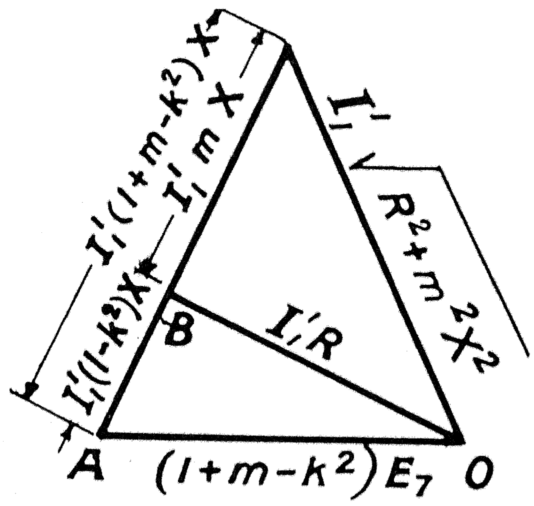


FIG. 11.

Fig. 9 shows the complete diagram, the electromotive force triangle $O\,E\,B$ being shown vertical so as to bring the circle into a more customary place.

From the foregoing it is easy to investigate the value of I_1' in the variable-speed motor, as I_{11} will be the same as before.

There is now another electromotive force $E_s = m\,E_7$. Assuming that it is in phase with E_7, we get $(1+m-k^2)\,E_7$ for the resultant electromotive force causing the current to flow. The three sides of the triangle of electromotive forces will then be as indicated by Fig. 10.

To prove that I_1' still moves on a circle, $I_1'\,(1-k^2)\,X$ may be written $I_1'\,(1+m-k^2)\,X - I_1'\,m\,X$, and this results in the diagram Fig. 11, which is self-explanatory.

Since the angle $\overline{O\,B\,C}$ is a right angle, $\tan\theta = \dfrac{R}{m\,X}$ and hence θ is constant.

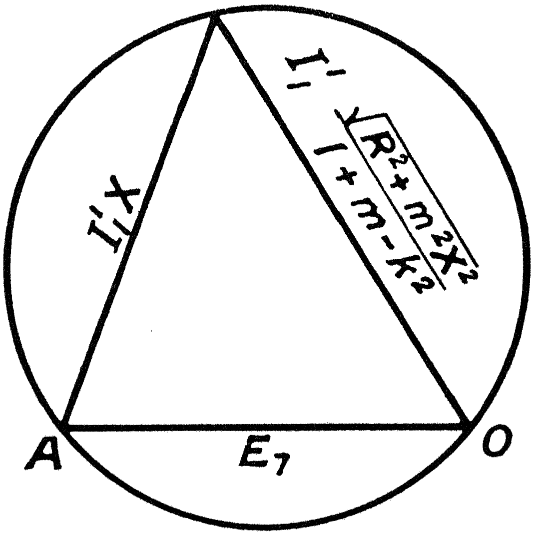


FIG. 12.

Since both $\overline{O\,A}$ and $\overline{A\,C}$ contain as a factor the quantity $(1+m-k^2)$, dividing all three sides by this will give the somewhat simpler values of Fig. 12.

Since $\overline{O\,A}$ and θ are constant while the ratio

$$\frac{\overline{A\,C}}{\overline{O\,C}} = \frac{(1+m-k^2)\,X}{\sqrt{R^2+(m\,X)^2}},$$

and therefore varies with the speed, it is clear that the point C must describe a circle. Since $\overline{O\,C}$ is proportional to the load current, it is evident that in the motor under consideration the secondary current vector moves on a circle.

The magnitude of the load current I_1' is given, at any speed, by the equation

$$I_1' = \frac{E_7 (1 + m - k^2)}{\sqrt{R^2 + (1 - k^2)^2 X^2}}$$

and when $k = 1$, then $I_1' = \frac{m E_7}{R}$; or, at synchronous speed the secondary load current is equal to the electromotive force fed in through the commutator, divided by the secondary resistance. The current at synchronism in this motor, therefore, is equal to the current taken by a single-phase induction motor at synchronous speed, together with a current E_s / R in phase with the electromotive force E_s.

The next step is to calculate the current at the free running

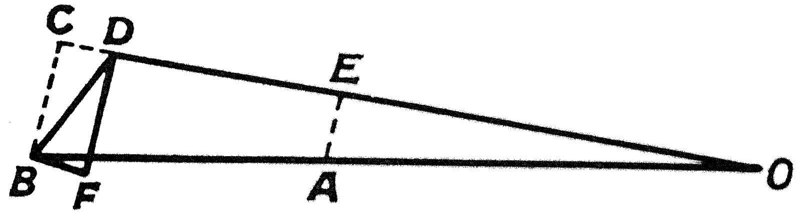


Fig. 13

speed. This speed occurs when the current on the $Y\,Y$ axis is in quadrature with the counter electromotive force E_y.

Fig. 13 is a new diagram in which $O\,A = E$, the electromotive force induced on the $Y\,Y$ axis by the primary flux. Then the triangle $O\,A\,E$ represents the conditions at synchronous speed. This triangle, as shown above, is identical with the electromotive force triangle for the $X\,X$ axis, and hence $A\,E$ is equal to $I_2\,R$ and is in quadrature with $O\,E$. If, however, an electromotive force $A\,B$ be added, making a total of $O\,B$, then, at a suitable speed, the counter electromotive force of the motor will rise to $O\,C$, and $B\,C$ will equal $\frac{\overline{O\,B}}{O\,A} \times A\,E = k^2 . A\,E$, since the speed is only proportional to the square root of the voltage. But it is only at synchronism that the rotor current is in phase with the vector difference between the impressed and

'unter electromotive forces. At the speed now under con-
tion the self-induction along the $Y\,Y$ axis comes into
We saw that at the speed $k \times$ synchronism the induct-
ilong this axis was $(1-k^2)\,X$, if X is its value at stand-
Above synchronism, accordingly, it will be negative, or in
words, have the effect of a capacity. Hence in the diagram
3, in which $\overline{B\,D}$ is the vector difference between the im-
d and the counter electromotive forces, $\overline{D\,F} = I_1\,R$ and
= $(1-k^2)\,I_1\,X$.
his diagram is to correspond to the free speed, $\overline{D\,F}$ must
rpendicular to $\overline{O\,D}$, since it is in phase with I_1, and in
that the torque may vanish I_1 must be in quadrature
$\overline{O\,D}$; $\overline{B\,F}$ is perpendicular to $\overline{D\,F}$ and therefore must be
el to $\overline{O\,D}$; $\overline{B\,C}$, as shown above, is also perpendicular to
Hence $B\,C\,D\,F$ is a parallelogram and $I_1\,R = \overline{D\,F} =$
: $k^2\,\overline{A\,E}$. Hence the secondary current on the $Y\,Y$ axis
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evidently will rise somewhat above its theoretical speed
load.

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$$I_s = \frac{I_0}{\sigma}$$

I_0 = magnetizing current of the machine or the current
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;sed electromotive force e while $\frac{E_s}{R}$ is in phase with it;
ɔre the addition must be made vectorially.

secondary current at the free speed was $k^2\,I_0$, and adding
imary magnetizing current, the primary current at the free
$= (1+k^2)\,I_0$.

Fig. 14 shows the circle diagram drawn through the three points calculated; here $O\,A$ is the standstill current, $O\,B$ that at synchronous speed, and $O\,C$ that at the free speed, $D\,P$ being the secondary current.

The torque will clearly be proportional to the product of the secondary current $D\,P$ and the flux along the $X\,X$ axis, multiplied by the cosine of the angle between them. This flux, like the counter electromotive force, is nearly in phase with the primary electromotive force, differing from it by a small angle due to the secondary resistance. It is shown in the diagram as $O\,F$ and is, of course, proportional to the speed. Hence if a line $D\,G$ be drawn perpendicular to $O\,F$ and passing through

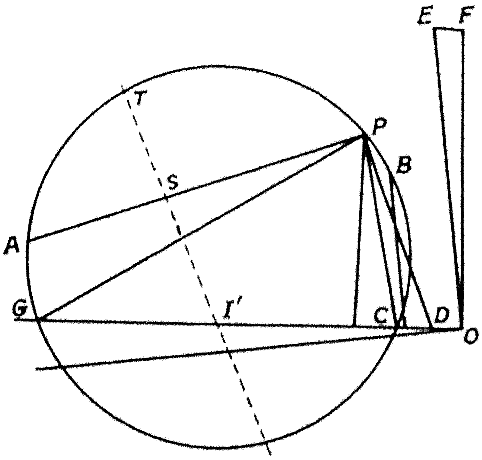

FIG. 14.

O_1D, we may say that the torque is proportional to $P\,N \times K$, the same construction as in the ordinary single-phase induction motor.

It is easy to show, morever, that $C\,P \div A\,P$ is approximately equal to $k_0^2 - k^2$ where k_0 is the free running speed $\div$ synchronous speed.

By setting out a line $S\,T$ so that the angle $A\,R\,T$ is equal to $A\,P\,C$, we get $k_0^2 - k^2$ as the intercept $Q\,R$ cut off on this line by the line $A\,P$. $S\,T$ will naturally be parallel to the tangent to the circle at C.

This is all we need know of the diagram in order to appreciate the characteristics of the motor.

Fig. 15 gives diagrams of three methods of connecting a commutator motor: (*a*) as a plain single-phase commutator induction motor with two pairs of short-circuited brushes; (*b*) as a single-phase induction motor with the speed raised above

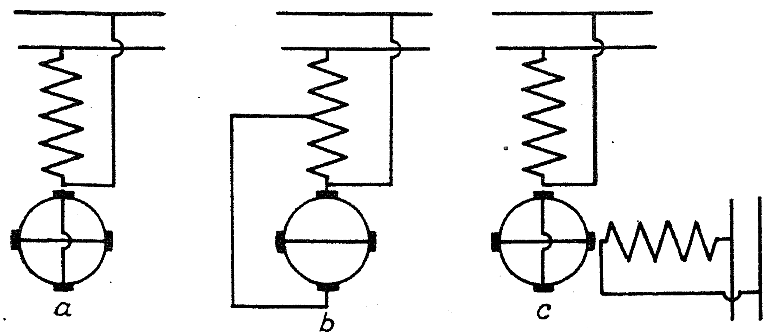

FIG. 15.

synchronism by Punga's method; (*c*) as a two-phase induction motor with two pairs of brushes short-circuited.

In Fig. 16, 1 is the well-known circle diagram of the single-phase induction motor; 2 is the better-known diagram of the two-phase motor, drawn out so that the current vector represents the

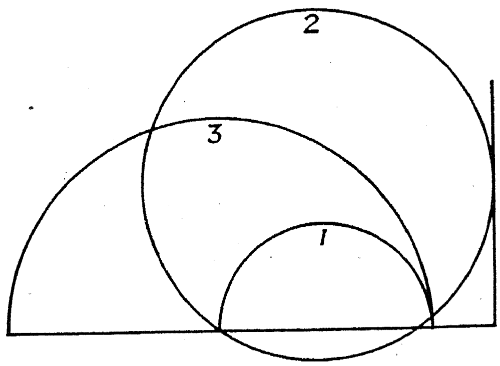

FIG. 16.

total current; that is, the arithmetical sum of the currents in the two phases. This is the only fair way of making a comparison. Under these circumstances the total no-load current of the polyphase motor will be the same as that of the single-phase machine, but the total short-circuit current will be doubled, giving the

diagram as shown. Diagram 3 is the circle diagram of the new motor. The no-load current is rather greater than that of the single-phase motor, while the short-circuit current is the same. The current at synchronous speed, $\frac{E_s}{R}$, is shown in the diagram.

It is clear that the shunt motor excels both the single-phase and the polyphase motors enormously as regards both overload capacity and power-factor. Moreover the overload capacity may be adjusted at the pleasure of the designer without changing the flux. In fact, this motor is more closely analogous to the direct-current motor in that the output is not limited by the break-down point but by heating and sparking. The power-

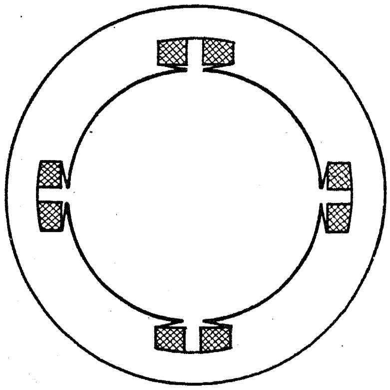

FIG. 17.

factor also may, of course, be controlled by the same means as the break-down point viz. by varying E_s.

The principal lesson which the comparison of circle diagrams teaches, then, is that in this type of motor the power-factor and overload capacity are independent of the leakage coefficient, which determines both in the polyphase motor and the ordinary single-phase induction motor. This gives the designer a great deal of freedom in the choice of the number of slots, air-gap, etc. In fact, in the writer's opinion, it is probable that a motor of this type might be made to operate fairly well if built with salient poles, somewhat as shown in Fig. 17. Such a motor, no doubt, would be somewhat heavier than the usual distributed-winding type, and would probably encounter a good deal of prejudice at first. The dispersion coefficient would undoubtedly be very

high, say 0.25, but, as already shown, this affects neither the overload capacity nor the power-factor.

It is true that the theory given above does not take account of the primary leakage in detail, but it may be pointed out that if the primary current leads, then the electromotive force of self-induction due to this current tends to increase the primary counter electromotive force; that is, the electromotive force to be balanced by the main or air-gap flux will be greater than the impressed electromotive force. It can very easily be arranged to make the primary current lead, so primary leakage need not cause trouble.

A good many difficulties would, no doubt, have to be met before such a radical departure from present-day practice could be successfully introduced. However, such a motor would be so simple and robust, almost comparing with a direct-current machine, that the introduction would probably be well worth the effort. It would also present possibilities of standardization probably excelling those offered by any other type. To adapt a given frame for different speeds, frequencies (within limits), voltages or horse-powers (also within limits), all one has to do is to change the coils, since the armature is standard, not being connected to the mains, and the speed can be changed without changing the number of stator poles. However, the salient-pole machine is at present a speculation, as one naturally would try a machine with a distributed winding, first it would probably be worth while, even in a standard machine in which speed variation is not required, to run the machine at, say, 7 per cent or 8 per cent above synchronism on account of the very much increased power-factor and overload capacity thereby obtained. Such a machine would then drop to about synchronous speed at full load, and one should get improved commutation as well as the other advantages mentioned. All this holds in the case where the motor runs above synchronism.

If this method of speed variation be employed to reduce the speed, very different results are obtained. In this case, the electromotive force E_g is reversed, and the circle is as shown in Fig. 18, where $\overline{OC}$ = no load current, $\overline{OA}$ = short-circuit current, and $\overline{OB}$ = current at synchronous speed. In this diagram the motor part (that above $\overline{AC}$) almost disappears. Hence the machine will not carry any load, and the conclusion is that this method is not suited for reducing the speed below synchronism.

THE USE OF SELF-INDUCTION

We now come to the second type of motor in which the speed is varied by inserting self-induction in the "field" or $X\ X$ circuit. A simple modification of the diagram used before will serve to elucidate the properties of this type of motor. In this diagram (Fig. 19) we have E as before, but the speed is now above synchronism. Hence E_1, the voltage on the $X\ X$ axis, will be greater than E, as shown. This electromotive force is balanced, as before, by $I_2\ R$ and $I_2\ (X_1+X_0+X)$, R being the secondary resistance and X_0+X+X_1 the total secondary reactance. This must now be divided into two parts, the reactance X_0+X, due to the flux along $X\ X$ and the reactance X_1 which is external to

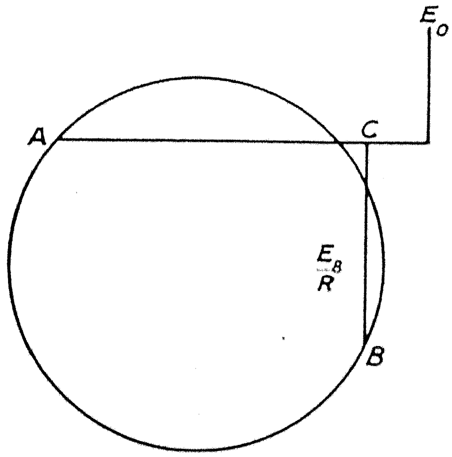

FIG. 18.

the motor. The counter electromotive force E_2 will, of course, be k times $I_2\ (X_0+X)$ only. This is shown in the diagram, and the free speed will occur as before when the current on the $Y\ Y$ axis is in quadrature with E_2.

Now, following through the various reactions due to the leakage flux along the $Y\ Y$ axis in exactly the same way as before, a diagram quite similar to Fig. 19 may be derived and it will be found that the reactance is balanced exactly at the free speed of the motor. Hence the current on the $Y\ Y$ axis must be in phase with the vector difference between E and E_2 at that speed. Thus the relations of transformer electromotive force, counter electromotive force, and current on the $Y\ Y$ axis are just the same in the new motor at its free speed, as in the ordinary

single-phase induction motor. The current, therefore, will have the same value; that is, the current on the $Y\,Y$ axis will be equal to that which exists on the $X\,X$ axis at synchronous speed. The diagram then is identical with that of the single-phase induction motor save that the free speed is different.

If k_0 be the ratio of free speed to synchronous speed, then the construction which in the ordinary motor gives $(1-k^2)$ will give $(k_0^2-k^2)$. The torque characteristics, overload capacity, etc., will be just as in the ordinary induction motor. Hence this machine will work best when compensated.

A capacity connected across the $X\,X$ circuit would clearly be just as effective in lowering the speed, but it is not much use going into the theory of this in detail, as the condenser required for such a method of speed regulation would be far too costly for practical use.

One caution is very important when testing these motors, or, indeed, any type of commutator induction motor. The presence

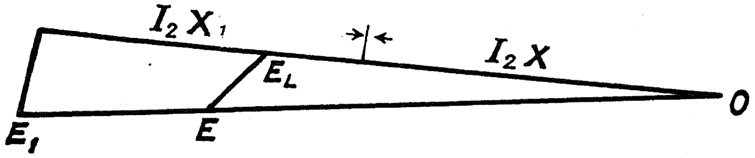

FIG. 19.

of four brushes on the commutator, opposite pairs being joined together, gives an opportunity for the circuit along $X\,X$ to differ in resistance from that along $Y\,Y$. If the $X\,X$ circuit has a high resistance, say, double that of the $Y\,Y$ circuit, then the difference between E and E_2 at synchronous speed will be double what it would be if the two resistances were equal; hence the no-load current on the $Y\,Y$ axis will be doubled and the power-factor of the motor correspondingly reduced. It is very important, therefore, to keep the resistance of the $X\,X$ circuit down, or very unfavorable conclusions may be drawn as to the power-factor of these machines.

We now come to the third type of motor, in which there is a coil parallel to the $X\,X$ circuit and connected in series therewith. Fig. 20 is the diagram for this case; it is quite similar to those already drawn. The voltage E_1 is larger than E, as before, because the speed is assumed to be greater than synchronism. This is balanced by $\overline{O\,E}$, the electro-

motive force of self-induction due to the flux along $X X$ interlinking the whole circuit along that axis, both on the stator and the rotor. $O E''$ is the electromotive force induced by the field flux in that portion of the $X X$ circuit which lies on the armature alone; of course, $\frac{E''}{E'} = \frac{T}{T_1}$, if T is the number of turns on the armature and T_1 that on the armature and auxiliary coil combined. The counter electromotive force, E_2, will be k times E''. The free speed occurs as before when $E - E_2$ is perpendicular to E_2.

An examination of the subsidiary diagram for the leakage flux will show that the self-induction in the $Y Y$ axis is also balanced at the free speed, since the diagram is identical with the one just given. Hence, as in the last type of motor considered, the current on the $Y Y$ axis at the free speed is equal to the stator magnetizing current, and, consequently, the stator

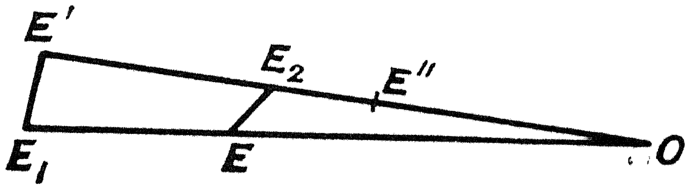

FIG. 20.

no-load current is equal to double this latter, just as in the ordinary single-phase induction motor. The circle diagram also is identical with that of the induction motor, as with the last type discussed, save that the speed construction gives $(k_0^2 - k^2)$ instead of $(1 - k^2)$.

The present method supplies probably the best method of lowering the speed, but its scope is limited by the following considerations.

Suppose it is desired to reduce the speed of a motor to half of the synchronous speed. In order to do this, the "field" flux must be twice that at synchronous speed. But in order to get twice the flux in the $X X$ axis, one-half the number of turns would be required, even if the voltage induced in the $X X$ circuit remained constant at all speeds. Since it is halved at half speed, only one-quarter of the number of turns required at synchronism will be needed to give double the flux, and since the required ampere-turns are twice those at synchronism, the

current must be eight times that at synchronism. The current in the $X X$ axis is proportional to the inverse cube of the speed. The decrease of speed obtainable by this method is limited, therefore, by the abnormal increase in the current on the $X X$ axis.

Unless the $X X$ brush gear is specially designed, probably a drop of 20 per cent in speed, which will double the current on the $X X$ axis, is as much as will be possible.

The foregoing investigation indicates that there is no thoroughly practical method of lowering the speed, although there are several good ways of raising it.

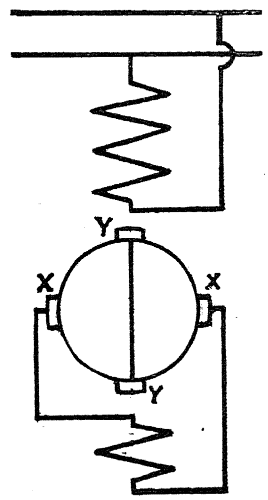

FIG. 21.

Before leaving the theory of these motors, it will be convenient to consider briefly the theory of phase compensation as applied to the single-phase induction motor, since in many cases phase compensation produces a great improvement. A diagram of connections of a compensated motor is shown in Fig. 21.

In order to avoid constructing an entirely new theory of this motor, the best way to consider it will be as follows: Add along the $X X$ axis an electromotive force, E_0^1, of constant value and in phase with the impressed electromotive force. Here E'_0 may be regarded as superposed on the others and as producing its own effects undisturbed by any of the other actions going on in the motor. This will give rise on the $Y Y$ axis to another

electromotive force in quadrature with it and proportional to the speed, and this second electromotive force will lead E' by 90 degrees if the compensating coil be correctly connected. The current due to this electromotive force will be determined by the resistance and inductance of the $Y\ Y$ circuit in the usual manner; in fact, this current will be

$$I_3 = \frac{k\ E_0{}^1}{\sqrt{R^2 + (1-k^2)^2\ X^2}}.$$

It is easily proved that if a constant electromotive force be applied to a circuit of constant resistance and variable self-induction, the current vector will move on a circle, as in Fig. 22. If it were not for the factor k in the numerator, I_3 would clearly flow in such a circuit, the resistance being R and the inductance $(1-k^2)\ X$, and therefore varying with the speed, but in the present

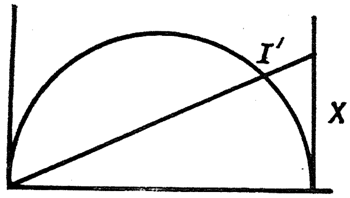

FIG. 22.

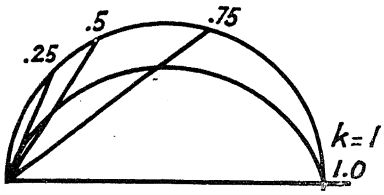

FIG. 23.

case the values obtained from such a circle have to be multiplied by k as shown by the equation, giving the diagram shown in Fig. 23. This current then has to be added to the ordinary induction-motor current to obtain the current in the compensated motor, and the diagram of Fig. 24 is obtained. The very simple theory on which this diagram is based is also the basis of the treatment of the ordinary single-phase induction motor and will be found in accord with the more complex theories which have been published from time to time, in which the compensated motor is treated independently.

The very considerable increase in the overload capacity is clearly shown by Fig. 24; it usually amounts to about 40 per cent. The locus of the current vector is now not strictly a circle, but for the purpose of calculating the overload capacity it may conveniently be assumed to be such.

In order to get satisfactory compensation, there must be a current on the $Y\ Y$ axis at synchronism capable of producing

a flux along that axis sufficient to balance the primary electromotive force, thereby rendering the primary magnetizing cur rent unnecessary; therefore, the normal current on the $Y\,Y$ axis must be reversed, and in order to do this, a current of twice the magnitude and of opposite direction must be superposed on it.

At synchronism the compensating current on the $Y\,Y$ axis is $\frac{E^1}{R^1}$, as shown above, E^1 being the compensating electromotive force. For good compensation, $\frac{E^1}{R} = 2\,I_0$, or $E^1 = 2\,I_0\,R$. This furnishes a means of getting an idea of what the compensating electromotive force should be, but the best value can really only be found by experience.

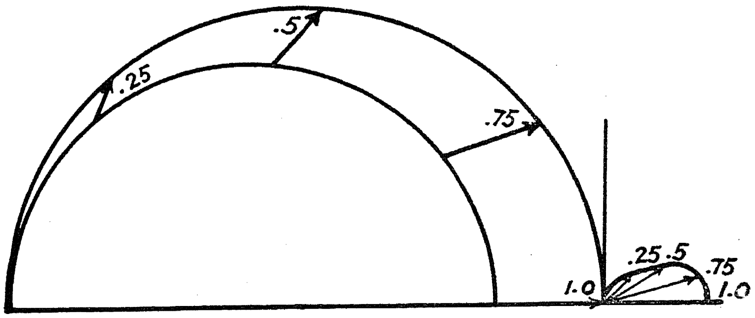

FIG. 24.

COMMUTATION

The commutation of the various types of motor described above must be discussed before one can decide that they are entirely feasible. Consider first the commutating coils under the $Y\,Y$ brushes. There will be three electromotive forces induced in these: the reactance voltage, proportional to and in phase with the current; the " electromotive force of rotation " or that induced by the commutating coil cutting the flux along the $Y\,Y$ axis, and the " transformer electromotive force " or electromotive force induced by the alternation of the flux along the $X\,X$ axis, which flux clearly threads the commutating coil. The flux along the $Y\,Y$ axis also induces an electromotive force of rotation on the $X\,X$ axis similar to that induced in the commutating oil. This electromotive force, obviously, will differ from thatc

induced in the commutating coil merely by the ratio of transformation between the rotor and the commutating coil.

The flux along the $X X$ axis induces a similar transformer electromotive force in the $X X$ circuit, likewise differing from that induced in the commutating coil merely by the transformation ratio. The two electromotive forces induced in the $X X$ circuit balance one another at every speed, of course, and it therefore follows that the similar electromotive forces induced in the commutating coil do so also. This leaves only the reactance voltage, which exists also in the direct-current machine.

We may, therefore, say that as long as the $X X$ brushes remain short-circuited, the commutation under the $Y Y$ brushes is the same as that of a direct-current machine. Now the $Y Y$ brushes carry the main current while the $X X$ brushes only carry a small magnetizing current, so that when speed variation is obtained by voltage control we may expect good commutation. If, however, extra turns or a reactance coil be inserted between the $X X$ brushes, the difference between electromotive force of rotation and transformer electromotive force will be equal to the actual external voltage between the $X X$ brushes multiplied by the ratio of transformation between rotor and commutating coil.

Now, consider the commutating coils under the $X X$ brushes. The axis of these coils is parallel to the $Y Y$ axis, just as the axis of the coils under the $Y Y$ brushes was parallel to the $X X$ axis. The "electromotive force of rotation" and transformer electromotive force induced in these coils then correspond to the induced and counter electromotive force, E and E_2, discussed above. These, we saw approximately balanced one another at the free speed in the simple induction motor. Hence, it may be considered that in the simple commutator induction motor, the commutation under the $X X$ brushes is like that of a direct-current motor at and about the free speed. If, however, an external electromotive force, E_8 be inserted in the $Y Y$ axis, say, such as to increase the induced electromotive force. E, then the counter electromotive force $E_2 = E + E_8$ or $E_2 - E = E_8$. As before, the resultant of the transformer and rotation electromotive force in the commutating coil on the $X X$ axis is equal to $E_2 - E = E_8$ multiplied by the transformation ratio between the rotor and the commutating coil. We may, therefore, generalize from the statement made above. as follows:

The resultant induced electromotive force (excluding reactance voltage) in the commutating coils under two opposite brushes of a commutator induction motor is equal, near the free speed, to the electromotive force between the alternate pair of brushes multiplied by the transformation ratio between the rotor and the commutating coil; hence, a potential difference between any two opposite brushes will tend to make the alternate pair spark. To obtain a given rise of speed, therefore, with the least sparking voltage, one should have equal potential differences between both pairs of brushes.

The foregoing theory cannot claim any great elegance or high degree of perfection. It represents merely the writer's attempt to get a clear understanding of this type of motor so as to be able to design it with some certainty, and it is hoped will be received as such.

Experimental Results

A considerable amount of experimental work has already been done on adjustable-speed single-phase motors, both in this country and abroad, and there can be little doubt, I think, that a commercial machine will shortly be placed on the market either in Europe or here, or perhaps both. I propose to describe herein a number of tests which are at present at my disposal and to indicate briefly in what directions further tests are desirable.

Experiments of Mr. F. Punga and the Author

Motor No. 1. The first motor built was a small machine giving 1.25 horse-power on a circuit of 40 cycles and 220 volts. It was arranged as shown in the diagram, Fig. 4, having a cross winding divided into two parts so that three speeds were available; viz., synchronism and two speeds above synchronism. A compensating coil was also employed on this motor. The results of the tests on this machine for the three different speeds are shown in Fig. 25. It will be evident that the power-factor is very high, the slip from no load to full load considerable, and the efficiency comparatively low although tolerably good for so small a motor of this type. Constant compensation was employed at all speeds and, as the charts show, the power-factor remained very nearly constant, falling off slightly at the higher speeds. The commutation was thoroughly satisfactory at all speeds as, of course, it should be in so small a motor.

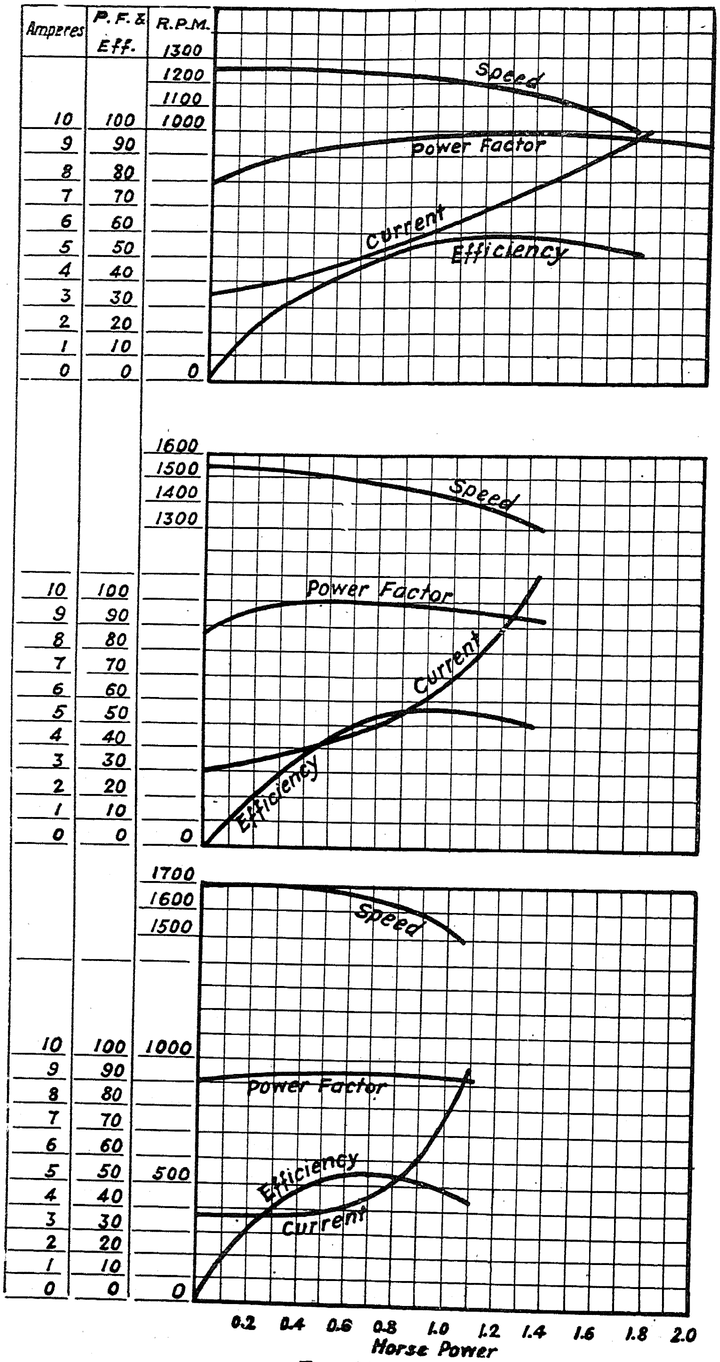
Amperes
P.F. & Eff.
R.P.M.
Speed
Power Factor
Current
Efficiency
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0
Horse Power

FIG. 25.

Below synchronism satisfactory results were not obtained, as a totally different compensating voltage would have been required. This could not be conveniently arranged for experimentally at the time. The results obtained by Dr. Alfred Fraenckel, however, given below, show that a considerable reduction in speed below synchronism is possible by suitable arrangements. In fact, his results below synchronism are superior to any that I have yet been able to obtain.

The results obtained on motor No. 1 were considered sufficiently satisfactory to justify further experiment and one or two motors were even built commercially for driving small printing presses and the like. These had a speed range of about 1.5 to 1.0 and a capacity about the same as that of the experimental motor.

Motor No. 2. This was a 5 horse-power machine operating at 200 volts and 50 cycles. It was arranged to obtain some data on the method of varying the speed by the use of self-induction. Some speed curves on this motor are given in Fig. 26. It was found that the use of inductance in the field circuit diminished the power-factor; hence variable compensation should be arranged for when it is desired to adjust the speed of a motor by this means.

The commutation of this motor was satisfactory at all loads and speeds, but the full-load efficiency averaged only 63 per cent to 65 per cent. The slip between no load and full load was high.

In spite of the comparatively poor results of tests on this motor the writer believes that the use of inductance in conjunction with a small adjustable compensating transformer offers one of the most promising means of varying the speed at our disposal. The two may be combined so that movement of the lever of the field-regulating controller simultaneously varies the inductance and adjusts the compensation. In this case the controller will be hardly more complicated than the regulating rheostat of a direct-current motor.

Motor No. 3. On this third motor a considerable number of tests have been made. This was a four-pole machine operating at 400 volts and 25 cycles. Arrangements were made for adjustable compensation in case it was needed, but it was finally found that very satisfactory results could be obtained without it. Arrangements were also made to try the method of speed variation both by armature voltage control and by field varia-

tion. The curves in Fig. 27 show the different speed curves, and those in Fig. 28 the power-factor at one-half, three-quarters and full load. A very wide speed range was covered, *viz.*, about $2\frac{1}{2}$ to 1 and it may be noted that the high slip characteristic of the older motors has been eliminated.

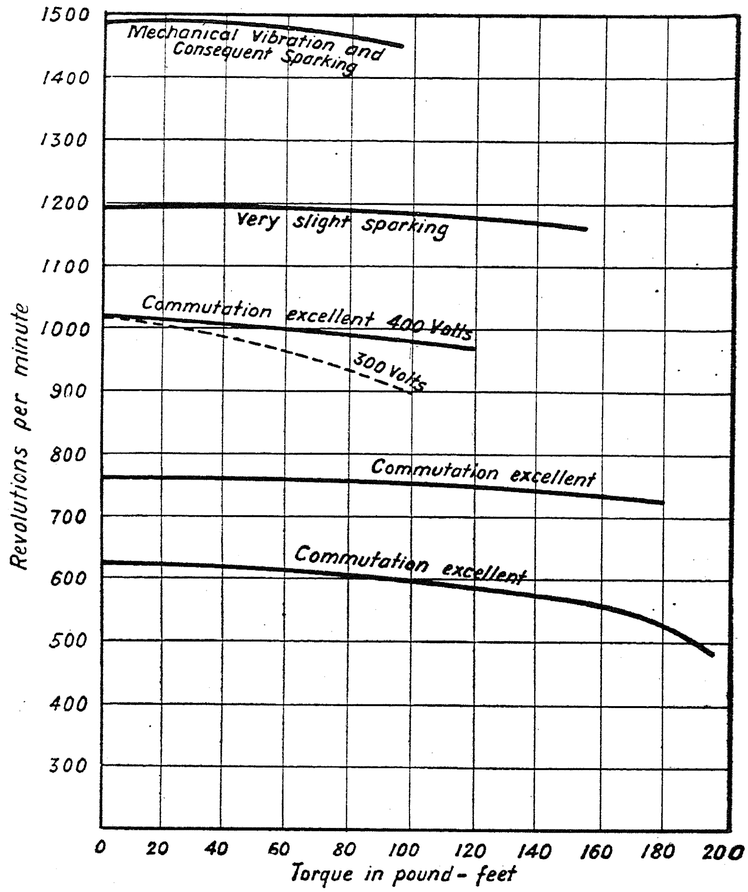

FIG. 26—Motor No. 3, 15 horse power, 400 volts, 4 poles, 25 cycles.

The upper limit to the speed was set in this case by the circumferential speed of the commutator, which, above 1400 rev. per min., gave rise to a good deal of vibration and consequent sparking. The lower limit was due to the abnormal increase of the current in the cross or magnetizing brushes which, below about 600 rev. per min., began to give rise to excessive heating and severe sparking

these brushes. A further reduction of the speed was ɿpracticable for another reason. Below synchronism the ɜ-turns of the auxiliary cross winding oppose those of the ⱱinding; hence the field circuit composed of these two

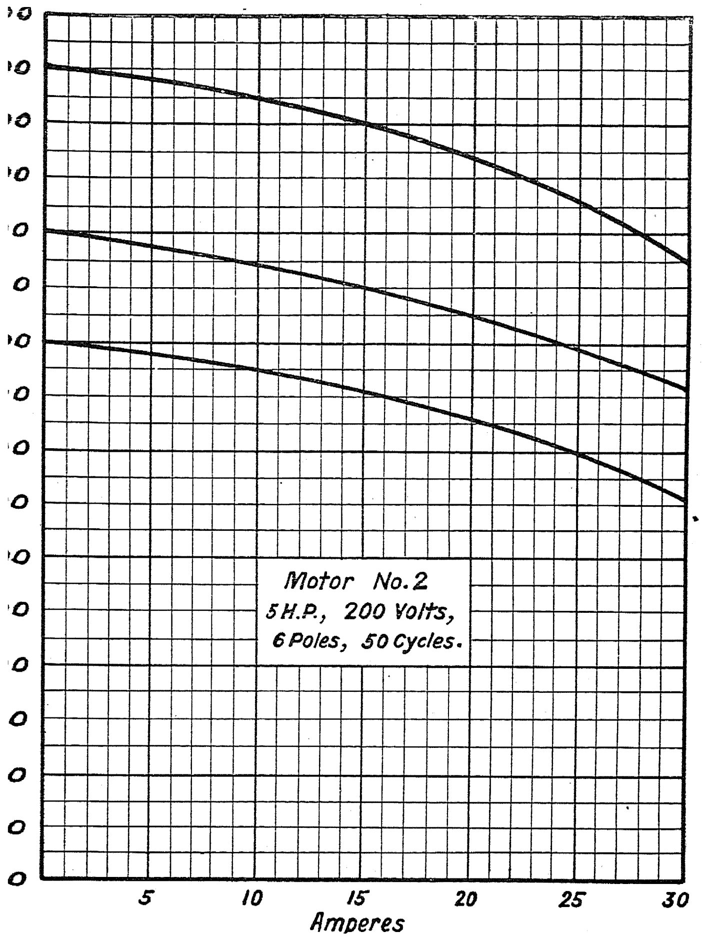

Fig. 27.

ɘs, has a comparatively low inductance. The correctness phase of the field flux depends upon the reactance of the ircuit being largely in excess of its resistance. At very eeds, this will no longer be the case and the torque will ɔo fall off.

The speeds between 600 and 1000 rev. per min. were obtained by field regulation while those above 1000 rev. per min. were obtained by using armature voltage control in addition. A constant compensation was used for all speeds except that of 600 rev. per min. where it was experimentally inconvenient to employ compensa-

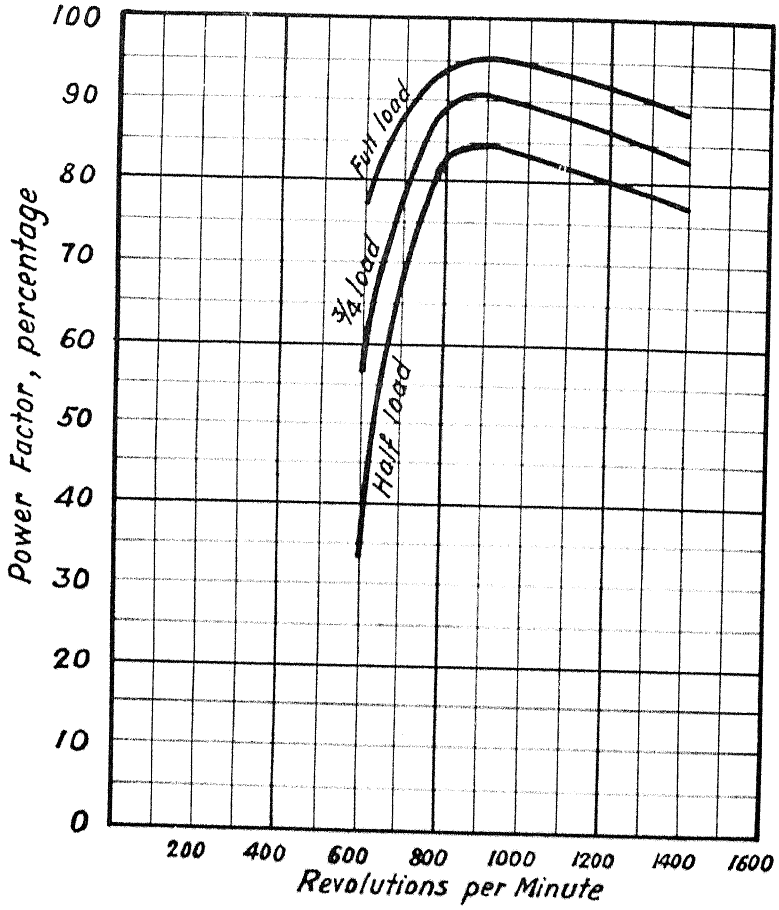

FIG. 28.

tion. This was adjusted so as to give somewhat less than the maximum power-factor as it was found that at high speeds the compensating voltage applied to the exciting brushes produced quite a noticeable effect on the commutation under the armature or load brushes. Hence, only an amount of compensation sufficient to give the results of Fig. 28 was employed. The

power-factor below synchronism could undoubtedly be brought up to that at other speeds by suitable compensating arrangements.

The efficiency of the motor has not yet reached a commercial figure, as it did not much exceed 70 per cent. Careful analyses of the losses were made and the low efficiency was found to be largely due to heavy $I^2 R$ losses in the brushes, a rather high brush friction and heavy iron loss This latter feature, however, is probably peculiar to the motor tested, which had open rotor slots, the winding being secured by numerous bands. Single-phase commutator motors, especially those intended for high frequencies, are never remarkable for their high efficiencies, but a recalculation indicates that the motor tested could be redesigned to have a full-load efficiency not below 78 to 80 per cent at any speed above synchronism, the maximum efficiency probably being about 83 per cent.

These tests, taken at various times and places in the intervals of other work, are sufficient, I believe, to show something of the possibilities of the machine. Some experiments by other workers which still further confirm the foregoing conclusions are, however, described below.

Experiments of Dr. Alfred Fraenckel

The author would like to call attention to the pamphlet of Dr. Alfred Fraenckel entitled "Der einphasige kompensierte Nebenschlussmotor" published in 1908, which contains a theoretical discussion of the compensated shunt induction motor running at synchronism and of the same motor when the speed is adjusted by means of a coil at right angles to the main primary winding. It also contains a number of experimental curves on both of these motors, of which I have taken the liberty of reproducing those relating to the adjustable-speed motor. This was a machine built for 8 horse-power at 200 volts and 50 cycles, and it ran at a synchronous speed of 1500 rev. per min. The armature was approximately 10 x 6¼ inches.

The motor ran sparklessly at all speeds represented by the curves in Fig. 29 to 32, except the very highest (1900 rev. per min. with no load) when there was considerable sparking at full load and overloads.

It will be seen from Figs. 30, 31, and 32 that the power-factor was very near to unity at all speeds, while the full-load efficiency varied between about 70 and 76 per cent.

For a 50-cycle motor this must be considered a very good performance. It will be seen that speed adjustment was obtained between about 1200 rev. per min. and 1900 rev. per min. or from

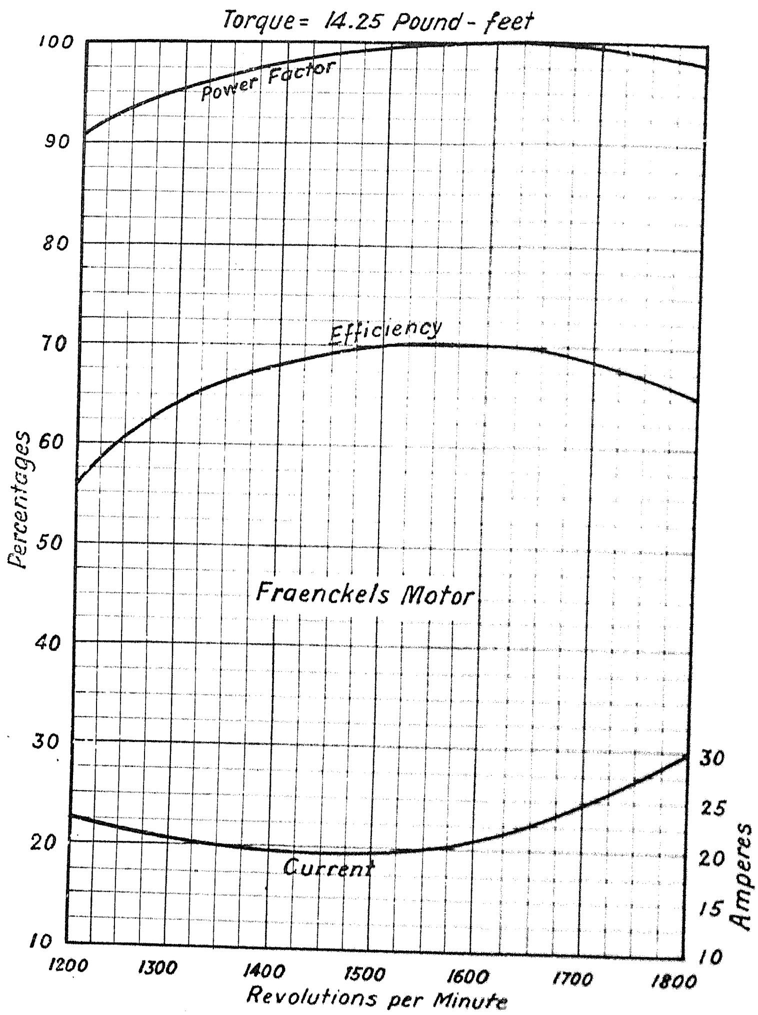

FIG. 29.

about 20 per cent below synchronism to 25 per cent above, a total range of 1.58 to 1.0. The percentage of reduction below synchronism agrees very closely with the writer's motor No. 3, while the latter, by the use of both field and armature control,

could be brought up to twice the synchronous speed before the sparking became prohibitive.

It may be confidently stated, therefore, that a speed range of

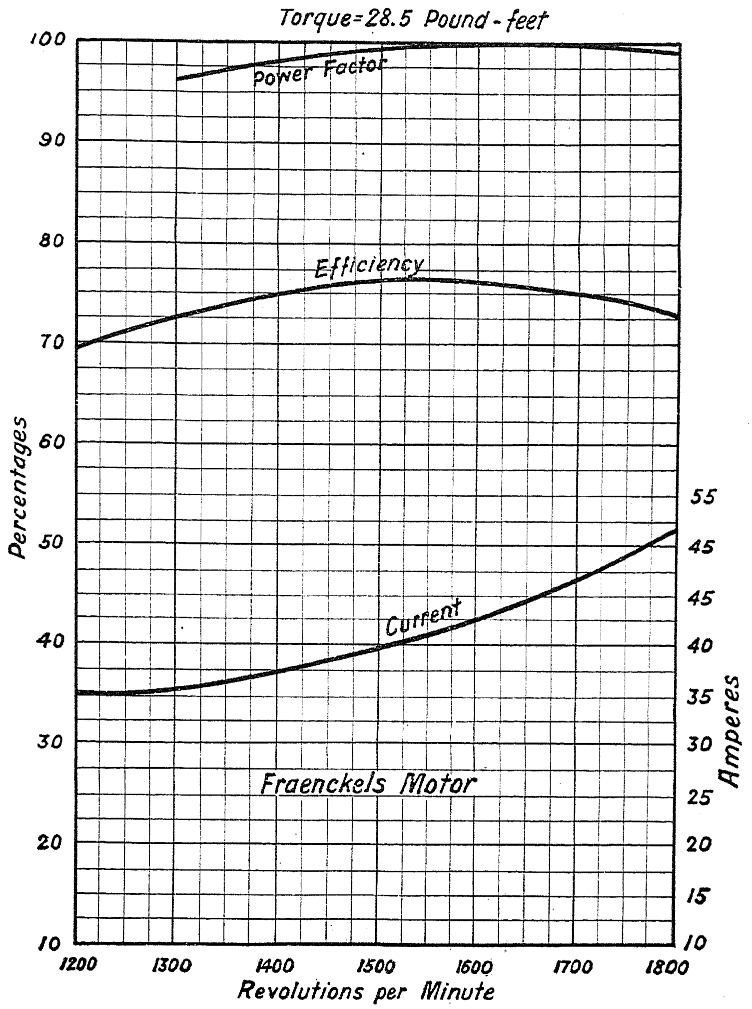

FIG. 30.

at least $2\frac{1}{2}$ to 1 is possible on 25 cycles, while a speed range of 3 to 1 could certainly be attained with very little difficulty. If the frequency be increased, raising the speed of the motor in the same proportion and keeping the rotor voltage constant, the

sparking voltage will be the same as before. In other words, if the 25-cycle motor be run at 60 cycles, keeping it at the same terminal voltage, the flux will decrease in inverse proportion

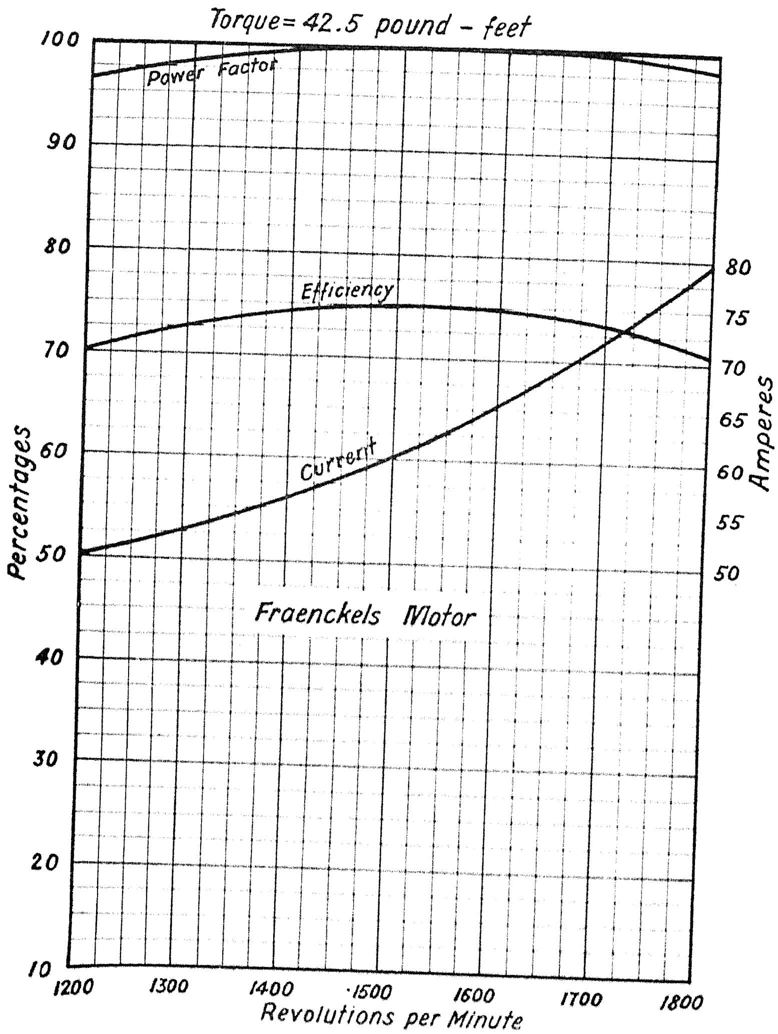

FIG. 31.

to the frequency and the speed will rise in direct proportion thereto. The power developed by the motor, of course, remains the same at all frequencies. Assuming that the commutator gives no trouble from vibration, etc., at the higher speed,

the commutation will be just the same as before, for the flux goes down in direct proportion to the increase in frequency. Hence the problem of building high-frequency motors reduces to that of building high-speed commutators.

An 8-pole 60-cycle machine could probably be built to run successfully between about 700 and 1600 rev. per min., and probably

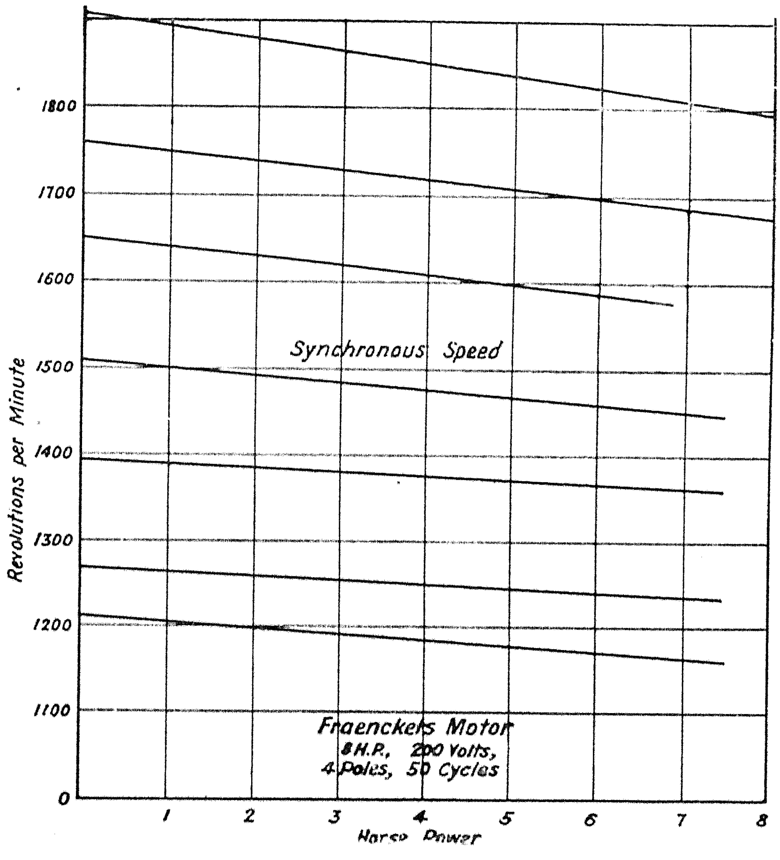

FIG. 32.

up to 1900 or 2000 rev. per min. if equipped with a mechanically good commutator. Such a machine would naturally be rather expensive but not more so than the compensated single-phase induction motor running near synchronism, as built by one American and several European firms. The efficiency would also be rather low, having about the same value as in the above-mentioned motor.

List of Symbols

E_0' = Compensating electromotive force.

E' = Total electromotive force induced by the field flux in the $X\ X$ circuit (motor of Fig. 4).

E'' = Electromotive force induced by the field flux in that part of the $X\ X$ circuit on the armature (motor of Fig. 4).

E = Transformer electromotive force in a single-phase induction motor; *i.e.*, the electromotive force induced in the secondary at standstill by the primary flux.

E_1 = Electromotive force of rotation induced in the $X\ X$ axis.

E_2 = Counter electromotive force induced by the cross flux on the $Y\ Y$ axis.

E_3 = Resultant of E and E_2.

E_4 = Electromotive force of self induction on the $Y\ Y$ axis.

E_5 = Electromotive force of rotation induced in the $X\ X$ circuit due to the leakage flux on the $Y\ Y$ axis.

E_6 = Counter electromotive force opposing E_4 due to extra current in the $X\ X$ circuit produced by E_5.

E_7 = Counter electromotive force at synchronous speed.

E_8 = Electromotive force introduced into the $Y\ Y$ axis by external means, such as a transformer.

X_1 = External reactance used for varying the speed.

X = Leakage reactance of $Y\ Y$ circuit.

R = Resistance of either $X\ X$ or $Y\ Y$ circuit.

$X_0 + X$ = Total reactance of the $X\ X$ circuit.

I_3 = Extra compensating current on the $Y\ Y$ axis.

I_s = Standstill current.

I_0 = Primary magnetizing current.

I_{1s} = Current on the $Y\ Y$ axis at synchronism.

I_1 = Total current in the $Y\ Y$ axis.

I_1' = Load current on the $Y\ Y$ axis.

I_2 = Current on the $X\ X$ axis.

k = Actual speed $\div$ synchronous speed.

k_0 = Free running speed $\div$ synchronous speed

m = $E_8 \div E_7$.

σ = Dispersion coefficient.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

REPULSION MOTOR WITH VARIABLE-SPEED, SHUNT CHARACTERISTICS

BY E. F. W. ALEXANDERSON

A characteristic feature of the electrical developments of late years has been the extension of the use of alternating-current to every field where it is practicable. The reasons for this preference for alternating current are so well known that reference need only be made: first, to the advantages of the distribution system; and second, to the fact that the best and most reliable of all motors is the induction motor.

Direct current is used in those cases in which the induction motor would not have the desired characteristics, and the natural consequence is that great efforts have been made to develop alternating-current motors with such characteristics as would enable them to take the place of the direct-current motor. The need most felt was for an alternating-current motor with the characteristics of the direct-current series motor. This requirement has been filled by the repulsion motor, the series motor, and certain modifications of these fundamental types. Another characteristic that was desired was a single-phase motor with constant speed and high starting torque. This requirement has been filled by the Milch motor, the Wagner motor, and some other similar types, all of which are of the repulsion-motor type in starting and run at constant speed as single-phase induction motors.

Although the advantages of alternating current for industrial plants are generally recognized, it is often necessary to recommend the use of direct current because many of the machines that are to be driven require a motor with adjustable speed. The direct-current shunt motor is the only one on the market

which fills this requirement. If it is not practicable to have the plant wired for direct current as well as alternating current; all the constant-speed work must be done with direct current, although induction motors would be preferable for this purpose. The investigations and tests described in this paper have been made in order that this need arising in the development of alternating current, may be met by providing a motor with the characteristics of the direct-current shunt machine.

Before entering into a detailed description of any specific form of motor, it may be of interest to present a theory of the alternating-current machine in such a way as to indicate the possibilities of developing a motor with required characteristics.

When an induction motor runs at synchronous speed, the armature winding is stationary in relation to the magnetic field, which is assumed to rotate at a uniform velocity. It is therefore apparent that the conductors do not cut any lines of force and no electromotive forces are induced.

In giving the theory of a motor with adjustable speed, the subject can be made clearer by departing from the theory of the rotating field. If the strength of a rotating field fluctuates so that the maximum strength always occurs when the poles are in a certain position and the minimum strength in another position, the field may be called elliptical instead of a plain rotating field; however, the elliptical shape of the field does not in itself convey any idea as to the reason why the motor should tend to run at any other speed than that of the rotating field.

In the following, the rotating field will be treated as if it were composed of two alternating-current fields at right angles to each other. If an armature with a commutator of ordinary construction is placed in such a field and two sets of brushes are arranged on the commutator in quarter-phase relation to each other, the following will take place: When the armature is at a standstill, a voltage will be induced between the positive and negative brush of each set, due to the alternating character of the two fields. Field A in Fig. 1, will induce a voltage between brushes B and vice versa. If the armature rotates at any speed, field A will induce an electromotive force between brushes A, and this electromotive force will be in opposition to the electromotive force induced by field B due to the alternating character of the field. In an ordinary induction motor, the two fields A and B are of equal strength, and in this case, the electromotive force of

rotation will be completely neutralized by the electromotive force of alternation when the motor has reached synchronous speed. The armature has a tendency to run at a speed at which the induced currents are a minimum. All motors of this kind have only one free running speed, which is that of synchronism. This is true with motors of the commutator type, as well as the squirrel-cage and collector type, provided that the brushes are short-circuited.

If the condition which determines the speed of an alterna-

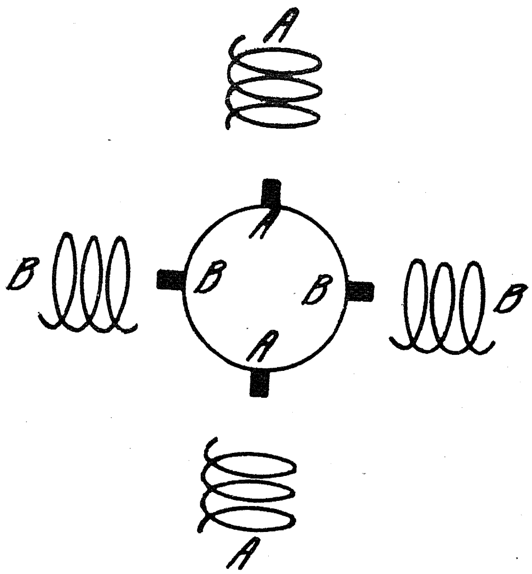

FIG. 1.

ting-current motor should be formulated broadly, the following might be stated:

The motor tends to run at a speed at which the sum of all the electromotive forces in each of the armature circuits is zero. The electromotive forces to be considered are particularly the electromotive force of rotation and that of alternation; these two electromotive forces are always in opposition. A third electromotive force may be induced artificially by impressing a voltage on the circuit from outside.

The two sets of brushes in a repulsion motor of the compensated type may be classified as the main brushes and the exci-

ting brushes; the main brushes carrying the energy current of the armature, the exciting brushes carrying the magnetizing current. If the main brushes as well as the exciting brushes are short-circuited, the motor will act like a single-phase induction motor. A practical modification of this motor has been developed by Milch, its principal feature being the power-factor regulation accomplished by impressing a suitable voltage on the exciting brushes. The impressed voltage is out of phase with the electromotive force of alternation and the electromotive force of rotation in the same circuit, so that it has nothing directly to do with the speed of the motor, which remains a constant-speed machine. It should be mentioned here that it is found advantageous in all commutator motors which will be described here, single-phase as well as three-phase, to introduce a correction of the power-factor like the one used in the Milch motor. However, the power-factor regulation has nothing to do with the theory of the speed variation, and reference to the power-factor regulation will, therefore, be omitted from the description of each particular form of motor in order that a more comprehensive explanation of the speed regulation may be given.

It is apparent from the preceding that the equilibrium of voltages that exists at synchronous speed when both sets of brushes are short-circuited, can be disturbed by introducing a voltage from the outside into either the circuit of the main brushes or the circuit of the exciting brushes; but it remains to be shown that in doing so another equilibrium can be established by changing the speed of rotation.

By a simple calculation, it can be shown that the relation between speed and field strength and the impressed voltages can be expressed as follows:

$$\text{Speed} = \text{Synchronous speed} \sqrt{\frac{E+e}{e}}$$

$$\frac{F_1}{F_2} = \sqrt{\frac{\text{speed}}{\text{synchronous speed}}}$$

where E is the voltage impressed on the stator winding reduced to number of turns in the armature, and e is the voltage impressed on the armature. F_1 and F_2 are the two components of the field in the axis of the exciting brushes and the axis of the

main brushes, respectively. The voltage which is impressed on the armature may be either positive or negative, and consequently the speed can be regulated below synchronous speed as well as above synchronous speed. The method for speed

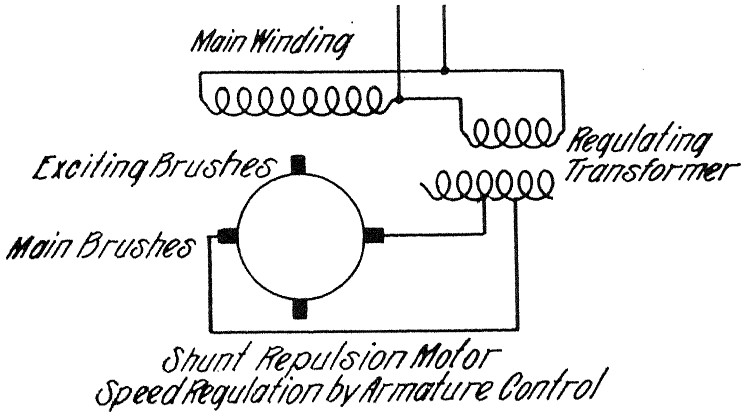

FIG. 2

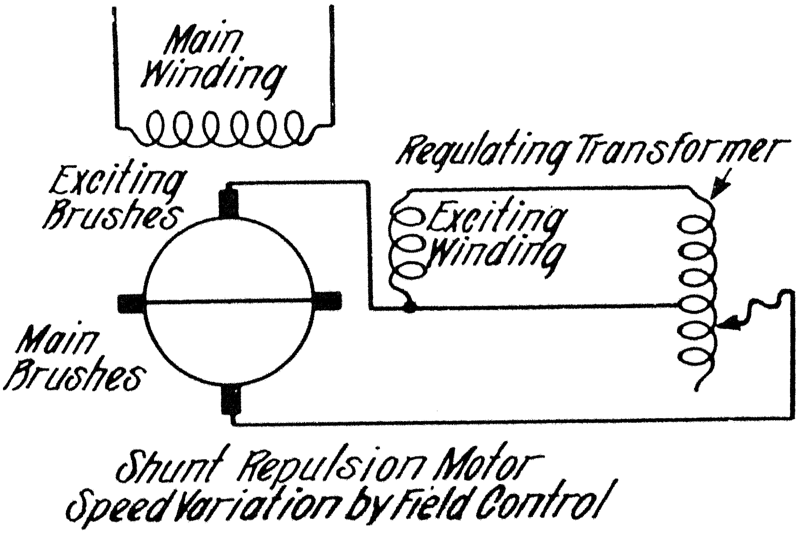

FIG. 2A.

variation described can be classified as armature control, because the functions correspond to those of a direct-current shunt motor when the speed is varied by raising and lowering the line voltage.

The second method for varying the speed of the repulsion motor follows logically. As shown in Fig. 2, the short-circuit is maintaned on the main brushes, while an external voltage is impressed upon the exciting brushes. In order to make the impressed voltage effective for speed variation, it should be in phase with the voltage of alternation and the voltage of rotation

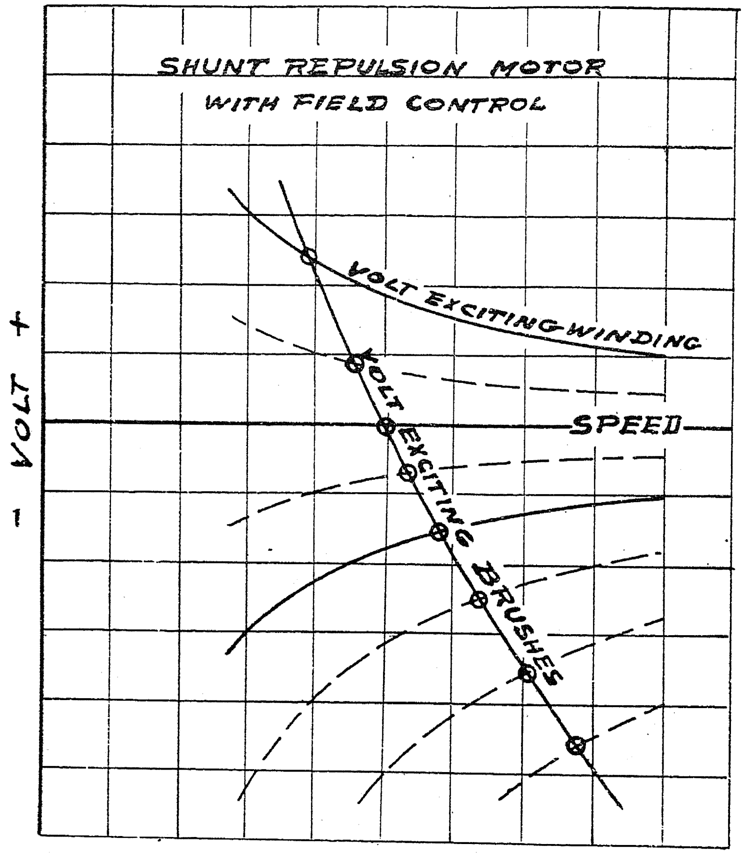

Fig. 3.

of the same circuit. These voltages in the exciting circuit are out of phase with the line voltage and, consequently, the line voltage cannot be used for varying the speed; however, if the motor has an exciting winding, like a plain repulsion motor, in addition to the exciting brushes, a voltage is found on the terminals of the exciting winding which has a suitable phase to be used for speed variation by field control.

The equilibrium of voltages is reached by a combined change of field strength and speed, for the same reason as in the case of armature control; however, the functions of the two fields are different; the entire energy supply is introduced through the stationary winding and the speed variation is obtained by variation of the torque-producing field, in the same way as the speed varies in a direct-current shunt motor with field control.

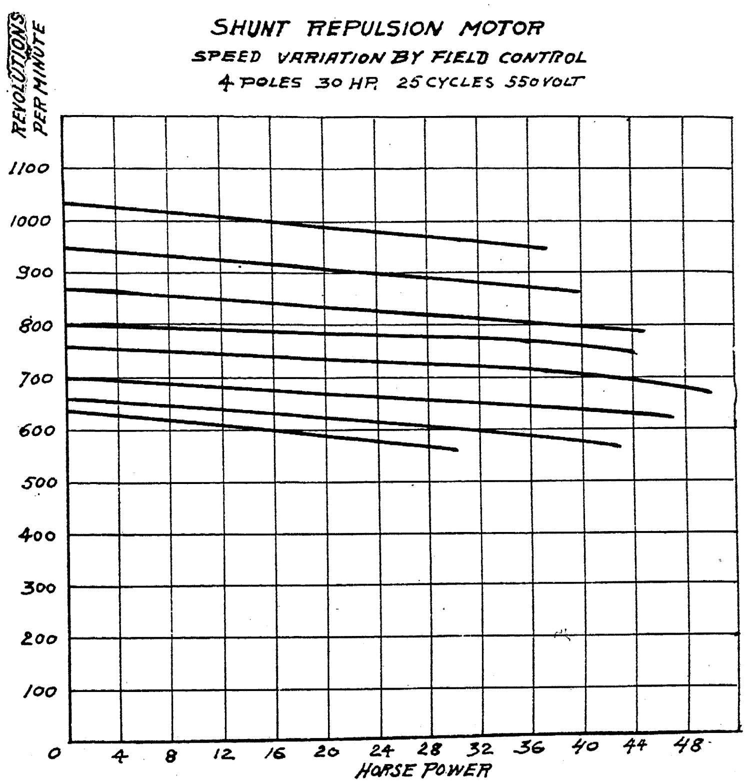

FIG. 4.

The relation of voltages which gives the equilibrium in the two brush circuits is not quite as simple to figure out as in the case of armature control; because the voltage impressed upon the brushes for the sake of speed variation is not derived from a constant source of potential, but from a winding which delivers a voltage that is in itself a function of the speed. However, the relations become comparatively simple by returning to the theory

of the plain repulsion motor and plotting curves for the voltage that would naturally occur at the exciting winding and the exciting brushes at different speeds. The voltage of the exciting winding is always positive, but varies inversely with the speed;

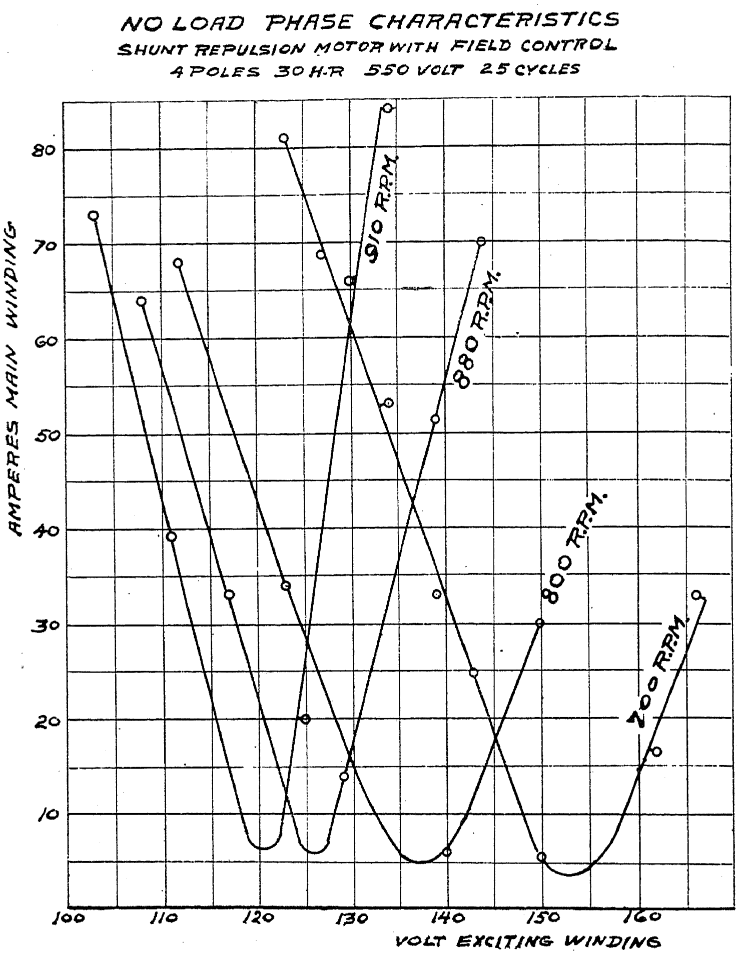

FIG. 5.

while the voltage of the exciting brushes is positive below synchronous speed and negative above synchronous speed. Fig. 3 shows curves of these voltages plotted on the same sheet. Only one curve is shown for the voltage of the exciting brushes, while the voltage of the exciting winding is plotted on the posi-

tive as well as on the negative side. The dotted curves indicate different voltages that may be derived from the exciting winding by the use of a step-up and a step-down transformer. The points marked in the intersections between the curves indicate equilibrium of voltages that may be obtained at different speeds by impressing on the exciting brushes the different voltages derived from the exciting winding.

The tests, which have been made on a 30-h.p., 25-cycle motor and a 3-h.p., 60-cycle machine in order to demonstrate the possibilities of the alternating-current shunt motor, confirm entirely

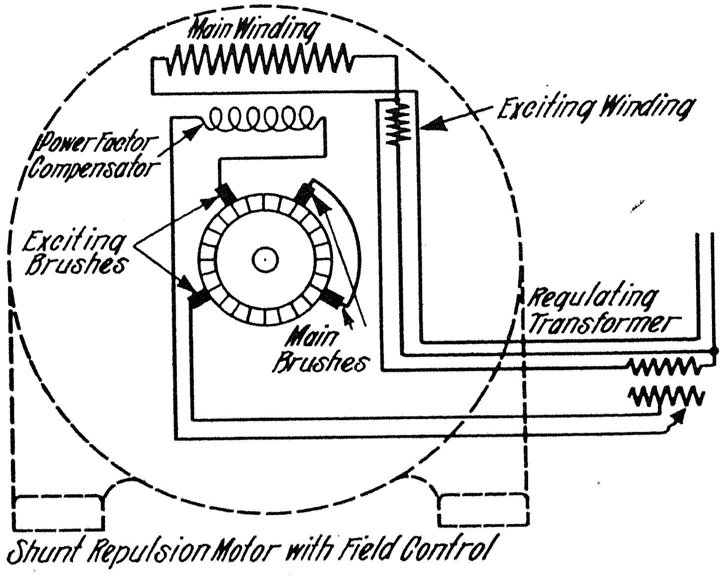

Fig. 6.

the theory given above. The curves in Figs. 4 and 5, which may be considered as typical, show the results of tests of the 30-h.p., 25-cycle motor. Fig. 2 shows a diagram of connections giving the principal features of armature control as well as of field control. The connections used for controlling the power-factor have purposely been omitted from these diagrams because they are apt to make confusion and have nothing to do with the point that is to be illustrated. A practical control diagram for the motor with field control is shown in Fig. 6.

The power-factor of any commutator motor with shunt characteristics, single-phase as well as three-phase, canbe regu-

lated in a manner similar to that of a synchronous motor. The functions of the power-factor regulation are the same as in the synchronous motor and can be best understood if they are described from the same point of view. The power-factor characteristics of a synchronous motor are usually illustrated by a curve showing the leading and the lagging current taken by the motor at no load with different field excitations. This curve is illustrated in most treatises on the synchronous motor and is recognized by a characteristic V shape. Fig. 5 shows the corresponding V-shaped phase characteristic for the shunt repulsion motor.

While a synchronous motor has only one no-load phase characteristic, the variable-speed shunt repulsion motor has one curve for each setting of the speed; in other words, the power-

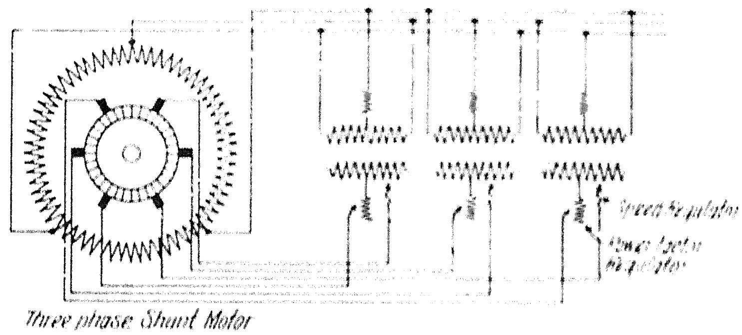

Fig. 7

factor can be varied at will at each speed by adjusting the excitation. In the commutator motor, as well as the synchronous motor, the power factor regulation consists in superimposing a wattless current upon the current that would otherwise be taken by the motor. It is known from the theory of the synchronous motor, that the power-factor can be maintained practically at unity between no load and full load with one fixed excitation, provided the excitation has been adjusted for the average load. The same is the case with the commutator motor. The regulation of the field for the purpose of adjusting the power-factor is accomplished in the commutator motor by a transformer or a coil in the stator winding, connected so as to introduce a fraction of the line potential in the exciting circuit. By varying the voltage introduced in this way, the amount

of wattless current can be regulated, as shown in Fig. 5. The complete diagram for the shunt repulsion motor with field control, illustrated in Fig. 6, shows the feature of power-factor regulation introduced by a coil of the stationary winding.

In connection with this subject, it will undoubtedly be of interest to refer to the possibilities of the three-phase motor with shunt characteristics. The possibility of such a motor was demonstrated theoretically by Georges about ten years ago, and tests of such a motor were made several years ago by Eichberg and Alexander. The speed variation of the Georges three-phase shunt motor is obtained by maintaining a constant field and adding or subtracting voltage to the armature circuit. The power-factor of the three-phase shunt motor can be regulated in the manner described above for the single-phase motor, and the comparison with the synchronous motor is applicable. Fig. 7 shows a diagram of the three-phase shunt motor with power-factor regulation.

In the theory given above for the alternating-current shunt motors, it has only been attempted to indicate the general principles, and the scope of this paper does not permit of entering into the details of application. However, it may be stated that speed variation by armature control is particularly applicable to those cases in which the torque increases with the speed, and field control to those cases in which the torque decreases with the speed. With armature control a speed variation can be obtained of 3 to 1; whereas with field control the variation is limited to a range of from 1.5 to 1. It will not be attempted to compare the merits of the three-phase and single-phase shunt motor; however; it may be stated as the impression of the author that the single-phase shunt motor will be preferred on account of the greater simplicity of control.

DISCUSSION ON "A SKETCH OF THE THEORY OF THE ADJUSTABLE-SPEED, SINGLE-PHASE, SHUNT INDUCTION MOTOR," AND "REPULSION MOTOR WITH VARIABLE-SPEED, SHUNT CHARACTERISTICS." FRONTENAC, N. Y., JUNE 28, 1909.

V. Karapetoff: I would like to ask Mr. Creedy the reason for calling this motor an induction motor? It seems to me that it would cause some confusion in our nomenclature. An induction motor, according to present nomenclature, is a motor without a commutator. I think that the title of the paper should be "A Sketch of the Theory of the Adjustable-Speed, Single-Phase, Shunt, Commutator Motor."

E. F. Alexanderson: The paper by Mr. Creedy on the shunt induction motor treats of the same subject as my paper on the shunt repulsion motor and the difference in the subject matter appears to be only the name. Mr. Creedy's paper therefore affords an excellent opportunity for comparing the viewpoints of similar investigations carried on independently.

In regard to the technical definition of certain terms used in both papers, it may be pointed out that the terms "adjustable-speed motor" and "shunt motor" are used to signify a motor having the characteristics of a direct-current shunt motor with field control. It is possible to obtain variation in speed by methods which do not give shunt characteristics. For instance, the speed of the ordinary induction motor can be varied by introducing resistance in the secondary winding. In certain cases the introduction of reactance will give the same results. A speed variation obtained in this way is similar to the speed variation of a series motor. Motors of various makes using this type of speed variation have been in the market for some time in Europe as well as in America.

The speed variation by introduction of resistance in the secondary has been used to some extent in the American built Milch motor in cases where speed variation by this method is applicable. The motor which Mr. Creedy refers to as being the only motor of a similar type built in this country, is apparently the motor which I dealt with particularly in investigating the possibilities of an adjustable-speed shunt motor. This motor has been on the market for several years and is known as the Milch motor. It is classified as a repulsion induction motor. The diagram in Fig. 21 of Mr. Creedy's paper illustrates this type.

In my paper it has not been attempted to describe early types of motor or to mention names of inventors, because so many have been working along the same lines that it is difficult to give credit to some without doing injustice to others. My investigation has been based on the assumption that the system for compensating the power-factor developed by Milch is essential for the success of any motors of this class. This conclusion Mr. Creedy has also reached and a direct statement to

that effect is made in his paper. The object of my paper is simply to give the results of an investigation as to what practical additions can be made to the constant-speed motor developed by Milch in order to introduce the adjustable-speed feature.

For this purpose not only the theoretical possibilities but also the established practice in manufacturing had to be considered. The aim has been to develop such a method of control that the constant-speed, repulsion, induction motor of a well-known make could be used for adjustable-speed work with the

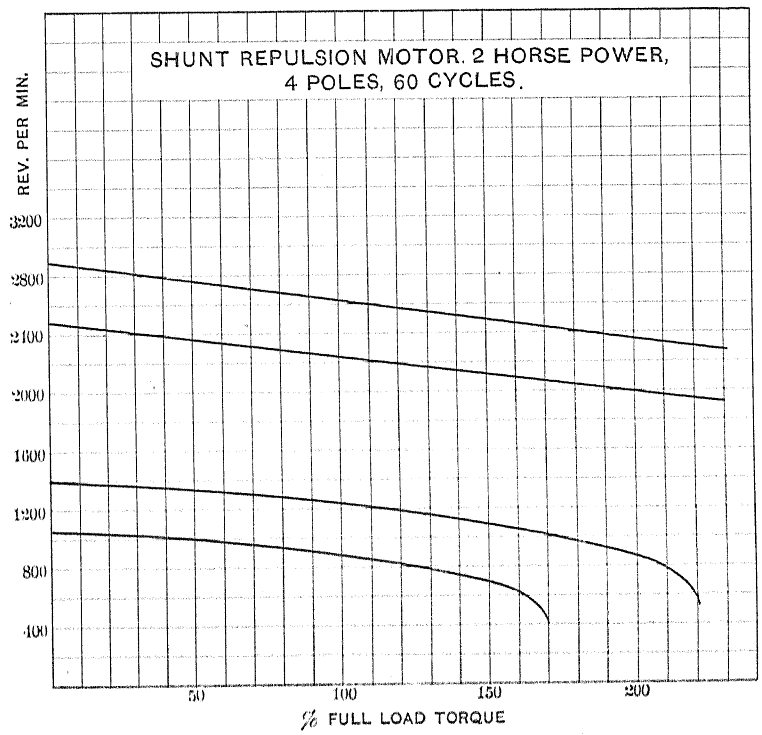

FIG. 1.

least amount of modification. In the first attempts it appeared as if speed variation by field control, illustrated in Fig. 6 in my paper and in Fig. 4 of Mr. Creedy's paper, would be the most applicable.

The experimental data obtained from a test of the 30-h.p., 25-cycle motor built for experimental purposes, were worked out with reference to field control. The results are shown in Figs. 4 and 5. Subsequently, a 3-h.p., 60-cycle motor was built, adhering as closely as possible to the manufacturing methods developed for the constant-speed motor. In designing

this motor it appeared that a motor of the original type could always be furnished by adjustable-speed field control, provided the motor was of the reversing type; in other words, provided it had a section of the field winding placed in quarter-phase relation to the main part of the winding. The method of speed variation with armature control was also reconsidered, and it was found that a suitable arrangement of the controlling circuit made it possible to apply a practical system of speed variation to the original type of motor which is being built at present for constant speed. The speed variation can be applied to the non-reversible as well as the reversible type without any modification of the motor itself. By this method it is practicable to lower the speed to one-half synchronous speed, and increase it to 50 per cent above synchronous speed.

The exact characteristic curves and diagrams for this combination were not available for publication at the time the paper was printed, and for this reason it was only possible to state that it is feasible to obtain a speed variation of 3 to 1 with armature control. In view of these results, I feel that exception should be taken to the statement in Mr. Creedy's paper that there is no thoroughly practical method of lowering the speed, although there are several good methods of raising it

In controlling the speed by armature regulation in the way mentioned, the commutation appears to be nearly as good for all the sub-synchronous speeds down to one-half synchronism as it is at synchronous speed. At over-synchronous speeds, the commutation remains good up to about 30 per cent above synchronous speed, whereas at 50 per cent above the synchronous speed the sparking becomes objectionable. The tests referred to were made on 3-h.p. and 2-h.p., 4-pole, 60-cycle repulsion induction motors of standard make, and the speed was varied from 2700 to 900 rev. per min.

In answer to Mr. Creedy's expectation that an adjustable-speed shunt motor would soon be placed on the market in either Europe or America or both, it can be said that the latest developments show that such a motor is already on the market; but this fact was not known and the motor has been considered as a constant-speed motor, due to the lack of a suitable controlling device. Owners of such motors may be interested to know that it will be possible to add such a controlling device for adjustable speed to their equipments as soon as such a device has been developed.

My paper was not intended to cover all the possible connections. The investigation I made was for certain practical purposes, in conjunction with established practice of manufacture, and therefore it might not be of sufficient interest at the present time to go into detail in answering questions relating to the different connections that may be suggested.

F. Creedy (by letter): I am interested in Mr. Alexanderson's paper, as it contains the results of an independent investigation

of a subject on which I also have done a good deal of work. At the stage at which this motor has at present arrived, one may say that the principal problem before us is to keep the efficiency up and the cost down. It will probably be found that any constant-speed compensated motor of the type shown in Fig. 21 of my paper can be modified to form an adjustable-speed motor. If the speed range is high say, 3 to 1, a somewhat larger commutator will be required, but the same frame can be used and rated so as to give the same horsepower at synchronous speed.

I would like some information in regard to the diagram of connections, Fig. 6. Redrawing this in a manner corresponding to the other diagrams it would appear as in Fig. A. I am inclined to think that such a motor, while it may be arranged for self-starting without change of connections, will be somewhat intermediate in its characteristics between shunt and series motors and will have a more drooping speed characteristic than

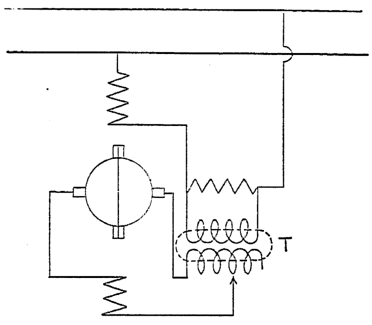

Fig. A

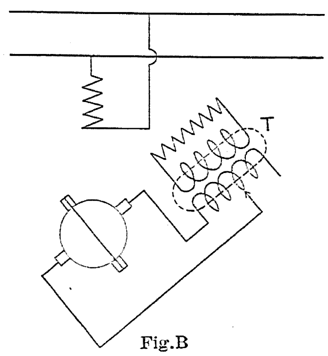

Fig. B

the plain motor without a quadrature coil in the primary circuit. It reminds me of a motor proposed some years ago in one of the early patents (see Fig. B) in which the coil usually wound in quadrature is wound at a certain angle with the primary coil, and the brushes are then moved so as to get the best ratio between the compensating and the speed-increasing electromotive forces. This motor has, I believe, similar characteristics, although the connection is quite different. I should be glad to know whether Mr. Alexanderson's experience bears out these views. When considering these single-phase, adjustable-speed motors the Görges polyphase adjustable-speed motor naturally comes up for consideration also. Like Mr. Alexanderson I have done some figuring on such motors, and, like him, I have come to the conclusion that the much greater simplicity and flexibility of the control will always give the advantage to the single-phase motor in spite of its somewhat greater weight per horse power.

I do not think that Mr. Alexanderson's remarks on my paper call for any very extended reply, as I find myself in complete agreement with nearly everything he says. I am much interested in the tests shown in Fig. 1 of his discussion, which show a wider speed range than any which have hitherto been published. I presume, however, that a commercial motor would be built with six poles having a range of from 600 to 1800 rev. per min. Since writing my paper, further investigation had begun to convince me also that I had somewhat underestimated the possibilities of speed reduction by armature control, so I am quite willing to modify the statement to which Mr. Alexanderson takes exception.

I think that in view of the results of these two papers we may fairly say that we are at least in possession of that complement to the induction motor which has been sought for so long; namely, a practical adjustable-speed, alternating-current motor. Its manufacture will probably be taken up in the near future.

With reference to Professor Karapetoff's remarks on nomenclature; this seems to be largely a matter of individual choice, but personally I usually endeavor to use the classification proposed by Atkinson* and further developed by V. A. Fynn in a little book he has published on the subject. This is the only logically thought-out system I have seen, which avoids self-contradiction. In it, motors are classified as *inductive*, where the power is induced in the armature, and *conductive* where it is led in through brushes. It is impossible to distinguish the motor of Fig. 1 from the single-phase, squirrel-cage, induction motor on any ground whatever except mechanical construction, and as all the others are developments of this, I feel justified in calling them all induction motors. I cannot understand, however, why they should be called repulsion motors, as according to my idea a repulsion motor is essentially a machine with series characteristics.

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* Proc. Inst. Civil Eng., London, 1898.

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THE HEATING OF INDUCTION MOTORS

BY A. MILLER GRAY

The object of this paper is to show the limitations imposed on the designer of induction motors by the heating of the machine, and also to show how this heating may be predetermined.

The highest temperature at which an electrical machine should ever be operated, is that at which the insulation begins to deteriorate owing to the action of heat. With the present practice in insulating, the temperature of the stator coils of an induction motor should not exceed 85° cent. for any length of time. The same limitation holds for the rotor coils of a wound rotor machine. The temperature of the rotor of a squirrel-cage motor is only limited by the fact that a hot rotor causes a hot stator, because the hot air from the rotor has usually to pass through the stator coils before it gets out of the machine.

Heating conditions at starting. The squirrel-cage induction motor, with full applied voltage at the terminals, gives a starting torque of about 1.8 times full-load torque, with about 6 times full-load current, or, for full-load torque, the current in the motor is 4.5 times full-load current.

The operation of such a machine can best be shown by the circle diagram Fig. 1.

$a\,b$—represents the loss in the rotor end connectors, with the rotor in the locked position.

$b\,c$—that in the rotor bars.

$c\,d$—the rotor stray loss, which cannot be calculated.

$d\,e$—the stator stray loss, which cannot be calculated.

$e\,f$—the stator I^2R loss.

$f\,g$—the windage, friction, and core loss, which are assumed constant.

At starting, the losses in a squirrel-cage motor are very large, while the speed, on which the power of the motor to get rid of heat depends to a great extent, is low, therefore the heat generated in the various parts of the machine at starting, must be absorbed by these parts.

To determine the temperature rise of the rotor bars at starting. The resistance of copper at the usual working temperature of a motor is 1 ohm per circular mil per inch, therefore the loss in the rotor bars $= I^2 R$ watts.

$$= \frac{I^2 \lambda n}{\textcircled{M}} \text{ watts.}$$

Where I = the effective rotor current in each bar.
λ = the length of the rotor bar in inches.
n = the number of bars.
$\textcircled{M}$ = the area of each bar in circular mils.

The heat absorbed by the rotor bars =
$n \times \lambda \times \square'' \times .32 \times .092 \times \theta$ calories per sec.

Where $\square''$ = the area of each bar in sq. in.
0.32 = the weight of 1 cu. inch of copper.
0.092 = the specific heat of copper.
θ = the temperature rise per sec. in degrees cent.

$$\frac{I^2 \lambda n}{\textcircled{M}} \text{ watts} = \frac{I^2 \lambda n}{\square'' \times 1.27 \times 10^6} \times \frac{1}{746} \times \frac{550}{1} \times \frac{1}{774} \times \frac{5}{9}$$

calories per sec.

$$= \frac{I^2 \lambda n}{\square'' \times 1.27 \times 10^6} \times 0.00053 \text{ calories per sec.}$$

$$= n \lambda \square'' \times 0.32 \times 0.092 \times \theta$$

therefore θ = temp. rise per sec. in deg. cent. $= 1.4 \times 10^{-8} \left(\frac{I}{\square''}\right)^2$

$$= 2.28 \times 10^4 \left(\frac{I}{\textcircled{M}}\right)^2 \qquad (1)$$

The value $\frac{\textcircled{M}}{I}$ is seldom less than 400 in the rotor bar at full load, and therefore for full-load torque at starting the $\frac{\textcircled{M}}{I} =$

the circular mils per ampere $= \frac{400}{4.5}$ approx. $= 89$; and the temperature rise per second from equation 1 = 2.9 degrees cent.

To determine the size of the rotor end connectors. Let P_c be the loss in each ring in watts at starting, required to give the necessary starting torque.

$$P_c \text{ watts} = P_c \times 0.00053 \text{ calories per sec.}$$
$$= \omega \times S \times \theta$$

Where ω = the weight of each ring in lb.
S = the specific heat of the rings assumed = 0.092.

therefore,

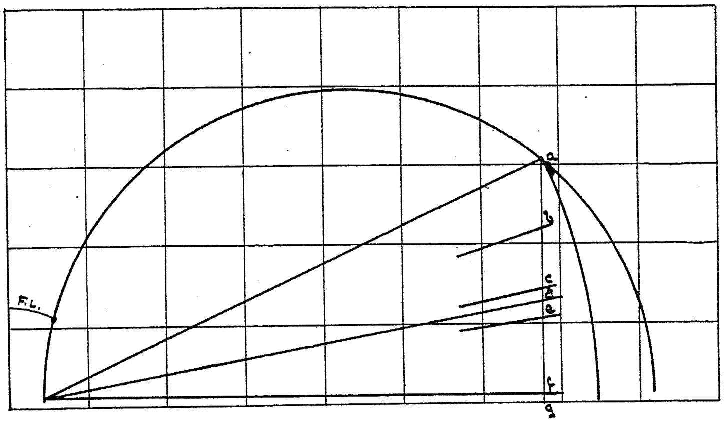


FIG. 1.

the temperature rise per sec. per watt per lb.

$$= \frac{0.00053}{0.092} = \frac{1}{173} \text{ degrees cent.} \tag{2}$$

or for 1 synchronous h.p. per lb. the temp. rise = 4.3° cent. per sec. The proper weight of ring to be used in any particular case depends on the starting torque required, and the time needed to bring the motor up to speed. For motors up to about 50 h.p., where the starting duty is not severe, 1.5 synchronous h.p. per lb. will give satisfactory results.

To determine the temperature rise of the stator windings at starting. This may be found in a similar way to that used for

the determination of the temperature rise of the rotor copper; namely:

$$\theta = \text{temp. rise per sec. in degrees cent.} = 2.28 \times \left(\frac{I}{(M)}\right)^2 \times 10^4$$

This temperature rise cannot be measured by thermometer, because the rate of increase of temperature is so rapid that the heat does not get through the insulation. In making static torque tests on induction motors, the stator coils may often be smoking and yet the coils not feel exceptionally warm to touch.

Test results. The following starting tests are of interest in connection with the previous discussion. A squirrel-cage induction motor was supplied with rings of extra high resistance

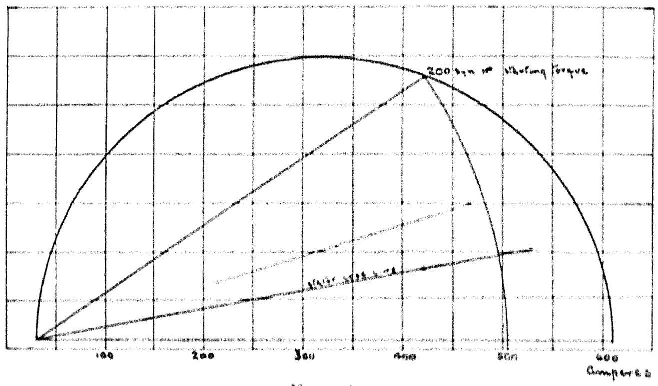


Fig. 2.

and was connected up for experiment to a rotating mass having a large moment of inertia and a small friction loss. Such a rotating mass is found in the rotating field of a turbine alternator.

The operation of the machine in question is shown by the circle diagram Fig. 2.

The moment of inertia of the rotating mass = 36,000 lb.-ft.²

The tests were made on half voltage, under which conditions the

$$\frac{(M)}{I} \text{ stator} = 249$$

$$\frac{(M)}{I} \text{ rotor} = 164$$

Weight of each ring = 23 lb.

P_c = watts lost in each ring = 13750 watts = 18.5 synchronous h.p.

H.p. per lb. of ring = 0.8

The calculated temperature rises per second from equations 1 and 2 are:

stator coils—0.37 °cent.

rotor bars—0.85 °cent.

rotor end connectors—3.4 °cent.

The speed-time and current-time curves of the machine are shown in Fig. 3. About 60 seconds after starting, smoke began to come from the stator windings, and the stator continued to smoke until the speed rose to 50 per cent of full-load speed, at which point the stator current had fallen to 83 per cent of the

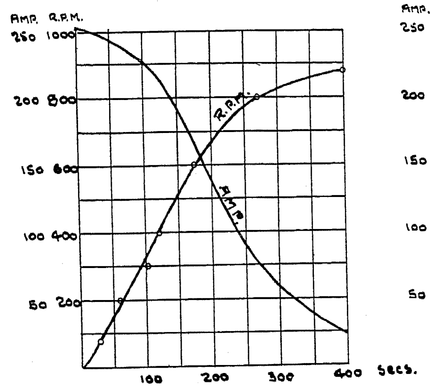


FIG - 3

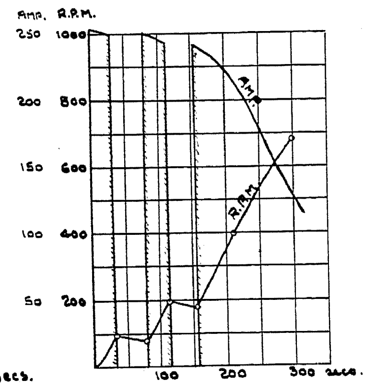


FIG - 4

starting current, and the ventilation was more than able to take care of all the heat generated.

At 253 amperes starting current, the circular mils per ampere is 249 and the calculated temperature rise 22° cent. in 60 seconds, which is not sufficient to cause the machine to heat up as it did. To test this point a direct current of 300 amperes was sent through all three phases of the stator; the rotor was stationary. Under these conditions the temperature rose from 16° cent. to 103° cent. by thermometer in six minutes, at which temperature smoke began to appear. The cause of the excessive heating when the motor was started up must therefore have been the high temperature of the rotor end connectors. The construction of this machine is similar to that shown in Fig. 5. The

rotor end connectors *H* are close to the stator windings *E* so that, if the former are very hot, the latter will be scorched on the outside. In the actual test the rings began to oxidize in about a minute after starting, and they were highly discolored before the motor was up to speed.

The total rotor loss at starting	=	37,000 watts.
	=	50 syn. h.p.
The weight of two end connectors	=	46 lb.
of rotor bars	=	66 lb.
of rotor laminations	=	200 lb.
Total rotor		312 lb.

If the heat generated in the rotor bars and end connectors had

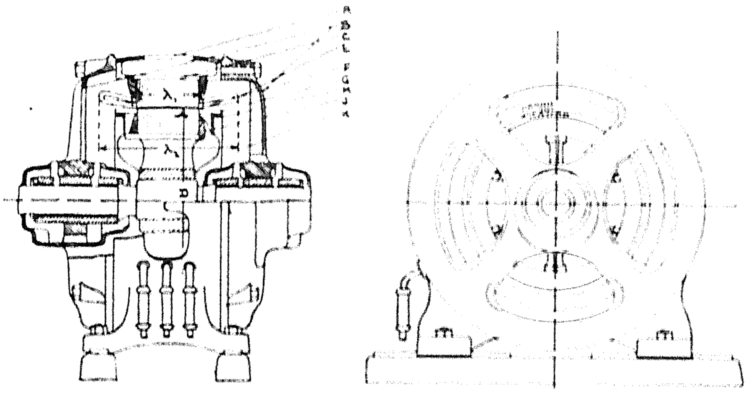


FIG - 5

time to diffuse all through the rotor, the rotor temperature rise would be from equation (2)

$$\frac{50}{312} \times 4.3 = 0.61^{\circ} \text{ cent. per sec.}$$

An attempt to approximate this condition was made by bringing the machine up to speed in steps as shown in Fig. 4. In this case the rings remained untarnished and the temperature rise on the windings was only 51° cent. by thermometer. The machine showed no signs of high temperatures.

Heating of induction motors under running conditions. Before discussing any particular type of construction or system of

ventilation, attention must be drawn to the fact that in an induction-motor stator, built up of laminations which are insulated from each other by paper or by varnish, the thermal conductivity of the core is not the same in all directions; for example,* in the case of a core built with alternate layers of iron 0.020 in. thick and of paper 0.002 in. thick, the conductivity was as follows:

Along the laminations—0.1365 gram calories per cm. per sec. per degree cent.

Across the laminations—0.0013 to 0.0025 gram calories per cm. per sec. per degree cent.

The latter quantity varied with the structure of the paper and the kind of scale on the iron.

It was also found, by blowing air across the radiating surface of the core, that the watts per sq. cm. radiating surface per degree cent. rise was equal to $0.0038\ (1+0.25\ v.)$ where $v =$ the air velocity in metres per sec.

The following discussion of induction motor heating is based on the type of machine shown in Fig. 5.

The system of ventilation of this machine is as follows: air is drawn in at K, passes between the rotor end head J and the rotor end connector H and then through between the rotor bars G.

Leaving the rotor, the moving air strikes the stator windings F and some of the air passes through the windings; the remainder of the air moves along the coils to E, then, as it still has a considerable radial velocity, some of the air strikes the inside of the housing at B, and is deflected back over the outer surface of the stator coils and then over the laminations C, before it leaves the machine at A.

An important feature about the construction of this machine is that the moving air is not passed out of the machine after cooling the stator coils, but passes over the back of the stator laminations with a fairly high velocity, in this way getting the effect, which the experiments on conduction just mentioned would indicate as desirable to obtain.

Table I gives a number of heat runs on a motor of this type and of the following dimensions:—

$D =$ dia. of rotor $=$ 33 in.
$\lambda_1 =$ length of core $=$ 4.5 in.
$\lambda_2 =$ length over coils $=$ 15.125 in.

* *London Electrician*, March 7, 1907.

TABLE I.

Test No.	Stator loss in kw.			Stator temp. rise in °cent.				I^2R – core loss / core loss	core loss – I^2R / I^2R	temp. rise copper / I^2R	temp. rise iron / core loss
				Machine running		After shut down					
	I^2R	Core loss	Total	Copper	Iron	Copper	Iron				
	1	2	3	4	5	6	7	8	9	10	11
1	2.09	1.6	3.69	21	22	21½	22	0.307	– 0.235	10.3	13.8
2	2.8	1.6	4.4	24½	23	27	26	0.75	– 0.428	9.65	16
3	3.08	1.6	4.68	26	25	27.7	26.5	0.925	– 0.48	9.0	16.6
4	2.9	3.7	6.6	26	29	30	31	– 0.216	0.275	10.4	8.4
5	3.64	3.7	7.34	35½	35½	36½	36½	– 0.016	0.0165	10	9.9
6	2.2	4.2	6.4	21	25½	22	26	– 0.475	0.91	10	6.2
7	3.6	4.2	7.8	33	34½	34	35½	– 0.143	0.166	9.5	8.4

$\sim$ = frequency = 60 cycles.
p = number of poles = 10.
synchronous speed in rev. per min. = 720
S = % slip at full load = 4% approx.

Column 1 gives the stator $I^2 R$ in kilowatts.

R is the resistance at the air temperature measured by direct current. The eddy-current loss is negligible, because the core is short, and the conductor in no case is more than 0.5 in. deep.

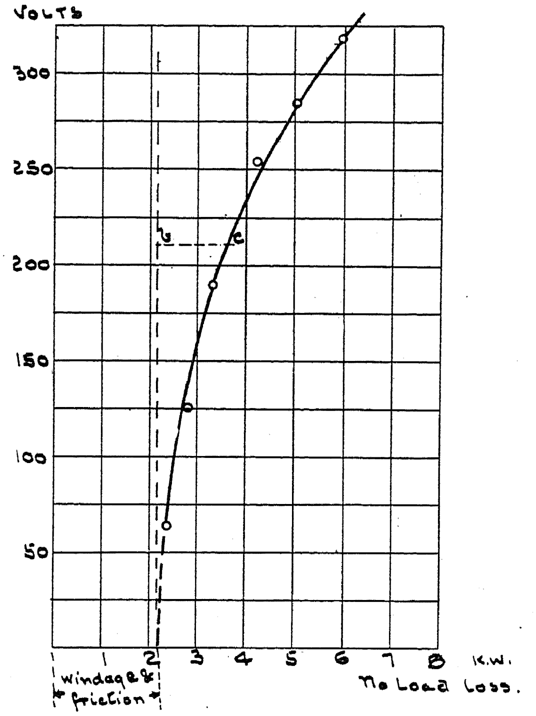


FIG.-6
Induction Motor Core Loss Curve

Column 2 gives the stator core loss in kilowatts. This is taken directly from the saturation curve Fig. 6 in the following way: The saturation curve in Fig. 6 is taken as low as possible and is then continued as shown by dotted part to the base line. The core loss at 220 volts then = $b\ c$ = 1.6 kw.; at 330 volts = 3.7 kw. and at 360 volts = 4.2 kw.

Columns 4 and 5 give the temperatures of the stator copper and iron while the machine is still running under load.

Columns 6 and 7 give the same temperatures after the machine has been shut down and the heat has come out, to use a shop phrase. The temperature always rises a little after shut-down because, under running conditions, the internal parts of the machine are hotter than the parts on which the thermometers

D : Rotor dia : 33"
λ₁ : length over coils : 15⅝"
R.P.M. : 120

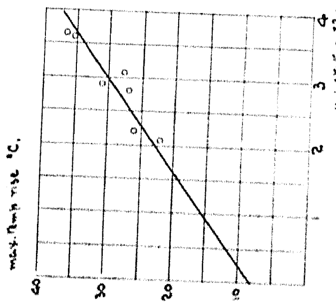


FIG-8

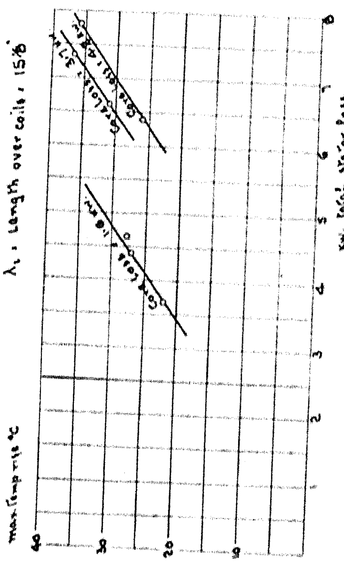


FIG-7

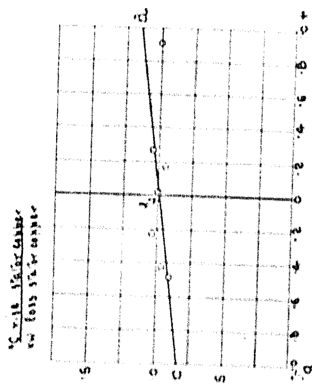


FIG-10

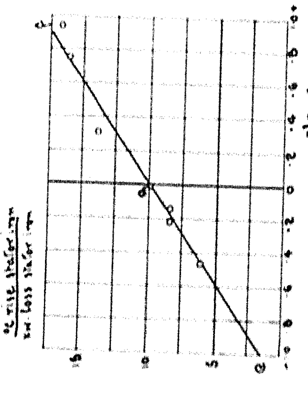


FIG-9

are placed; while after shut-down, the temperature tends to become uniform throughout the stator.

These temperatures are all measured by thermometer. In the case of the stator coils the thermometer bulb is pushed into the windings and is protected from the direct ventilation of the

machine. In the case of the stator iron, the thermometer bulb is laid on the back of the laminations at a point where it is protected from air blowing through the vent ducts. The bulb is then covered with a pad of waste. This pad is in such a position and is of such a size that it will not interfere with the natural ventilation of the machine.

Two important points are to be seen from these tests:

1. Irrespective of the distribution of the stator loss between the copper and the iron, the final temperature of the copper, after shut-down, is approximately equal to the final temperature of the iron, in any one test.

2. The temperature rise is not proportional to the total stator loss. It is more nearly proportional to the $I^2 R$ loss of the stator. This is well shown in curves, Fig. 7 and 8, plotted from the tests of Table I.

The former of these two points shows that there is a transfer of heat from the hotter parts of the machine to the cooler parts, and suggests the following method of plotting heating data. The method is purely an empirical one, but it gives good results.

The temperature rise of the stator iron $= t_2$
" " " " " " copper $= t_1$

The following assumptions are made

$$t_1 = K_1 (I^2 R) + K_2 (\text{core loss} - I^2 R)$$
$$t_2 = K_3 (\text{core loss}) + K_4 (I^2 R - \text{core loss})$$

or

$$\frac{t_1}{I^2 R} = K_1 + K_2 \left(\frac{\text{core loss} - I^2 R}{I^2 R}\right) = \frac{\text{temp. rise copper}}{\text{kw. copper loss}}$$

$$\frac{t_2}{\text{core loss}} = K_3 + K_4 \left(\frac{I^2 R - \text{core loss}}{\text{core loss}}\right) = \frac{\text{temp. rise iron}}{\text{kw. iron loss}}$$

The curve in Fig. 9 gives

$$\frac{t_2}{\text{core loss}} \text{ plotted against } \frac{I^2 R - \text{core loss}}{\text{core loss}}$$

The curve in Fig. 10 gives

$$\frac{t_1}{I^2 R} \text{ plotted against } \frac{\text{core loss} - I^2 R}{I^2 R}$$

from the tests of Table I.

TABLE II.

Stator loss in kw.			$\frac{I^2R - \text{core loss}}{\text{core loss}}$	$\frac{\text{core loss} - I^2R}{I^2R}$	temp. rise copper $\frac{I^2R}{}$ from curve—10	temp. rise iron $\frac{}{\text{core loss}}$ from curve—9	Calculated temperature rise	
I^2R	Core loss	Total					Copper	Iron
1	6	7	− 0.83	5.0	19° C.	3° C.	19	18
2	5	7	− 0.6	1.5	12.7	5	25	25
3	4	7	− 0.25	0.33	10.6	8	32	32
4	3	7	0.33	− 0.25	9.6	13	38	39
5	2	7	1.5	− 0.6	9.0	23	45	46
6	1	7	5.0	− 0.83	8.6	53	52	53

TABLE III.

Test No.	Amp. stator	Kw. stator loss		° cent. temp. rise					$\frac{I^2R - \text{core loss}}{\text{core loss}}$	$\frac{\text{core loss} - I^2R}{I^2R}$	temp. rise copper $\frac{}{I^2R}$	temp. rise iron $\frac{}{\text{core loss}}$
				Stator		Rotor						
		I^2R	Core loss	Copper	Iron	Cond.	Ring	Iron (inside)				
1	180	2.9	4.1	$25\frac{1}{2}$	30	21	21	19	− 0.292	0.414	9.8	7.4
2	227	4.6	4.1	41	43	31	33	26	0.122	− 0.108	9	10.4
3	180	2.9	4.1	39	41	34	35	33	− 0.292	0.414	13.4	10
4	227	4.6	4.1	59	58	50	50	46	0.122	− 0.108	12.8	14.2

The effect of the distribution of loss on the temperature rise is shown in a very striking manner by the following figures taken from curves 9 and 10. See Table II.

Analysis of the temperature loss diagram. It has been pointed out in discussing the tests of Table I, that the final temperature of the stator copper after shutdown is approximately equal to the final temperature of the stator iron in any one test. Therefore, $o\ a$, Fig. 9, the temperature rise of the iron per kw. iron loss at the point where core loss $= I^2 R$ loss, must equal $o\ b$, Fig. 10, the temperature rise of the copper per kw. $I^2 R$ loss, at the same point.

If then $o\ b$ can be predetermined, and also the slope of the copper line $c\ d$, Fig. 10, the iron line $e\ f$, Fig. 9, can be drawn in, because the copper and iron temperatures are about equal at any load.

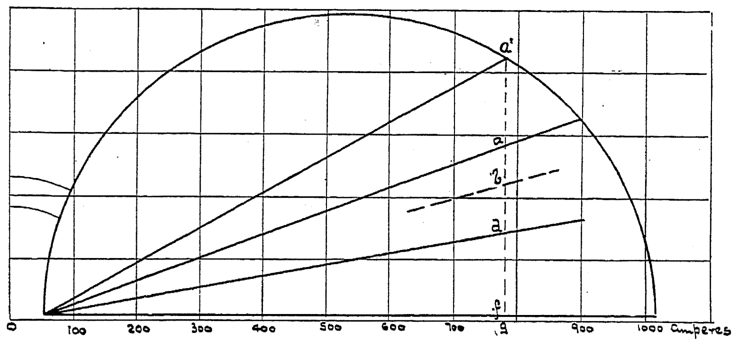


FIG - 11

The reason why it is desirable to predetermine $o\ b$ rather than $o\ a$ is that, as has been shown in Figs. 7 and 8, the copper loss is a more important factor in determining the temperature rise than is the iron loss.

Effect of a high resistance rotor on the stator temperatures. Table III gives four heat runs on a motor of construction similar to Fig. 5 and of the following dimensions:

D = 33 in.

$\lambda_1 = 7\frac{1}{4}$ in. The machine has a 0.5-in. vent duct in the center of the stator and rotor cores.

λ_2 = 17.25 in.

$\sim$ = 60 cycles.

p = 12 poles.

synchronous rev. per min. = 600.

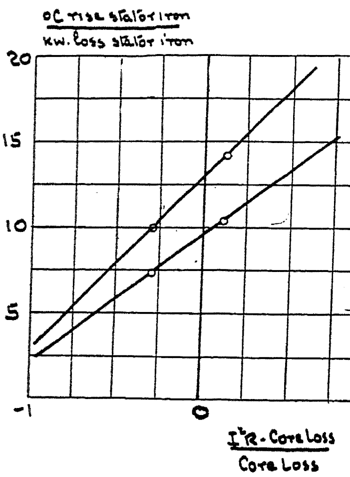


FIG- 12

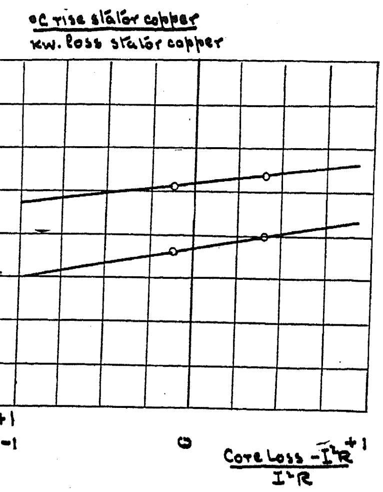


FIG- 13

EFFECT of HIGH ROTOR RESISTANCE

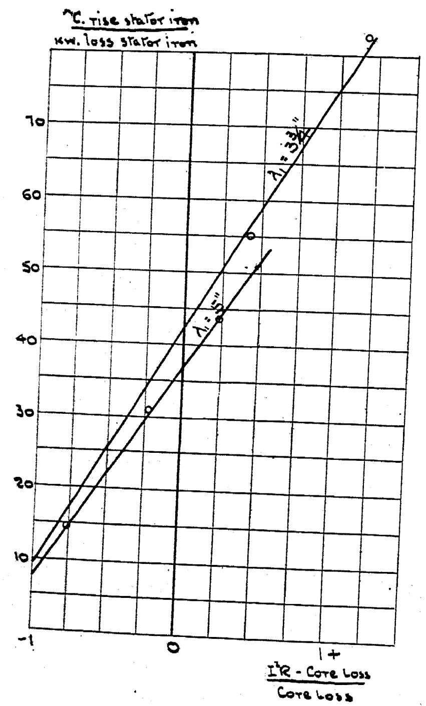
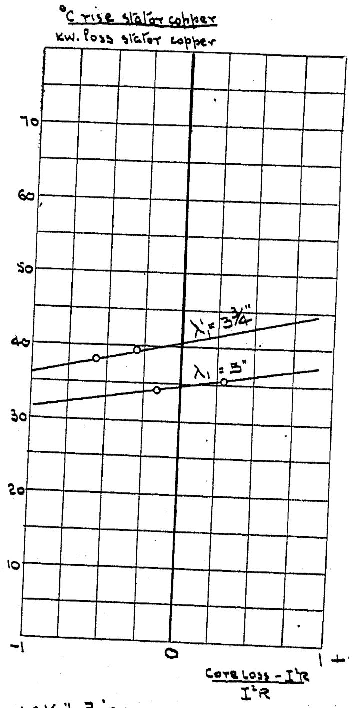


FIG- 14



Machine - 14½" dia.

FIG-15

This machine was tested with two different sets of rotor end connectors, all other things being unchanged. The effect of the change in the end connectors may be seen from Fig. 11, where

$a\,b$ = the rotor end connector loss at the lock point in tests 1 and 2, Table III.

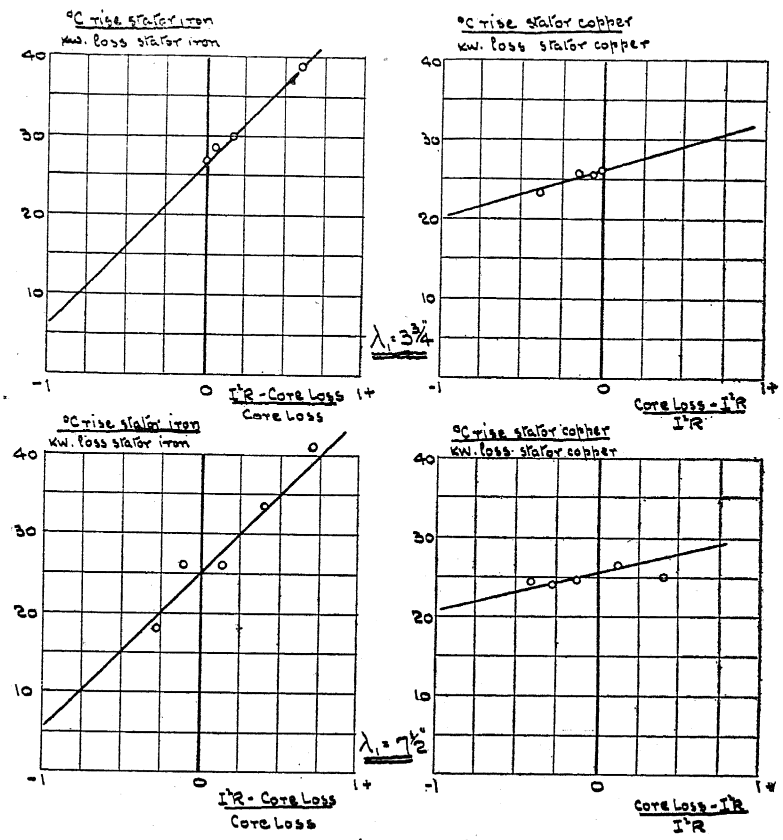


FIG-16 Machine-20" dia. FIG-17

$a^1\,b$ = the rotor end connector loss at the lock point in tests 3 and 4, Table III.
$b\,d$ = the rotor bar loss, which was unchanged.

These results are plotted in curves, Fig. 12 and 13, and show that with an increased rotor loss of 100 per cent there is an increase in temperature rise of 40 per cent.

TABLE IV.

D''	λ_1''	λ_2''	Kw. stator loss		° cent. Stator temp. rise		$\frac{I^2R - \text{core loss}}{\text{core loss}}$	$\frac{\text{core loss} - I^2R}{I^2R}$	$\frac{\text{temp. rise copper °cent.}}{I^2R}$	$\frac{\text{temp. rise iron}}{\text{core loss}}$
			I^2R	Core loss	Copper	Iron				
14½	3¾	11⅜	0.72	0.5	28½	27½	0.44	−0.305	39½	54
"	"	"	1.15	0.5	44	41½	1.3	−0.56	38	83
"	5	12⅝	0.7	0.9	25	28	−0.22	0.29	35½	31
"	"	"	0.96	3.2	40	45½	−0.67	2.34	42	14.2
"	"	"	1.12	0.9	38	39	0.245	−0.196	34	43.5
20	3¾	11⅜	0.69	0.67	18	18	0.005	−0.003	26	26.8
"	"	"	0.71	0.67	19	18	0.06	−0.056	25.3	28.4
"	"	"	1.08	0.67	26	26	0.61	−0.38	23.2	38.8
"	"	"	0.93	0.8	24	24	0.175	−0.15	25.7	30
20	7½	16½	1.37	1.2	34	31½	0.142	−0.124	24.7	26
"	"	"	2.04	1.2	50	49½	0.7	−0.41	24.5	41
"	"	"	0.93	1.3	23½	23½	−0.285	0.4	25	18
"	"	"	1.6	1.15	38½	38½	0.39	−0.28	24	33.5
"	"	"	1.6	1.8	43	47	−0.11	0.125	26.5	26

Effect of changing the length of machine. The tests in Table IV, were made on four machines built as in Fig. 5; the dimensions of these machines are shown under columns D, λ_1 and λ_2.

The results of these tests are plotted in curves Fig. 14 and 15 for the machines 14.5 in. diameter, and Fig. 16 and 17 for the machines 20 in. diameter. These latter curves are plotted separately for the two lengths of machine because they lie so nearly on the top of each other.

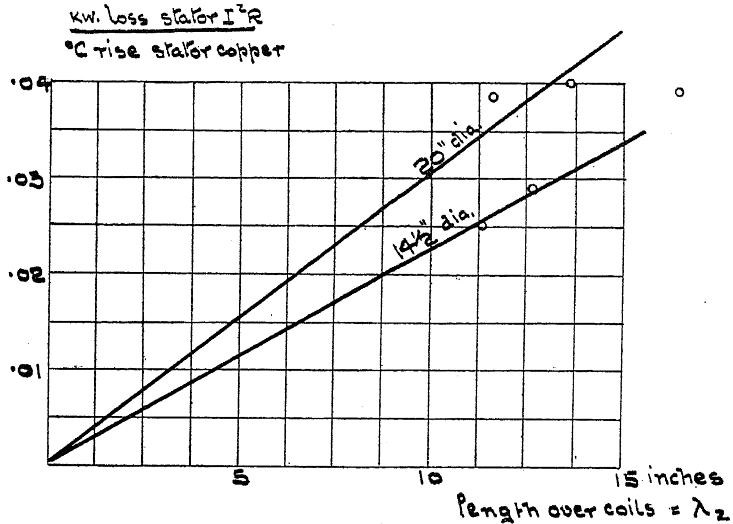


FIG.-18

From Figs. 15 and 17, at the point where core loss = stator I^2R, the value of

$\frac{\text{deg. cent. rise stator copper}}{\text{kw. loss stator copper}}$	D	λ_1	λ_2	$\frac{\text{kw. loss stator copper}}{\text{deg. cent. rise stator copper}}$
40.5	14½	3¾	11⅜	0.025
34.5	14½	5	12⅝	0.029
26	20	3¾	11⅞	0.0385
25 see table VII	20	5½	13⅝	0.04
25.5	20	7½	16½	0.0393

These results are plotted in the curve shown in Fig. 18.

It will be noted in the case of the 14.5 in. diameter machine that $\frac{\text{kw. loss stator copper}}{\text{deg. cent. rise stator copper}}$ is directly proportional to λ_2.

In the case of the 20-in. machine, however, the ratio is nearly constant, that is, there is nothing gained, so far as heating is concerned, by lengthening the machine.

There is a reason for this, however, which adds interest to the results. In the 14.5-in. machine the same bearings and fans are used on both lengths. In the 20-in. machine, on the other hand, as the machine is lengthened, the size of the bearings is increased, and thereby the air inlet is diminished in size. The fans have to be made smaller in order to clear the larger bearing housings, and the hub of the spider has to be increased in order to take care of the larger shaft. It is therefore reasonable to assume that for equally good ventilation

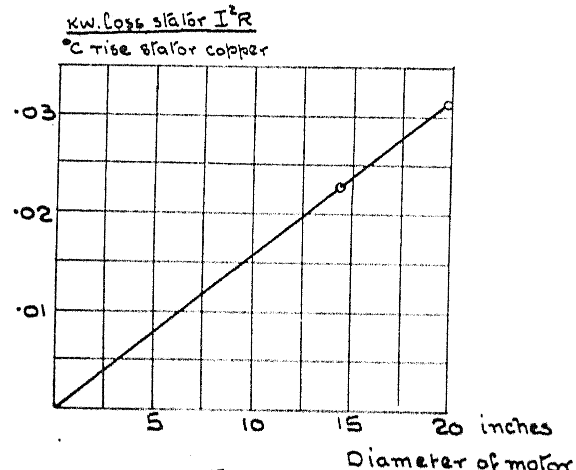


FIG.-19

$$\frac{\text{kw. loss stator copper}}{\text{deg. cent. rise stator copper}}$$

is directly proportional to λ_2.

Effect of changing the diameter of machine. From the curve in Fig. 18, assuming a constant length over the coils $= \lambda_2 = 10$ in.

$$\frac{\text{kw. loss stator copper}}{\text{deg. cent. rise stator copper}} = 0.0225 \text{ where } D = 14.5 \text{ in.}$$

$$0.031 \quad \text{''} \quad D = 20 \text{ in.}$$

These two figures are plotted in curve Fig. 19 from which it will be seen that:

$$\frac{\text{kw. loss stator copper}}{\text{deg. cent. rise stator copper}} \text{ is directly proportional to } D.$$

Effect of the diameter of bearing. It has already been suggested in the case of the 20-in. diameter machine that the size of the bushing may affect the heating to some extent.

The following tests bear this out.

A machine with dia. D = 17 in., λ_1 = 13.5 in. with four 0.5-in. vent ducts, $\sim$ = 25 cycles, p = 4, was tested first with a set of bearings 3 in. by 9 in. and again with a set of bearings 4 in.

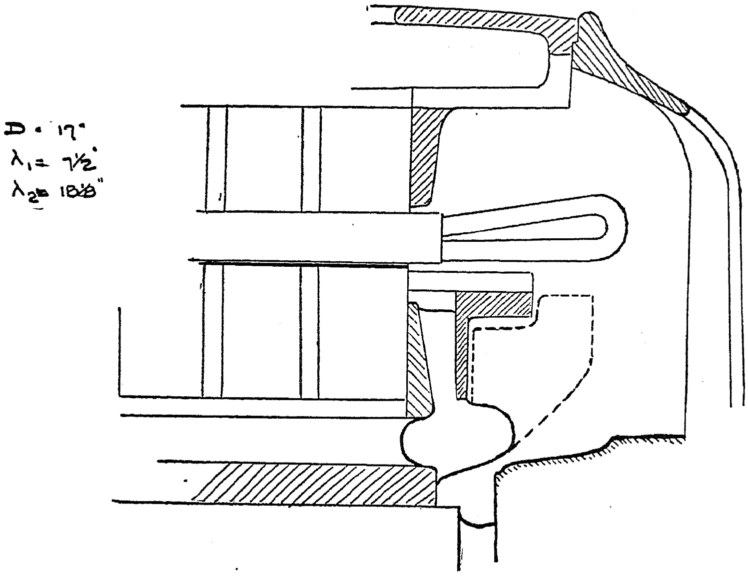


FIG. – 20

by 10 in. the fans and spider hub being unaltered, with the following results:

Bearings	Kilowatt stator loss		Temp. rise in deg. cent.	
	Copper	Iron	Copper	Iron
3 by 9	1.8	2.3	43	39
3 " 9	2.78	2.3	58	53
4 " 10	1.8	2.3	49	45.
4 " 10	2.78	2.3	65.5	57

TABLE V.

~	Syn. r.p.m.	Kw. stator loss		° Cent. Stator temp. rise		$\frac{I^2 R - \text{core loss}}{\text{core loss}}$	$\frac{\text{core loss} - I^2 R}{I^2 R}$	$\frac{\text{temp. rise copper ° cent.}}{I^2 R}$	$\frac{\text{temp. rise iron ° cent.}}{\text{core loss}}$
		$I^2 R$	Core loss	Copper	Iron				
25	750	1.06	1.4	40	36½	-0.242	0.32	37.5	26
"	"	0.975	1.15	37	32½	-0.152	0.18	38	28.5
"	"	0.815	1.4	33½	32	-0.417	0.72	41	23
40	1200	0.91	1.1	27	25	-0.173	0.208	29.5	22.7
60	1800	0.97	2.8	25	25	-0.65	1.88	25.8	8.9
"	"	1.38	2.8	32	32	-0.505	1.02	23	11.4

TABLE VI.

~	Syn. r.p.m.	Kw. stator loss		° cent. Stator temp. rise		$\frac{I^2 R - \text{core loss}}{\text{core loss}}$	$\frac{\text{core loss} - I^2 R}{I^2 R}$	$\frac{\text{temp. rise copper ° cent.}}{I^2 R}$	$\frac{\text{temp. rise iron ° cent.}}{\text{core loss}}$
		$I^2 R$	Core loss	Copper	Iron				
25	750	1.24	1.05	25	24	0.18	-0.153	20.2	22.8
"	"	1.95	1.05	39	34	0.86	-0.46	20	32.5
"	"	0.18	2.1	10½	16	-0.91	10.7	58	7.6

The increase in temperature rise, due to the use of the larger bearings and shaft, is 14 per cent.

Effect of peripheral speed on heating. A machine of the dimensions shown in Fig. 20 was run under the following conditions shown in Table V.

The above tests were made without the rotor fans which are shown dotted. These results are plotted in curves Fig. 21 and Fig. 22.

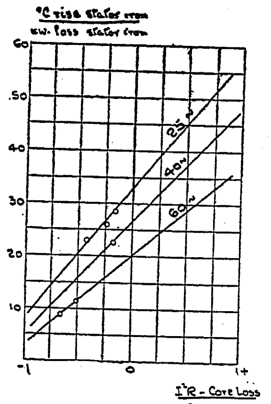


FIG-21

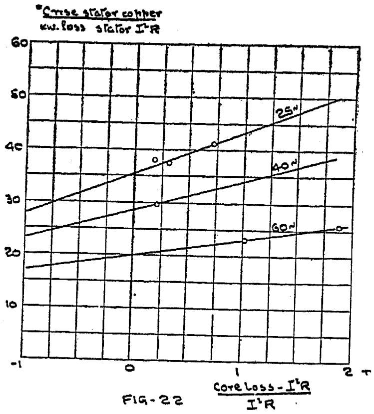


FIG-22

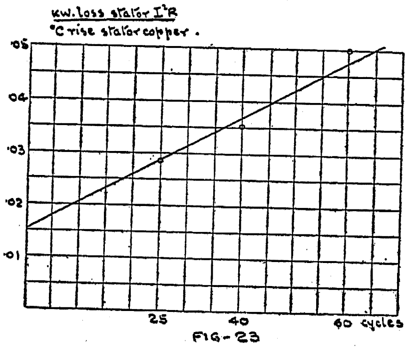


FIG-23

TESTS on an INDUCTION MOTOR

RUN at DIFFERENT FREQUENCIES

From curve Fig. 22 at the point where $I^2 R =$ core loss, we have

$$\frac{\text{deg. cent. rise stator copper}}{\text{kw. loss stator } I^2 R} = \begin{array}{lll} 20 & \text{for } 60 & \text{cycles.} \\ 28.5 & \text{“} \quad 40 & \text{“} \\ 35.5 & \text{“} \quad 25 & \text{“} \end{array}$$

or the reciprocal, viz.:

$$\frac{\text{kw. loss stator } I^2 R}{\text{deg. cent. rise stator copper}} = \begin{array}{lll} 0.05 & \text{for } 60 & \text{cycles.} \\ 0.035 & \text{“} \quad 40 & \text{“} \\ 0.028 & \text{“} \quad 25 & \text{“} \end{array}$$

These latter results are plotted in Fig. 23, which curve shows

that the heat is got rid of by radiation and by convection; the former is independent of the speed of the machine; the latter is directly proportional to speed.

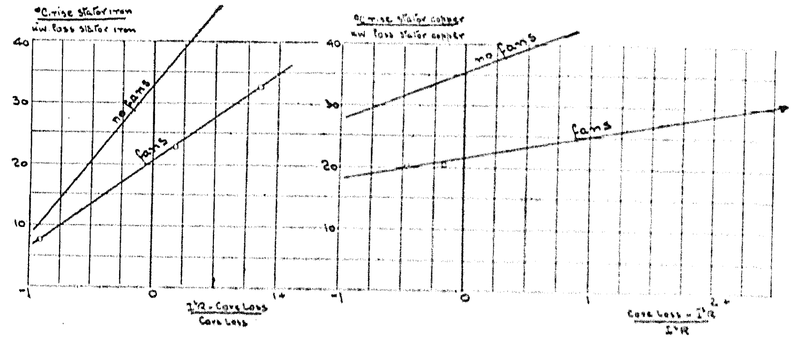


FIG-24

To find the constant of radiation.

$$\text{At standstill } \frac{\text{kw. loss stator } I^2 R}{\text{deg. cent. rise stator copper}} = 0.015.$$

Watts $I^2 R$ per square inch taken care of by ventilation per one degree cent. rise.

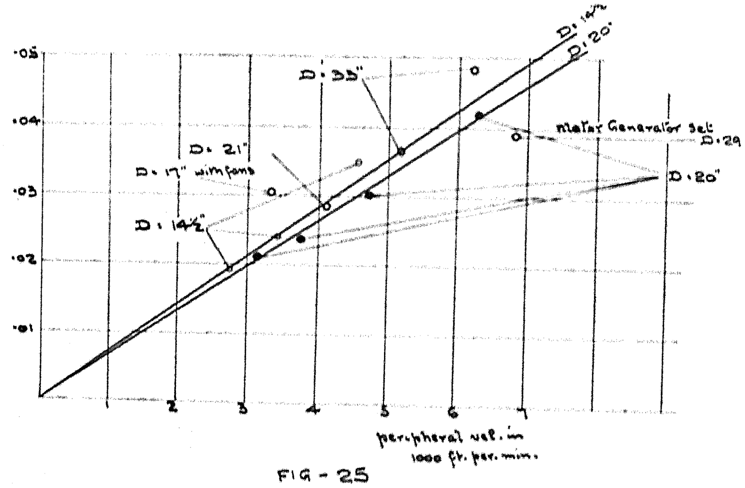


FIG - 25

The radiating surface of the machine is taken as

$$\pi \times \text{diameter} \times \lambda_2 = \pi \times 17 \times 18\tfrac{1}{8} = 970 \text{ sq. in.}$$

therefore,

deg. cent. rise per watt $I^2 R$ per sq. in. radiating surface

$$= \frac{1 \times 970}{0.015 \times 1000} = 65 \text{ deg. cent.}$$

This value has been checked on other diameters and lengths of machine.

Effect of fans on heating. The same machine was supplied with sheet metal fans as shown by the dotted lines, Fig. 20, with the results as shown in Table VI.

These results are plotted in curves in Fig. 24.

General summary of results. Table VII gives a summary of the tests already quoted. The table is self-explanatory, and the results are plotted in the curve in Fig. 25.

Expressing the results in the form of an equation, the temperature rise of an induction motor similar to Fig. 5, at the point where the stator $I^2 R$ loss = stator core loss, is given by:

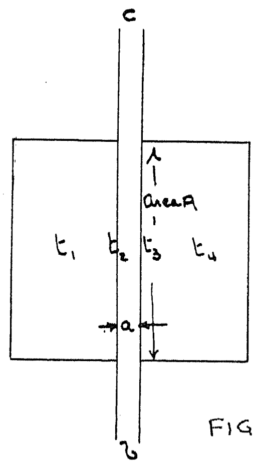


FIG-26

Watts $I^2 R$ for 1 deg. cent. rise

$$= \frac{1000}{N} = \frac{1000}{M} + \frac{1000}{Q}$$

$$= \frac{\pi\ D\ \lambda_2}{65} + 0.0065 \times V \times \pi\ D\ \lambda_2$$

$$= \pi D \lambda_2 \,(0.015 + 0.0065 V)$$

where V = peripheral velocity in 1000 ft. per min.

This equation determines $o\,b$, Fig. 10, and $o\,a$, Fig. 9.

TABLE VII

Poles	~	Syn. rev. per min.	D	λ_1	λ_2	$\pi D \lambda_2$	Radiation from motor at rest. Assume 65° cent. rise per watt I^2R per sq. in. $M = \frac{1000 \times 65}{\pi D \lambda_2}$ ° cent. rise per kw. I^2R radiated	$\frac{1}{M}$ Kw. I^2R radiated per 1° cent. rise	Heating under load conditions N ° cent. rise per kw. I^2R loss = ob Fig. 10	$\frac{1}{N}$ Kw. I^2R loss per 1° cent. rise	Effect of ventilation $\frac{1}{Q} = \frac{1}{N} - \frac{1}{M}$ Kw. I^2R loss taken care of by ventilation per 1° cent. rise	$\frac{1}{Q} \times \frac{1000}{\pi D \lambda_2}$ Watts I^2R per sq. in. taken care of by ventilation per 1° cent. rise	Peripheral velocity in ft. per min.	Remarks
6	60	1200	14½	5	12⅝	575	113	0.0088	34.5 fig. 15	0.029	0.0202	0.035	4560	
8	"	900	"	"	11⅝	530	122.5	0.0082	48	0.0209	0.0127	0.024	3420	
10	"	720	"	"	11⅜	517	126	0.0079	56	0.0178	0.0099	0.019	2750	
6	"	1200	20	3¾	13½	850	76.5	0.0131	20.5	0.049	0.0359	0.042	6300	
8	"	900	"	5½	13¾	860	76	0.0132	25.5	0.0392	0.026	0.0304	4730	
10	"	720	"	6½	14¼	900	72	0.0139	28.5	0.035	0.0211	0.0235	3780	
12	"	600	"	3¾	10⅞	685	95	0.0105	40	0.025	0.0145	0.0211	3150	
10	"	720	33	4½	15⅛	1570	41.5	0.0241	10	0.1	0.0759	0.0485	6220	
12	"	600	"	5½	15½	1610	40.5	0.0246	12	0.083	0.0584	0.0363	5200	
8	"	900	29	7½	19¼	1750	37	0.027	10.5	0.095	0.068	0.039	6820	2 bearing motor generator set 3½ in. vent ducts
4	25	750	21	9½	23¼	1535	42.5	0.0235	15	0.067	0.0435	0.0284	4130	4½ in. vent ducts.
[illegible]	"	"	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Fig. 20 with fans.

It is still necessary to determine the slope of line $c\,d$ for a complete determination of the temperature rise.

In Fig. 10 $g\,c$ is the value of the deg. cent. rise of the stator copper per kw. stator $I^2\,R$ at the point where the core loss = 0. This quantity varies from 70 per cent to 90 per cent of $o\,b$, depending upon the rotor resistance, construction and size of machine; a good guess based on results of similar machines is the easiest way to determine it.

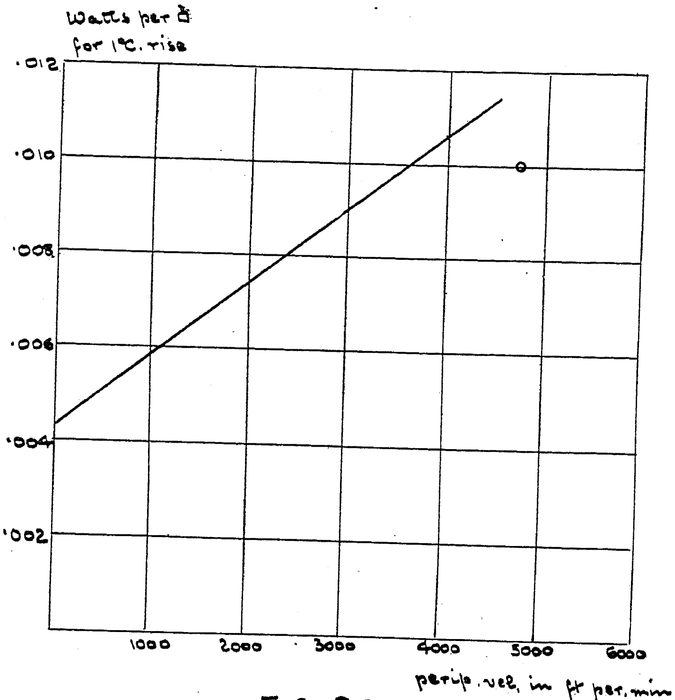


FIG-27

Heating of induction motors totally enclosed. Before discussing this subject the following digression will prove instructive. If $c\,b$, Fig. 26, be a plate of metal through which heat is passing from, say, steam at one side at a temperature t_1, to water at the other side at a temperature t_4, the flow of heat being steady, then if the plate be not too thick k = the thermal conductivity $= \dfrac{Q\,a}{A\,(t_2 - t_3)}$

Where Q = the quantity of heat passing through the plate per sec.

a = the thickness of the plate.

A = the area of the plate.

t_2 and t_3 = the temperatures of the two sides of the plate. This quantity k was measured by three different experimenters who gave the following values: Clément 0'.0057, Péclet 0.178, Angström 1.1,

The-last mentioned result is now recognized as being correct, and the cause of the great discrepancy is of interest in connection with the heating of enclosed machines.

Clément assumed that $t_1 = t_2$ and $t_3 = t_4$. He had steam at the one side at a temperature of 100 deg. cent. and water at

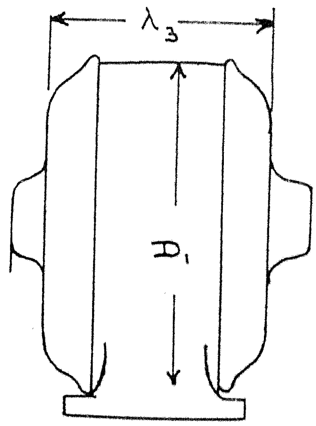


FIG. 28

the other side at a temperature of 28 deg. cent. He assumed that $t_2 - t_3 = 100 - 28 = 72$ deg. cent., whereas we now know it was only 0.36 deg. cent.

Péclet improved on Clément's method by putting stirrers in the two chambers, by which means he got 32 times as much heat through the plate; that is, stirring brings the temperature t_1 more nearly equal to t_2, and t_3 more nearly equal to t_4.

It will therefore be seen that ventilation, or stirring up the air, is of as much importance in an enclosed motor as it is in the open type of machine.

In curve, Fig. 27 is plotted the result of a number of tests on a line of well-ventilated direct-current enclosed motors; also one test on a squirrel-cage induction motor, similar to Fig. 5. The

radiating surface used to determine the value of watts per sq. in. is that of the total external surface of the frame. It equals

$$\pi D_1 \lambda_3 + \frac{\pi D_1^2}{2}$$

The temperature rise is that of the armature or stator coils.

TABLE VIII

D = 20 in. p = 8 poles. External Radiating Surface = 3260 sq. in.
λ_1 = $5\frac{1}{2}$ in. $\sim$ = 60 cycles.
λ_2 = $13\frac{3}{8}$ in. periph. vel. = 4730 ft. per min.

Losses
stator I^2R = 0.77 kw.
rotor I^2R = 0.9 "
stator core loss = 0.77 "

total loss 2.44 kw.

Heat runs temperature rise given in ° cent.

Standard motor Fig. 5	Same motor — Housing holes closed with perforated sheet metal $\frac{3}{8}''$ holes on $\frac{5}{8}''$ centres — Yoke openings A. open	Same motor — Housing holes closed with sheet metal — Yoke openings A. open	Same motor — Housing holes closed with sheet metal — Yoke openings A. closed with cardboard	Parts of motor
$18\frac{1}{2}$° cent.rise	22 ° cent. rise	46° cent. rise	74 ° cent. rise	Stator coils
$18\frac{1}{2}$	20	48	71	" laminations
15	21	46	71	Rotor cond.
14	21	44	71	" ring
14	20	41	69	" laminations
14	24	39	61	Oil in bearings
		25–30	52–57	Outside of yoke

The following figures, in Table VIII, show how nearly uniform the temperature becomes in the inside of such a machine. It is not the purpose of this paper to enter into the subject of the use of open motors with large sheet-metal cases, or the use of forced ventilation or water cooling, yet there is room for valuable discussion on this point.

DISCUSSION ON "THE HEATING OF INDUCTION MOTORS." FRONTENAC, N. Y., JUNE 28, 1909

Theodore Hoock (by letter): In calling attention to the large losses in the motor while the motor is starting, Mr. Gray is apt to give the squirrel-cage motor a worse reputation than it merits. It is true that the losses at the moment of starting are very large, but it is also true that these losses decrease rapidly as soon as the motor is under way. I believe that the squirrel-cage motor can sometimes be used under quite serious starting conditions where a wound secondary motor would be objectionable, as the amount of the starting losses at the first moment proves by no means a serious obstacle. In considering a proposal involving rather severe starting conditions, I proceeded as follows:

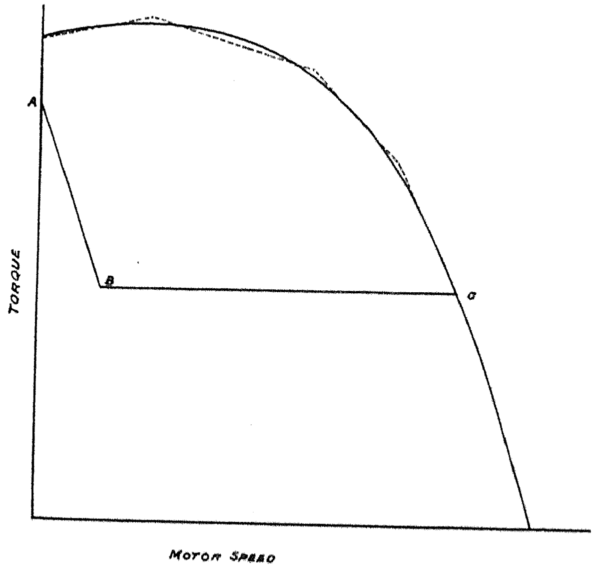


FIG. 1—Speed-torque curve.

The speed-torque curve of any motor can be replaced approximately by a number of straight lines, as shown in Fig. 1. From the total torque, the load torque, represented by the straight line BC, should be subtracted. Also a liberal allowance for the friction of rest should be made for the first part of the starting period, as indicated by the line AB. By subtracting the load and friction torque from the total torque, we obtain the curve of the torque available for acceleration, as shown in Fig. 2.

For each of the straight line parts, the following relations apply, see Fig. 3. The equation for any of the lines is

$$T = T_1 + \frac{T_2 - T_1}{v_2} v \qquad (1)$$

We know further

$$d\,v = p\,d\,t \qquad (2)$$

where $d\,v$ is the change of speed during the time $d\,t$ and p is the acceleration, we find

$$p = \frac{T}{m} = \frac{T\,g}{w}$$

where

m = mass to be accelerated.
w = weight " "
g = acceleration of gravity.

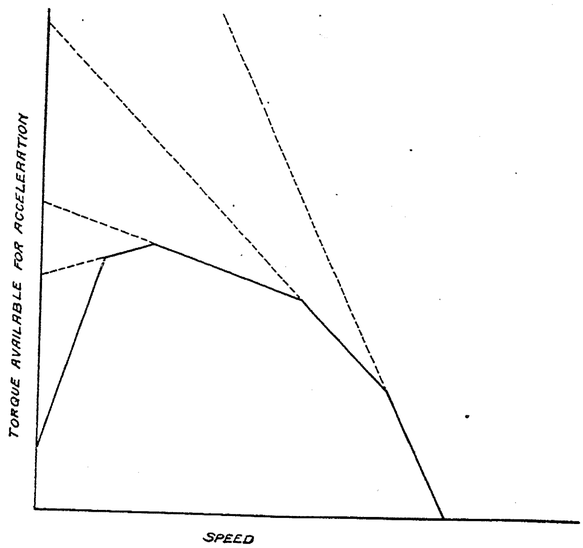


FIG. 2.

By combining equations 1, 2, and 3, we obtain the following differential equation,

$$d\,v = \left(T_1 + \frac{T_2 - T_1}{v_2}\, v \right) \frac{g}{w}\, d\,t$$

From this we obtain by integration,

$$t = \frac{w\,v_2}{g\,(T_2 - T_1)}\; l\,n \left[\frac{T_1 + \frac{T_2 - T_1}{v_2}\, v}{T_1 + \frac{T_2 - T_1}{v_2}\, v_1} \right]$$

For cases where the torque decreases, the following form of the formula is more convenient,

$$t = \frac{w\,v_2}{g\,(T_1 - T_2)}\; l\,n \left[\frac{T_1 - \dfrac{T_1 - T_2}{v_2}\,v_1}{T_1 - \dfrac{T_1 - T_2}{v_2}\,v} \right]$$

For constant torque, we obtain of course,

$$t = \frac{(v - v_1)\,w}{T\,g}$$

The speed-time curves can be directly calculated from these formulas.

From the motor characteristics it is possible to plot a curve

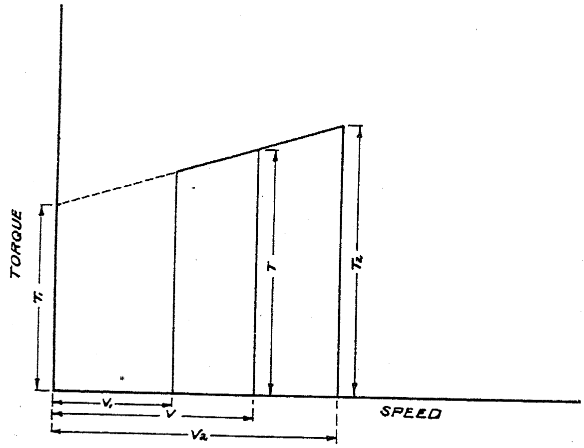


Fig. 3.

showing the relation between losses and speed. From such a curve, and the speed-time curve, a curve giving the losses at any time during the starting period may be plotted. When starting is not frequent, it is sufficient for one start to figure the heating of the various parts from the heat capacity of the parts under the assumption that no cooling takes place during the starting period. This assumption will of course give somewhat too high values, so the figures will be on the safe side. When the starting is frequent, the time-loss curves of the starting period may be used in combination with the curves for the running periods for figuring the continuous average losses. Figuring this way, it will be found in many cases that the heating is not at all serious. By properly designing the motors for the purpose, it has been possible to use squirrel-cage motors; for

Mr. Gray's tests show that the temperature rise of the primary copper is virtually independent of the iron losses. This has been my experience. I use a very simple method of estimating the temperature rise of an induction motor without knowing the dimensions of the winding.

The temperature rise of the primary copper can be found by the equation

$$T_1 = T_0 + \frac{F_1}{F_{vent} \times \left(1 + \frac{v}{1970}\right)} \times A\,S_1 \times S_1$$

T_0 = Temperature-rise at primary copper losses = 0.

F_1 = A constant largely depending upon the type and secondary losses; further upon length of iron, ventilating ducts, insulation and number of poles.

F_{vent} = A constant depending upon the velocity and volume of the cooling air.

v = Speed of rotor in feet per minute.

$A\,S_1$ = Ampere bars per inch of circumference.

s_1 = Copper density in amperes per square inch.

This formula will be a great assistance in beginning a design, as it compares the different types and ventilating devices.

If the temperature rise of the primary and secondary copper is plotted in the ratio of the primary and secondary losses, the points will lie on practically straight lines and cut the ordinate at watts = 0 at a certain temperature rise 3 to 10 degrees cent. (see Fig. 8 of Mr. Gray's paper).

For the calculation of the secondary copper temperature-rise a similar method can be used. In most cases of low-voltage motors the temperature rise of the iron may be said to be equal to that of the primary copper. The application of blowers allows an increase of from 2 to 3.5 times in the primary and secondary losses. For the machine given in Mr. Gray's paper, Fig. 20, the temperature rise may be figured as follows:

Assume for 1 kw. primary losses $A\,S_1 = 500$ $s_1 = 1800$ amperes per square inch.

$F_{vent} = 1$ without blowers.

25 cycles $T_1 = 10 + \frac{81.10^{-6}}{1\,(1 + 1.64)}\,0.500 \times 0.1800 = 37.6°$ cent.

40 cycles $T_1 = 10 + \frac{81.10^{-6}}{1\,(1 + 2.71)}\,0.500 \times 0.1800 = 29.7°$ cent.

60 cycles $T_1 = 10 + \frac{81.10^{-6}}{1\,(1 + 4.06)}\,0.500 \times 0.1800 = 24.4°$ cent.

with fans

$$25 \text{ cycles } T_1 = 10 + \frac{81.10^{-6}}{2.1\,(1+1.64)} 0.500 \times 0.1800 = 23° \text{ cent.}$$

It was tested $37.5° - 28.5 - 25°$ cent.

F_{vent} has been increased 100 per cent.

For high-voltage machines with heavy insulation, special attention should be paid to the *highest* temperature rise of the copper which is generally in the center of the core. The proposed temperature rise of 85 per cent, if measured on the coil-ends, seems to be far too high for cotton insulation.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

REDUCTION IN CAPACITY OF POLYPHASE MOTORS DUE TO UNBALANCING IN VOLTAGE

BY S. B. CHARTERS, JR. AND W. A. HILLEBRAND

It is well-known that polyphase motors of both induction and synchronous types overheat when supplied from circuits in which the ideal conditions of voltage balance and phase angle are not maintained. In order to determine definitely the performance under other than ideal conditions, and to ascertain the points at which overheating commences, a series of tests were made under varying conditions of voltage unbalance, unsymmetrical angular relations between phases, and combinations of these two.

The authors have not attempted to analyze the performance under these conditions in any but the most elementary manner, and are able only to present the results of their tests with such comment as will bring out the essential features developed.

The equipment for the test consisted initially of a laboratory motor built by one of the leading companies; this was a 7.5 h.p., six-phase motor, capable of operating either as a two-phase or three-phase machine. The terminals of all phases were brought to an external connecting block or tablet and all inter-connections were made there. On this motor the first series of tests were made in order to determine the essential features of the performance, and were then followed by characteristic runs on several different commercial machines. The principal data of the machines tested are given in Table I.

Power was supplied by a two-phase 60-kw. inductor alternator, with voltage regulators in each phase, through two 10-kw. multitap transformers. Three phases were obtained from the two-phase supply by means of the Scott connection. By the

use of the voltage regulators and various taps on the transformers any desired degree of unbalance could be obtained. When operating two-phase, the phase angle of one of the applied pressures was shifted by combining with it pressure from a quadrature source, as shown in Fig. 1. This connection also unbalanced the voltage as well, but, when desired, the two voltages could be brought to equality by means of regulators at the generator.

From any three-phase star or delta connection wherein the vector sum of the electromotive forces is zero, it is impossible

TABLE I.

Machine	h.p.	No. of phases	Rotor	Voltage	Frequency	Connection
Laboratory model as induction motor........	7½	2	squirrel cage	190	60	
Laboratory model as induction motor........	7½	3	squirrel cage	127	60	delta
Laboratory model as synchronous motor...	7½	2		310	60	
Commercial induction motor..............	10	2	squirrel cage	200	60	
Commercial induction motor..............	5	3	squirrel cage	220	60	Y
Commercial induction motor..............	2	3	squirrel cage	110	60	delta(?)
Commercial induction motor..............	5	3	Wound rotor with internal starting resistance	110	60	delta (?)

to obtain unbalanced voltages without a shift of the phase angle as well, so that in the three-phase runs no attempt was made to separate these factors.

One set of instruments, especially calibrated for this test, was used throughout. Instruments were applied to one phase at a time and then shifted to the next, two or more sets of readings being taken from each phase, the second serving as a check and to insure that conditions did not change during the time the readings were being taken.

The motors were loaded by belting to a separately excited direct-current generator.

The operating limit of a motor, as determined by the temperature rise in any one phase, is reached when the current in that phase has attained its normal full-load value. Throughout these tests the motor was loaded under varying conditions of voltage unbalance and phase shift until the normal rated current was reached in one phase, when the output at that point was determined. Curves were then plotted showing the relation between the output at this point and the percentage of voltage unbalance and irregularity of phase difference. The voltage in the high phase was maintained at normal value throughout. The per cent. of voltage unbalance is defined as the difference between maximum and minimum voltages divided by the maximum, whether operating two-phase or three-phase.

In the test with the laboratory model, delta connected, it

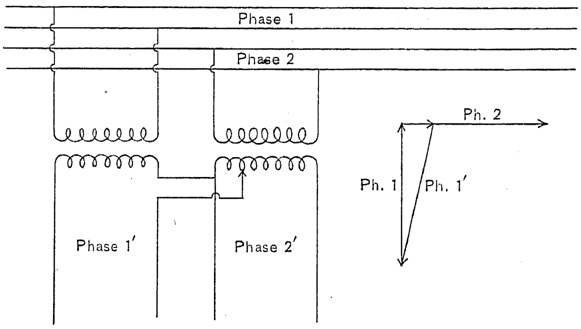


FIG. 1.

was found, on comparing the currents in the motor winding and in the lead wires, that with a condition of unbalance represented by two voltages high and of approximate equality and one voltage low, the current in the lead wire and that in the motor itself reached normal value at the same time, regardless of the percentage of unbalance. This fact was made use of in operating the two 110-volt, three-phase motors, for, not knowing whether they were connected Y or delta, we had no other means of determining when the current in the motor winding had reached normal value. Accordingly the motors were assumed to be delta connected and operated with two voltages high and one low, and, on the basis of the performance of the laboratory model, it was assumed that when normal current was reached in one of the lead wires one phase of the motor winding was also

carrying normal rated current. As indicated by Fig. 2, the motor would probably not behave very differently under any other condition of unbalance.

That the performance is practically independent of the nature of the voltage unbalancing, whether this takes the form of two voltages equal and the other higher or lower, or of all three at different values, is well brought out in Fig. 2, which shows the results of tests made upon a three-phase, Y-connected motor under all three conditions. This point, however, we do not regard as settled, because with our laboratory set delta connected, an appreciable difference has been observed between

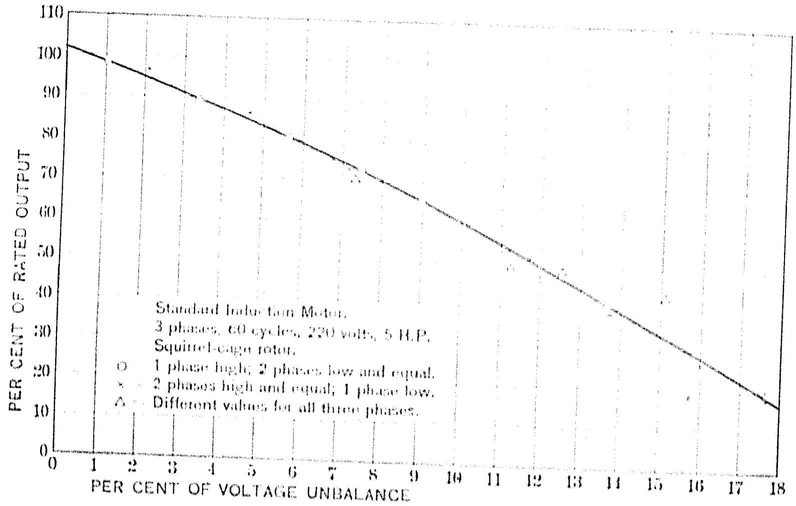


Fig. 2.

the performance with one voltage high and two low, and that with two high and one low, the latter being somewhat the worse condition. The difference, however, is not enough to alter the results materially, and we have not been able to determine as yet whether it is due to the fact that this is a special machine or whether it is more or less common to all motors.

The extent to which reduction in capacity is caused by voltage unbalancing is strikingly brought out by Figs. 2, 3 and 4. We wish particularly to call attention to the practical identity of the curve of Fig. 2 and curves C and D in Fig. 3, which represent the performance of three commercial motors differing in

ype, voltage or rating, in which a voltage unbalance of 10 per cent caused a reduction in capacity of 40 per cent.

The worst performance of all is shown in Fig. 4, which is that of a 2-h.p. three-phase 110-volt motor in which an unbalance of 10 per cent caused a reduction in capacity of 54 per cent or 44 per cent, according to which curve is used. On the name plate 13.7 amperes was given as the full-load current, whereas our test, with balanced voltage, showed 11.1 amperes at the rated output of 2 h. p.

Voltage unbalance, however, is not the only factor to be considered in the case of two-phase motors, for departure of the

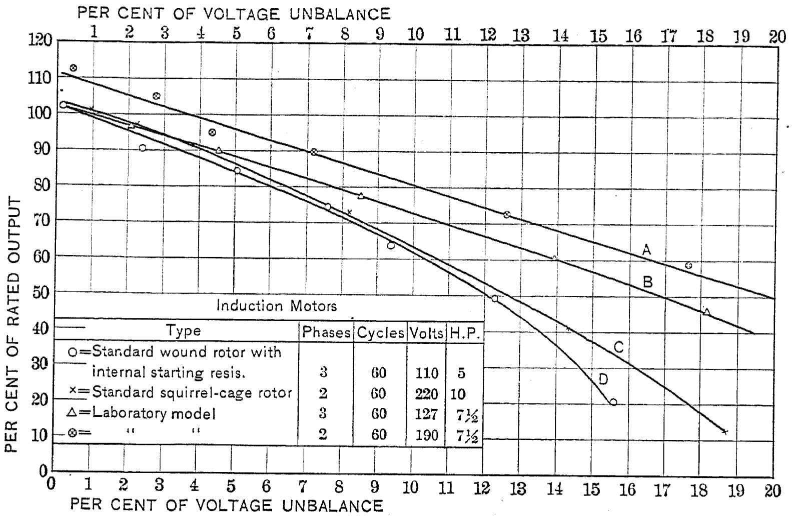


FIG. 3.

supply voltages from the quadrature relation, or phase shift, has an even greater effect. By measurement it was found that our generator gave practically true quadrature phases under all conditions of unbalance, the difference being less than 0.5 degree, so that the results from voltage unbalance alone are not complicated by phase shift.

The extent to which phase shift causes reduction in capacity is shown in Fig. 5, where in the one commercial motor tested a departure of 5 degrees from the quadrature relation caused a loss in output of 26 per cent.

Next to the extent to which reduction in capacity is caused

by voltage unbalance or phase shift, the most significant fact is that the reduction seems to vary with either irregularity approximately according to a straight line law. With a combination of the two, however, in the case of a two-phase motor, the effect seems to be cumulative and certainly worse than the sum of the two individual results. This is indicated by the results given in Table II.

In cases 1, 2 and 3 the laboratory model was operated as a two-phase motor. Cases 4 and 5 were for the commercial 10-h.p. two-phase motor, and furnish evidence along the same line, though the result is not so striking for the reason that, at the particular values used, the effect due to phase shift greatly exceeded that due to voltage unbalance, which therefore contributed a much smaller proportion to the combined result.

TABLE II.

Case	Unbalance alone		Phase shift alone		Unbalance and shift combined			Sum of *A* and *B*
	Per cent unbalance	*A* Per cent reduction	Degrees shift	*B* Per cent reduction	Per cent unbalance	Degrees shift	Per cent reduction	
1	5.38	16.2	10° 1′	20	5.38	10° 1′	50.2	36.2
2	9.05	27	14° 9′	30	9.05	14° 9′	72.7	57
3	8.16	24.3	21° 37′	42	8.16	21° 37′	98.7	66.3
4	2.1	3	5° 27′	31	2.1	5° 27′	36.9	34
5	4.38	10.5	6° 49′	38	4.38	6° 49′	52.4	48.5

Figs. 3 and 5 bring out the fact that the performance of the laboratory model with either voltage unbalance or phase shift was much superior to that of the commercial motors. For instance, with 10 per cent voltage unbalance its capacity was reduced 30 per cent as against 40 per cent for three commercial motors, and with a 5-degree phase shift the figures are 10 per cent for the laboratory model and 26 per cent for the commercial motor. By reference to Fig. 6 it will be seen that the efficiency of the laboratory model is about 3 per cent lower than the average of the efficiencies of the four standard motors. With greater proportions of unbalance and phase shift the difference in favor of the special machine is even greater. This shows in convincing manner how design may affect performance under other than ideal conditions.*

* See Appendix A.

It should be borne in mind that in all the foregoing tests the voltages recorded are those at the motor when operating with full-load current in at least one phase, and that they do not represent line conditions, no account having been taken of transformer regulation or voltage drop in the lead wires.

The effect of transformer regulation is to improve the conditions at the motor, for our experiments showed that the high phase always furnished the heaviest current, which would mean a greater reduction in the voltage of that phase, tending to bring the motor voltages to equality. In the three-phase motor the effect of transformer regulation is not so easy to foresee, because one phase can not be altered without affecting both of the others. However, it appears from observations made to determine this point that the three-phase case does not differ

TABLE III.

No load			Full load in one phase		
Voltage in high phase	Voltage in low phase	Per cent unbalance	Voltage in high phase	Voltage in low phase	Per cent unbalance
115.42	91.22	20.95	109.45	91.67	16.52
115.17	97.17	15.61	110.75	97.02	12.4
113.6	100.25	11.76	109.47	99.05	9.52
115.65	108.65	4.475	110.32	105.37	4.48

materially from the two-phase, in the tendency of transformer regulation to improve conditions of unbalance at the motor. The results of these observations are given in Table III.

In this run the primary voltage was held constant under both no-load and full-load conditions.

These figures must be understood as applying to a particular case, because the inherent regulation of our transformers, as connected, was poor. However, the results may be doubtless taken as indicating the general effect of transformer regulation. It is the line unbalance at no load which is responsible for the performance of the motor, so that this performance will be largely dependent on local conditions. If the motor is operated from its own individual transformers the regulating effect will be much greater than if it is only one of a large number of motors supplied from transformers of large capacity on which the effect

of any individual motor will not be appreciable. Therefore, in a large shop driven by polyphase motors supplied from a common set of transformers, if unbalance exists the performance of the motors will be much better when all are running than when only a few are in operation.*

The improvement due to transformer regulation means a general lowering of voltage, most of which, however, takes place in the high phase. The general lowering of voltage means a reduction in the capacity of the motor, but this is more than counter-balanced by the improvement in the condition of un-

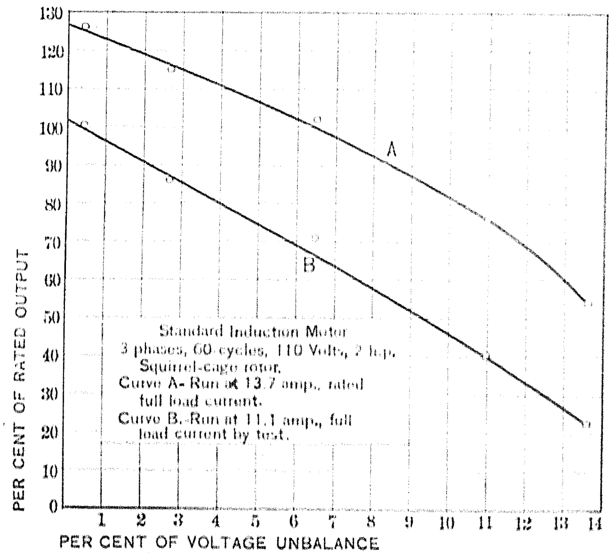


FIG. 4.

balance which enables each phase to approach more nearly its proper share of the power. For example, in our laboratory set a 20 per cent unbalance at the motor with normal voltage in the high phase means a reduction in capacity of about 65 per cent. Assuming a transformer regulation which would give 15 per cent unbalance at the motor, the reduction in capacity would be only 43 per cent, provided the voltage in the high phase could be maintained at normal. If this 15 per cent unbalance at the motor is accompanied by a 10 per cent reduction in the voltage of the high phase, with a corresponding reduction in the other phases, the additional reduction in capacity is only

* See Appendix B.

6 per cent, as shown by curve C, Fig. 7, giving a total of 50 per cent reduction for a combined 15 per cent unbalance and 10 per cent reduction below normal of the voltage in the high phase, as compared with 65 per cent reduction in capacity for 20 per cent unbalance, normal voltage being maintained in the high phase.

Curve C, Fig. 7, shows the reduction in output or torque in the laboratory set, operating three phase and delta connected, as the voltage is lowered at a uniform unbalance of 14.8 per cent, current being maintained at normal value in the high phase. This unbalance of 14.8 per cent is the ratio of the difference between maximum and minimum voltages divided by the

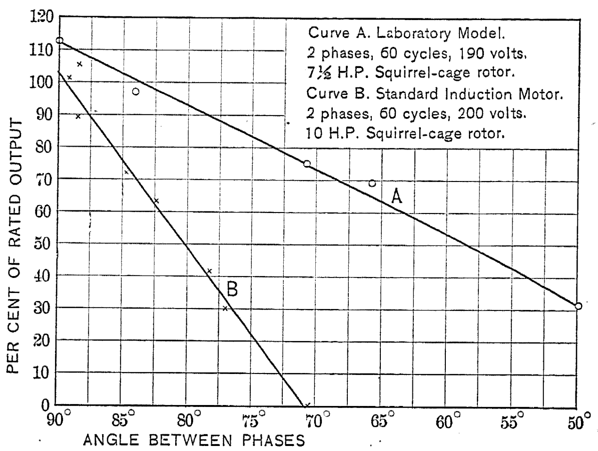


FIG. 5.

maximum voltage and not by normal voltage. Curve A, Fig. 7, shows results obtained with the same motor under conditions of balanced voltage, and curve B those obtained under conditions of balanced voltage but with a different motor.

Curves A and B would represent the performance due to line drop, with balanced voltage. Curve C would approximately represent the performance of a three-phase motor with unbalanced voltage as affected by line drop alone, for the observations recorded in Table IV indicate that with a three-phase motor the line drop does not tend to improve materially the conditions of unbalance.

With the two-phase motor the effect of line drop is similar

to that of transformer regulation, improving the conditions of unbalance at the motor, since the higher phase, where reduction is most needed, invariably furnishes the most current.

Curves A and B, Fig. 7, are of interest in their bearing on the commonly accepted statement that the torque of an induction motor varies as the square of the voltage. This is true provided the slip remains constant. If the slip is allowed to vary and the current held constant, the torque will be found to vary with the first power of the voltage, and this condition is the one usually found in practical operation. The familiar torque equation is

$$Q_p = k \frac{s\, r_2\, E}{r_2^2 \quad s x_2^2} \tag{1}$$

wherein Q_p = torque per phase.

TABLE IV.

Transformer voltage				Motor voltage				Motor voltage
Intermediate phase	High phase	Low phase	Per cent unbalance	High phase	Low phase	Per cent unbalance	Per cent load	Intermediate phase
112.7	130.0	112.0	13.88	115.95	101.1	12.81	83.6	104.9
102.4	128.45	102.0	20.6	113.95	90.25	20.8	71.8	97.4
127.3	127.35	111.1	12.7	120.05	106.15	11.53	69.3	116.25

k = a constant, depending upon units in which torque is expressed.
r_2 = secondary resistance.
x_2 " reactance.
E = primary induced e.m.f.
s = slip.

also

$$s = i_2^2 \frac{r_2 k}{Q_p} \tag{2}$$

wherein i_2 = secondary current.

Substituting the equivalent of s from equation 2 in equation 1, and reducing,

$$Q_p = k\, i_2 \sqrt{E^2 - i_2 x_2^{\,2}}$$

Keeping i_2 constant, this equation represents approximately a straight line, as borne out by the results shown in Fig. 7. Holding the primary current constant, the secondary current will also be nearly constant, a slight change being introduced by the decrease in magnetizing current with the drop in pressure, but not enough to affect materially the straight line relation of torque and voltage.

Synchronous Motor

The test on the synchronous motor was performed in exactly the same manner as on the induction motor except for the fact that the field excitation of the synchronous motor was adjusted at every observation so as to give minimum current in the high phase under the existing conditions.

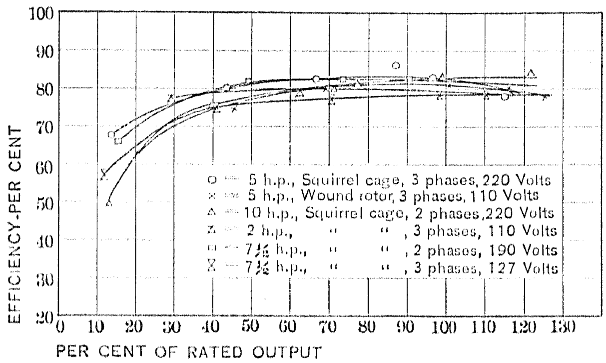


Fig. 6.

The results obtained with the synchronous motor are submitted not on account of their quantitative value but because of the indication they afford of the manner in which synchronous motors and converters behave when supplied from unbalanced circuits.

We wish to call attention to the striking similarity between synchronous and induction motor performances in that the reduction in capacity due to either phase shift or voltage unbalance varies in both cases approximately according to a straight line law, and that shift of phase is a much more serious factor than voltage unbalance. (See Fig. 8.)

The synchronous motor, however, differs materially from the induction motor in its behavior under conditions of combined

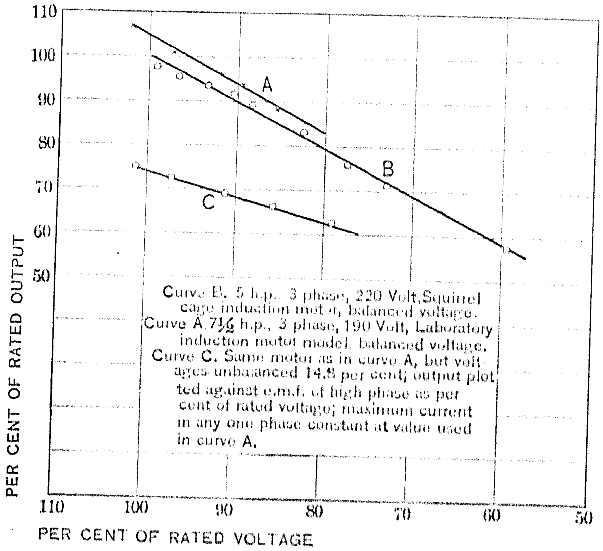

110
100
90
80
70
60
50
A
B
C
Curve B. 5 h.p. 3 phase, 220 Volt, Squirrel cage induction motor, balanced voltage.
Curve A. 7½ h.p., 3 phase, 190 Volt, Laboratory induction motor model, balanced voltage.
Curve C. Same motor as in curve A, but voltages unbalanced 14.8 per cent; output plotted against e.m.f. of high phase as per cent of rated voltage; maximum current in any one phase constant at value used in curve A.
PER CENT OF RATED OUTPUT
110 100 90 80 70 60 50
PER CENT OF RATED VOLTAGE

Fig. 7.

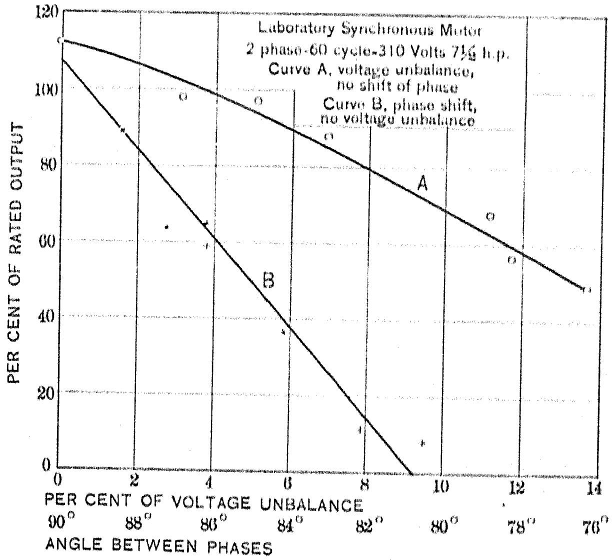

Laboratory Synchronous Motor
2 phase-60 cycle-310 Volts 7½ h.p.
Curve A, voltage unbalance, no shift of phase
Curve B, phase shift, no voltage unbalance
A
B
120
100
80
60
40
20
0
PER CENT OF RATED OUTPUT
0 2 4 6 8 10 12 14
PER CENT OF VOLTAGE UNBALANCE
90° 88° 86° 84° 82° 80° 78° 76°
ANGLE BETWEEN PHASES

Fig. 8.

phase shift and voltage unbalance, in that the two effects seem to neutralize one another, and an improved performance may result, as shown in Table V.

TABLE V.

Synchronous Motor Performance.

Unbalance alone	Reduction in capacity	Phase shift alone; degrees	Reduction in capacity	Sum of reductions	Effect of combined unbalance and phase shift
8.4%	34%	4.0	48%	82%	42%
3.6%	11%	2.1	24%	35%	10%

In general, the effect of voltage unbalancing is to circulate a quadrature current component in one phase. The effect of phase shift is to circulate an in-phase current in both phases, one phase often pumping back into the line. In either case, one phase takes most, if not quite all, of the power. When voltage unbalancing and phase shift exist together, with field excitation adjusted to give equal currents in the two phases, approximately similar conditions exist in the two phases and each is enabled to take more nearly its proper share of the power.*

Summary and Conclusions

Summarized briefly, the results of our tests are the following:

1. Unbalancing of voltage and shift of phase tend to cause serious overheating in polyphase motors.
2. Where it is possible to separate the effects due to phase shift and voltage unbalance, phase shift is found to be relatively of more importance.
3. The reduction in capacity due to either phase shift or voltage unbalance varies with the cause approximately according to a straight line law.
4. In the induction motor the existence of phase shift and voltage unbalance causes a worse performance than would be expected from either irregularity alone, whereas in the synchronous motor the reverse may be the case.
5. The performance of the induction motor supplied from an unbalanced circuit is materially affected by its design.

* See Appendix C.

6. The commonly accepted statement that the torque of an induction motor varies with the square of the voltage should be changed to read, "varies directly with the voltage," in order to make it more consistent with conditions of ordinary operation.

The authors hope that this paper will promote further investigation and discussion of the subject of line unbalance, which is continually increasing in importance as transmission lines increase in the amount and variety of service which they furnish.

APPENDIX A

An induction motor will endeavor to carry the load put upon it, regardless of the condition of supply voltage, so it is important

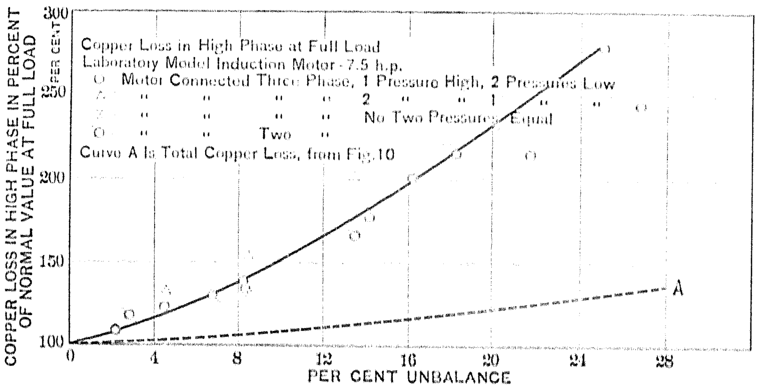


FIG. 9

to know not only what load the motor will carry with a given amount of unbalance without overheating in any phase, but also what the effect will be upon the motor if it is forced to carry full load when supplied from unbalanced circuits.

Fig. 9 shows for the laboratory model, the ratio of copper loss in the phase which carries the high current under varying conditions of unbalance, when the motor is delivering its rated load, to the copper loss at full load with balanced voltage. With this motor an unbalance of 10 per cent means a 50 per cent increase in copper loss in one phase, and with a good commercial motor the increase would probably be greater. As shown in Fig. 10, however, the total increase in copper loss for moderate unbalancing is not serious.

APPENDIX B

Transformer Regulation—Open Delta

An exception to the statement that the effect of transformer regulation is to improve conditions at the motor should be made in the case of three-phase service from two transformers connected as an open delta. When supplying current to a balanced, non-inductive load the power-factor in each of the two transformers, assuming balanced primary voltages, is 86.6 per cent, leading in one transformer and lagging in the other. The regulation in each transformer is different, with consequent un-

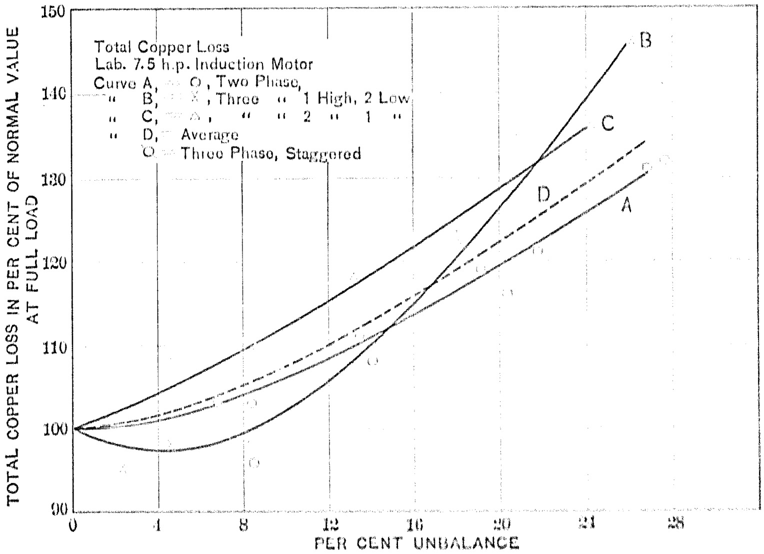


Fig. 10.

balancing and circulation of unequal currents to bring the voltages back to approximate equality.

When supplying current to induction motors operated at, say 85 per cent power-factor, the phase-angle of the current in each transformer is shifted about 30 degrees. One transformer is operating now at almost unity power-factor and the other at a power-factor of about 50 per cent. When the pressures are unbalanced, if the transformer with inherently the higher power factor has also the higher voltage and current, the impedance drop in each transformer may be about the same, the low power-factor of one compensating for the heavier current in

the other, with little or no improvement in the conditions of unbalance. This is illustrated in Table VI.

TABLE VI

Volts			Amperes			Watts		Power-factor		Per cent unbalance	
A C	*B C*	*A B*	*A*	*B*	*C*	*A C*-A	*B C*-B	*A C*	*B C*		
226.0	224.3	222.5	25.8	26.3	27.2	6055	3435	1.04	0.583	1.55	Load
233.6	230.2	232.1								1.45	No load
226.3	197.2	211.3	24.4	13.9	25.7	5670	1660	1.03	0.606	12.9	Load
232.6	202.4	220.0								13.0	No load
225.0	212.2	218.6	23.6	18.5	25.35	5440	2300	1.02	0.586	5.7	Load
230.1	216.8	223.8								5.8	No load
244.8	225.5	232.9	24.6	18.95	25.9	6240	2520	1.04	0.59	7.9	Load
248.8	232.4	244.3								6.6	No load
246.4	229.4	235.8	24.0	17.5	25.2	6140	2280	1.04	0.568	6.9	Load
251.9	230.9	244.4								8.33	No load

A diagram of connections is given in Fig. 11. The open side of the delta is *A B*. Full load current for each transformer is 45.4 amperes.

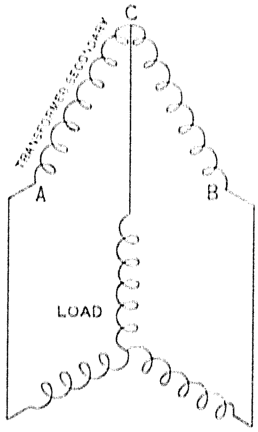


Fig. 11.

APPENDIX C

Synchronous Motor

In Fig. 12 is shown the performance on unbalanced voltage of the same motor connected three phase. The scattered location of some of the points is in part to be accounted for by the

fact that the current in the high phase was not exactly the same for all observations, the points above and below curve B corresponding in general to values of high and low current. The conditions of unbalance were chiefly two pressures high and one low with two or three staggered values, for the transformer regulation was so great that we could not get one pressure high and the other two low with the motor loaded.

After each observation with the motor loaded, the motor was immediately disconnected and the line pressures at no load were read to determine the effect of transformer regulation, or the

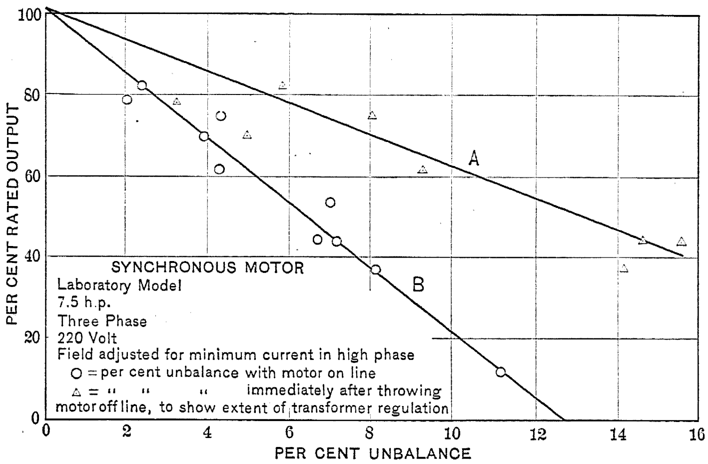


FIG. 12.

power of the motor in balancing up the line. The effect in this case is extreme because our particular transformers when transforming two-phase to three-phase, to furnish 220 volts at the secondary, regulate poorly. Curves A and B may be of interest, however, as showing the balancing effect in a particular case. The capacity of the motor was somewhat less than half the combined capacity of the transformers from which it was supplied.

That the chief regulating effect was in the transformers was shown by reading generator pressure with the motor on and off the line. The change in generator pressure with load was small.

Discussion on "Reduction in Capacity of Polyphase Motors Due to Unbalancing in Voltage." Frontenac, N. Y. June 28, 1909

R. E. Hellmund (by letter): All of the interesting figures regarding the harmful effect of unbalanced voltage upon the rating of polyphase motors given in this paper are based on the assumption that the largest current in any of the phases should not be larger than the rated full-load current. This assumption would lead to correct conclusions if there were no interchange of heat between the various parts of the stator; it is my experience, however, that there is a comparatively free interchange of heat between the various stator parts. This is brought out by the fact that the final temperature of the iron and coils is usually pretty much alike, no matter how the losses are distributed between iron and copper.

If the interchange of heat were complete, the tests for determining the influence of unbalanced voltage should be based on the total losses in the stator; that is, we should have in a two-phase motor $I_1^2 + I_2^2 = 2\, I^2$, and in a three-phase motor $I_1^2 + I_2^2 + I_3^2 = 3\, I^2$ where I_1, I_2, and I_3 are the unbalanced currents, while I is the normal rated current. If the tests were based on this assumption, the effect of unbalanced voltage would not by any means be as bad as that indicated by the curves in the paper. The interchange of heat is of course not perfect, so the practical result will be somewhere between the values given by the curves of the paper and curves based on the above equations.

Moreover, most motors are designed to allow for certain irregularities in the working conditions, and will stand a certain amount of unbalanced current without any reduction in the rating. Another thing that tends to reduce the danger caused by unbalanced currents is that the amount of unbalancing usually decreases with the load. It is, therefore, the smallest for the overloads; that is, for the condition for which increased heating through unbalancing is the least desirable.

It is well to mention these facts, for it is just as undesirable to overestimate harmful effects as it is to underestimate them. Overestimating harmful effects would induce an engineer to go to an expense out of proportion to the value of the results obtained thereby. Nevertheless I agree that unbalanced currents in polyphase motors are undesirable. In certain extreme cases they are even dangerous to the motor. If possible they should be avoided. As mentioned in the paper, the amount of unbalancing in the current depends not only upon the unbalancing of the voltage but also upon the design of the motor. The question arises, therefore, whether an attempt should be made to obtain motors that give for a certain unbalancing of the voltage a minimum unbalancing of the current. This question may be answered by the following considerations.

The current of an induction motor may be considered to consist of three parts: 1. The load current. 2. The leakage reactance current. 3. The no-load reactance current. It is the latter current that is chiefly affected by unbalanced voltages. This may be best explained by referring to a single-phase induction motor. In this type of motor a field is induced by the primary in the direction of the primary-coil axle, while certain secondary currents in the light running motor carry the field around and induce also a field at right angles to the first axle. At no load the size of this field at right angles is only a few per cent smaller than the primary field, the difference being chiefly caused by the ohmic drop in the secondary. If, therefore, the primary is provided with a second winding equal to the energizing winding, but at right angles thereto, the field carried around by the secondary will induce in this winding a potential only a few per cent smaller than the impressed single-phase potential and shifted 90 degrees against it. If upon the second primary there is impressed a potential which is opposite and equal to the potential induced by the secondary, there will be no current in this second winding. The motor can now be considered to be a two-phase motor with unbalanced potentials impressed; and it will be seen that while the unbalancing of the potential is only a few per cent, the unbalancing of the magnetizing current is as much as 100 per cent. If we further decrease the potential impressed upon the second winding, the motor will furnish current to the line through this second phase, which then acts as a generator. If, however, we increase the potential impressed upon the second winding, this winding will take current, and the current in the two primary windings will be equal as soon as the two primary potentials are equalized.

It has been shown above that a 100 per cent unbalanced condition is obtained if the unbalancing of the potential is equal to the ohmic drop of the secondary. This means that for a given unbalancing of the primary currents the unbalancing of the primary potentials may be the larger, the larger the ohmic drop in the secondary. In other words, the tendency for unbalanced magnetizing currents in the primary may be decreased only by increasing the secondary resistance.

It is at once obvious that this means should not be used as a remedy, since the increased losses caused thereby in the secondary are more harmful than the usual unbalancing of the total primary current. The unbalancing in the load current and the leakage reactance current is not quite so serious as that of the magnetizing current. Moreover, I do not think the design of the motor has much influence on it. It might be suggested that since the no-load magnetizing current is chiefly affected by the unbalanced voltage, the unbalancing of the total current may be kept small by designing the motors for small magnetizing currents. While this is true, it is hardly of any importance in this connection, since the designer always re-

duces for other reasons the magnetizing current as far as other considerations allow.

I have considered in the above an unbalancing in the intensity of the voltage only. The consideration of the case of incorrect phase displacement leads to similar conclusions. In this case the unbalancing of the current may be reduced by increasing the primary resistance and leakage reactance, while both of these factors should be kept small for other more important considerations. I believe it is usually safe to say that any unbalancing effect can be only reduced by adding resistance or reactance somewhere in the motor circuit; that is, by harmfully affecting the motor performance in other respects.

It appears, therefore, that the only remedy that may be applied to unbalanced currents in the motors is to improve the line conditions. Precaution is advisable regarding the statement made by the authors of the paper about the influence of the potential upon the torque of induction motors. It is stated in the paper that the torque is directly proportional to the potential; this statement is based on the assumption that the size of the current alone is limiting the motor rating. It should be remembered, however, that if the potential decreases, the core losses decrease also, and therefore larger copper losses and a larger current are allowable. For this reason it is difficult to make any correct statement concerning the continuous torque rating. The usual statement, that the torque is proportional to the square of the potential, applies only to the starting torque and the maximum torque, and is approximately right for these cases.

A. E. Averett: I do not think that the statements made in Conclusion 6 are correct. The ordinary condition of operation in the case of induction motors is at constant voltage. The torque of an induction motor is proportioned to the square of the voltage.*

Regarding the balancing of induction motors; I have found the same thing in a number of tests. I would express it in terms perhaps a little different, as follows: a motor with 15 per cent unbalanced voltage will heat almost double what it would at normally balanced voltage. It operates very much as it would single phase, (in which case the maximum output of the motor would be about 40 per cent of the three-phase) plus a supplemental quadrature component.

A. M. Dudley: I do not understand the conditions under which the writer wishes us to assume that the torque varies directly as the voltage; for under the conditions ordinarily met with, I think it is true that the torque varies as the square of the voltage. In the usual conditions of operation the current is not regulated, but automatically adjusts itself as the load is varied. For this reason the torque will certainly vary as the square of the voltage.

*See Steinmetz " Alternating Current Phenomena," p. 248.

John C. Parker: I wish to call attention to the commercial phases of the feature of unbalancing brought out in the paper. Where isolated industrial plants are used the distribution is almost always by means of conduits wherein line conditions do not obtain to unbalance the voltage, and wherein heavy single-phase loads are not carried in the plant to unbalance the voltage. Where long-distance power transmissions are used there is a possibility of unbalancing of the phases. With central station companies the same thing almost always occurs, owing to the carrying of lighting loads and single-phase motor loads. As a matter of experience, this current unbalancing in the different phases of induction motors has been found to be quite serious. Many of the transmission companies selling hydroelectric power incorporate into their contracts a clause requiring the balancing of the loads taken by the different phases; under such conditions, if a heavy motor load is carried and the voltage is unbalanced, the user of the power may be penalized by virtue of that fact. The motor struggles to deliver uniform counter electromotive force on all the phases, the line does not do equally well, and the difference must be made up in the circulating of wattless currents in some phases of the motor. So much for the matter of the purchase of power from a transmission line.

Taking the case of the ordinary distributing companies, distributing to retail customers. In the effort to improve the load-factor, most of the central station companies are adopting a basis of power sale in which the customer having the better load-factor receives the better rate. To determine what the maximum demand is, some form of indicator is used. Up to the present date the most commonly used maximum indicator is a meter or indicator depending on the heating effect of the current. The meter takes no cognizance whatever of the circulating current taken by the motor in its struggles to maintain mormal conditions, as to whether the circulating current is in one direction or the other. If the maximum demand meter unfortunately happens to be in the phase taking the high current, the customer is penalized; if in the phase taking the lower current, the company suffers. If maximum demand indicators are installed in all phases and read once a month, the conditions may change between readings, so once a month a customer might still be penalized as the meter records only the maximum demand which has been pulled.

I am aware, as we all are, that there are now under development maximum demand meters not depending on the current drawn by the line, but again the commercial consideration comes in that it is very desirable to penalize the customer whose demand is at a poor power-factor, and hence there are strong commercial reasons why a maximum demand wattmeter rather than a maximum demand ammeter should be employed. Hence we do not get away from the difficulty introduced by the fact that an unbalance of the phase angles or of the voltage

produces a deviation from balance in the current which is all out of proportion to the cause. This brings home very significantly the importance to distributing companies of keeping their load balanced as far as possible, and, where that is not possible, of the balancing of voltage by single-phase rather than by polyphase potential regulators.

Charles P. Steinmetz: In reading this paper I was astonished by the statement that the torque of the induction motor varies directly in proportion to the voltage, until I found the cause of this mistake in the assumption that the current must not exceed a definite maximum value. The torque of the induction motor varies with the load, so that in considering the relation of the torque to the voltage or anything else it is necessary to state what torque is meant. There may be under consideration the maximum torque of the motor; that is, the torque where the motor drops out of step and comes to a standstill. Or it may be the torque that the motor can maintain continuously with constant heating, the safe heating of the motor. Both torques are important. The maximum torque of the induction motor varies with the square of the voltage. To send a three-phase locomotive over a long-distance railway with twenty per cent drop in the line, trusting that the maximum decrease of torque in an emergency will be only twenty per cent, will result in disappointment, because the maximum torque drops with the square of the voltage. Again, the torque which the motor can maintain continuously at constant heating does not vary proportionally to the voltage, nor proportionally to the square of the voltage, because the heating depends not only on the current but also on the magnetic flux, that is on the voltage. If the voltage is increased, with the same current, the losses, and thereby the temperature, are also increased; to maintain the same temperature the current must be lowered correspondingly, while at lower voltage the current can be raised. It will be found, then, that for the same heating of the motor the maximum torque which the motor can carry varies with the voltage. The torque is greatest at a certain definite voltage, while at a lower voltage as well as at a higher voltage the torque which the motor can give with the same heating, is less.

Applying this to the case of unbalanced voltage: if the voltage in one phase is high and in the other low, the energy loss is shifted in the copper by raising one current and lowering the other, and also to some extent the losses are shifted in the magnetic field, and the simple relation given in the paper does not hold true; but it will be found that at constant heating the permissible motor-torque is decreased only very little by an unbalancing of voltage. A considerable unbalancing of voltage must occur before the torque is appreciably decreased at the same heating, or the heating is increased at the same torque.

These relations of the torque to voltage and unbalancing, as determined by the heating of the motor, are still further com-

plicated by the design of the motor, because not merely the total energy loss counts, but local heating must be considered. In one motor the loss of power in the coils may be more objectionable, because the heat cannot escape rapidly through the coil insulation. In another motor the coils and the iron may, through the entire range, maintain practically the same temperature; for instance, in a low-voltage motor. Therefore the relation is a more complicated one than appears at first sight; it depends on the design of the motor, the character of the insulation, its thickness; that is, on the voltage for which the motor is intended, etc. But speaking broadly, we must say that *the maximum torque of the motor varies with the square of the voltage*; that the *safe heating torque is a maximum at some definite voltage*; that the *unbalancing of the voltage is relatively harmless*, up to a certain amount, but becomes objectionable and harmful only when excessive.

Chas. F. Scott: This paper takes up an important and interesting feature of the induction motor. The basis of comparison, however, in which the normal current from one of the motor circuits is taken as a standard of comparison, is not a proper standard of comparison for ordinary practical operation, as has been pointed out by several of the speakers. There would be some danger, therefore, in applying the results given in the paper promiscuously to average circumstances. The practical point in operation is to determine whether a motor operating on a commercial circuit is getting into difficulty, and is going to have its safe operating conditions exceeded by an unbalancing in voltage. The safe capacity of the motor is not determined by the current through one of its circuits. That does not determine its temperature. Sometimes the heating of the motor is not the limit. It may be the regulation, the starting conditions, or something else. Some induction motors run at very low temperatures, so that a considerable unbalancing would not lead to an excess in the temperature conditions. The motor not merely receives more energy on one circuit than on the other, but it may act as a transformer. Synchronous motors and induction motors upon an operating circuit tend to lessen unbalancing between phases and hence have an equalizing effect.

That motors differ among themselves in their characteristics should be taken into account before applying the results of this paper to every condition. Motors differ not only in their temperature distribution, but also in various other characteristics. When acting as an equalizer between unbalanced circuits a large motor or one which is a poor transformer may not have any trouble in equalizing, whereas a little motor or one which is an efficient transformer may overexert itself. The summary of my comments is that the tests, while interesting, are based on certain laboratory conditions which do not permit their indiscriminate application to practice.

H. L. Wallau: Referring to unbalanced voltage, I have in mind some readings taken in a case of trouble. A 110-volt,

three-phase motor had been connected to a circuit consisting of two transformers arranged in open delta, one of them, however, cut over to a secondary voltage of 230 volts, and the other to 115 volts, so that the voltage applied to the motor was 230 volts across one phase and 115 volts across another phase, and 200 volts across a third phase. When this motor was started on its load, the voltage dropped in one phase from 230 to 220, in another from 200 to 196 volts, and jumped from 115 volts to 122 volts in the third phase. It confirmed the fact that motors will work as transformers when the supply voltage is unbalanced and fed back into the line.

S. B. Charters, Jr.: The only point I wish to make is in regard to the matter of torque. We probably did not make clear what we meant. Our idea was this: a man is operating a motor under such load that he is drawing practically the limiting current of the motor as far as heating goes. If, under these conditions the voltage is reduced, the torque of the motor will be reduced; that is, provided each time the full-load current is considered as the limiting factor. As the voltage goes down, less torque will be developed; it will vary approximately as a straight line loss, as far as our results go, as represented in Fig. 7, curves A and B. However, as has been said, to a certain extent these are laboratory results, and that should be taken into consideration before applying them to commercial circuits.

As stated by Mr. Scott, the difficulty in starting is an important point, because it was a case of that kind which first drew our attention to the difficulties introduced by unbalancing. In a plant operating three-phase, certain motors at considerable distance from the transformers would not start without the rest of the plant, because under the conditions of unbalancing which existed at the plant they were unable at times to develop enough torque to start under load. With the plant operating, the other motors balanced these conditions so that these motors would operate. When they were operating alone they were unable to affect the transformer conditions sufficiently and were not able to start.

I. E. Hanssen (by letter): The results obtained by the authors of the above paper agree closely with observations made by others, as may be seen by comparing the results obtained with the value given on the attached curve Fig. 1 which shows results of tests on a fairly large commercial motor. Of course, different motors will behave differently, depending upon the ease with which heat is conducted from the hottest places to the cooler ones.

It should be borne in mind that in the majority of cases the amount of unbalancing existing in well designed circuits is very small. A very small number of cases of trouble are reported from this cause. The writer believes it would be a serious mistake to build motors designed to minimize the effect of voltage or phase unbalancing by increasing the impedance

of the windings, as such motors would have a low maximum torque, low starting torque, low starting efficiency, and probably low power-factor as well. All that is necessary and desirable is to take all reasonable precautions to prevent unbalancing of the circuits; that is, all conductors should be arranged as nearly symmetrical as possible and unbalanced transformer connections such as open $\triangle$ and two-phase—three-phase, should be avoided whenever possible.

It goes without saying that motors which are themselves unbalanced should not be tolerated.

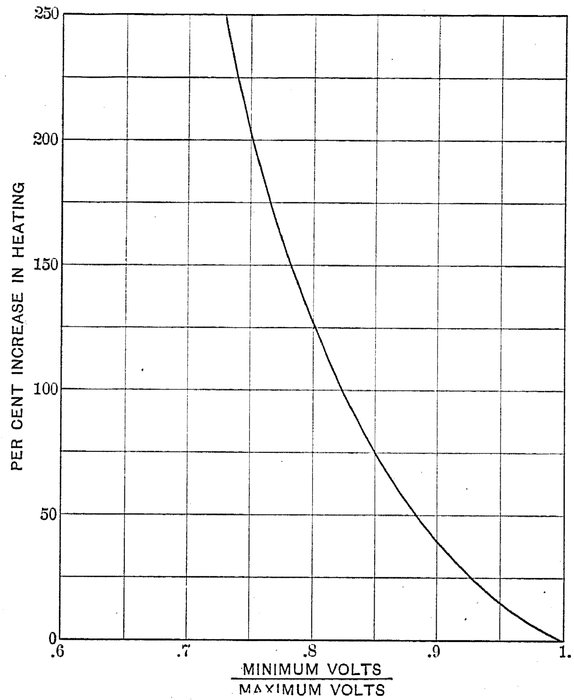


FIG. 1.

Such inherent unbalancing of a motor may be due to an unequal number of turns in the different phases, improper angular displacement between phases, or to unsymmetrical end-connections, the existence of none of these causes being justified.

That circulating currents may easily be of such magnitude as to be very objectionable, will be seen from the following. Take the case of a machine which has two or more parallel circuits per phase displaced from each other by say 10 electrical degrees. Then the voltage tending to set up circulating currents will be approximately $2 E \sin 5$ degrees $= 0.174 E$, when E is the impressed electromotive force.

Assuming the short-circuit current at rated voltage to be six times full-load current, then the circulating current will be about equal to one-half the full-load current.

If the impedance of a motor and its supply circuit are known,

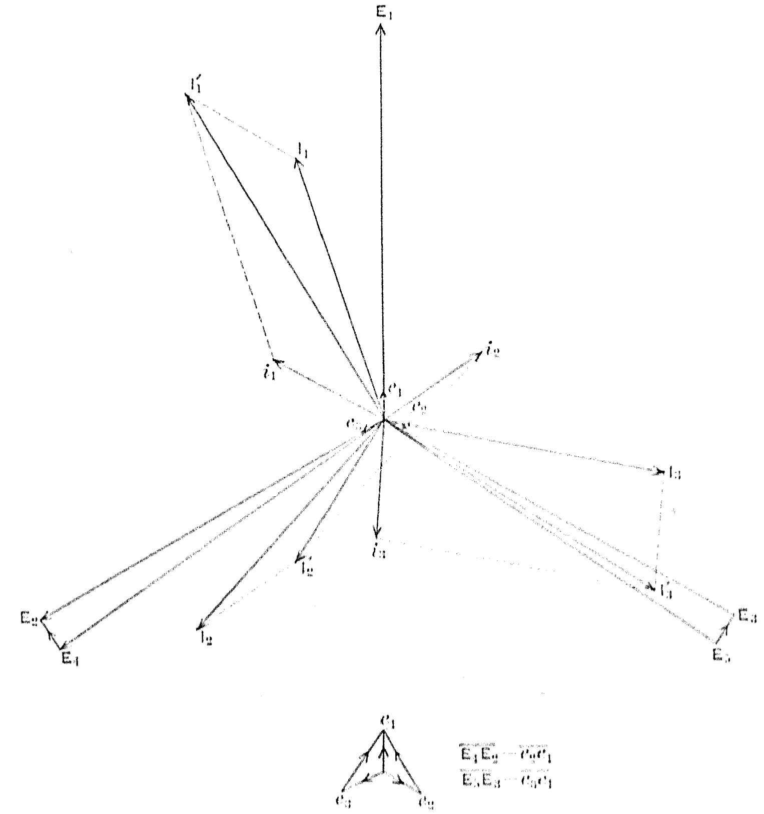


FIG. 2.

E_1 E_4 E_5 = unbalanced impressed electromotive forces
E_1 E_2 E_3 = three-phase counter electromotive forces of the motor
I_1 I_2 I_3 = full-load current on balanced circuit
i_1 i_2 i_3 = circulating currents due to e_1, e_2 and e_3
I_1' I_2' I_3' = resultants of normal full-load current and circulating currents

The circulating currents i_1 i_2 and i_3 are inversely proportional to the sum of the motor impedance and the circuit impedance per phase and lag behind their respective electromotive forces by angle $\tan^{-1}\phi = \frac{x}{r} = \frac{\text{rectance (inductive)}}{\text{resistance}}$

it is a simple matter to determine approximately the amount of unbalancing for any given load and voltage or phase unbalance. As an illustration Fig. 2 shows a graphical method applied to a three-phase motor.

S. B. Charters, Jr. and W. A. Hillebrand (by letter): Those who have discussed this paper agree that unbalancing is undesirable, but carefully refrain from stating the point at which the consequences of this unbalancing may become serious. This point naturally depends upon the construction of the motor and the nature of its load, but the authors believe it to be reached earlier than most of those who have expressed an opinion seem willing to admit. Our belief is based upon the performance of motors which have burned out when operated steadily from regular commercial circuits that were unbalanced, and upon the showing of Fig. 9 and 10, printed in Appendix A. These represent the stator copper losses in the motor giving the best performance on unbalanced voltage, when carrying its rated load. The abnormally high copper loss in one phase is in accord with our experience, that serious injury to the motor may occur at a much smaller degree of unbalance than would be expected from a consideration of the total heating alone.

With regard to the regulating effect of induction and synchronous motors, dwelt upon by Messrs. Hellmund, Wallau, and Scott, we wish to call attention to the fact not emphasized by them, though mentioned by Mr. Scott, that this regulating effect is obtained only at the expense of the motor, through the circulation of additional wattless current, and that the extent of this balancing effect in any particular case will depend entirely upon individual conditions. For instance, if it is a small motor operating from large transformers at a time when other motors on the same circuit are idle, the regulating effect will be nil.

The offhand proposal to balance up the line as a remedy for the evils of unbalancing may be easy of application on a system of limited extent, but becomes an altogether different matter when applied to a network covering hundreds of miles, subject to different climatic conditions in its various parts, and furnishing a varied and miscellaneous service to meet the demands of the territory it traverses.

In stating that the torque of an induction motor varies as the voltage, the authors had in mind the torque at constant heating, which though of the greatest operating importance, is not commonly considered. Although our original contention may not be strictly correct, we feel that we have made our point in showing the necessity for stating which torque is referred to—that at starting, pull-out, constant or synchronous speed, or constant heating. In the first three cases the torque admittedly varies as the square of the voltage. It is the torque at constant speed which is referred to on page 248 of Steinmetz's *Alternating Current Phenomena*, referred to by Mr. Averett. This torque we do not consider, in general, to be of practical operating importance.

The torque at constant current varies approximately with the voltage. The iron losses vary about as the 1.6 power of the

voltage. Therefore, below normal voltage the current and torque may be increased somewhat, and above normal voltage the current and torque must be decreased to some extent, to maintain constant heating.

Thus, below normal voltage the torque will vary somewhat less than the first power of the voltage, and above that point it will vary at a slightly lower rate. Throughout the operating range of any particular motor, however, we feel that our statement that the torque of the induction motor, for constant heating, varies as the first power of the voltage, is substantially true, and in support thereof quote from Dr. Steinmetz's contribution to the discussion to the effect that:

> It will be found, then, that for the same heating of the motor, the maximum torque which the motor can carry varies with the voltage.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

THE CURRENT LOCUS OF THE SINGLE-PHASE INDUCTION MOTOR

BY A. S. LANGSDORF

The theory of the single-phase induction motor has, until recently, been complicated by the general use of Ferraris' expedient of resolving the impressed alternating magnetic field into forwardly and backwardly rotating components; this conception, though it leads to correct results if consistently handled, is open to the objection that it departs unnecessarily from the physical facts, and gives rise to the notion that the ordinary laws of electromagnetic induction are not immediately applicable, as in the direct-current machine. Within the last few years, however, the tendency in treating this problem has been to go back to first principles, and several valuable contributions have appeared, notably those of Fynn and McAllister. The following treatment has been based largely on the theory advanced by Fynn in his several publications.

I. *General principles and vector diagram.* The physical behavior of the single-phase induction motor having a permanently short-circuited or squirrel-cage rotor is identical with that of the motor shown diagrammatically in Fig. 1. This represents a stator of the usual cylindrical form with an embedded winding, and a wound rotor provided with a commutator. Two sets of brushes, short-circuited in pairs, are provided, the one set being magnetically in line with the axis of the stator winding, and the other at right angles thereto. Employing the usual nomenclature, the rotor axis *a a* will be referred to as the "transformer" or "armature" axis, while the axis *f f* will be called the "speed" or "field" axis. From the assumed construction of the magnetic circuit it follows that the reluct-

ance will be the same along all axes, and that the resistance and leakage reactance of the rotor will be the same in both the transformer and speed axes.

Under standstill conditions the stator winding and the rotor winding along the axis *a a* constitute a short-circuited transformer whose magnetic circuit is " leaky " owing to the air-gap. But under any conditions, at standstill or when running, the usual transformer relations must be satisfied so far as this axis of the motor is concerned; the stator current must supply the magnetomotive force to produce the flux demanded by the impressed electromotive force (modified by the drop due to primary local impedance) and it must balance the magneto-

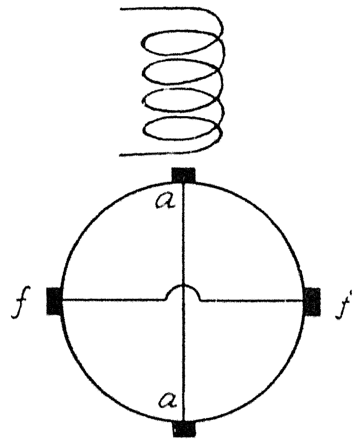


FIG. 1.

motive force of the rotor current in the transformer axis. Thus, at standstill, if Φ_t, Fig. 2, is the flux of mutual induction in the transformer axis there will be induced in both primary and secondary an electromotive force E_t lagging 90 degrees behind Φ_t (assuming unity ratio of transformation or that the secondary has been reduced to terms of the primary). A current i_t will then flow in the axis *a a* of the rotor, being determined in magnitude and phase by the local impedance of the rotor. The primary current, i_1, will then be the resultant of i_0, the magnetizing current, and of i_t reversed. The impressed electromotive force, E_1, will be the resultant of a component equal and opposite to E_t and of a component $i_1 z_1$ to overcome the local impedance, z_1, of the stator.

If the motor be brought up to speed in some way, there will be generated* an electromotive force E_t' (Fig. 3) of line frequency, in the speed axis, ff. The magnitude of E_t' will be proportional to Φ_t and the speed, and it will be either in time-phase with, or time-phase opposition to, the flux Φ_t, depending upon the direction of rotation. Let it be assumed to be in time-phase with Φ_t. The rotor current, i_t, in the transformer axis, also sets up a leakage field (ϕ_t) in phase with itself, stationary in

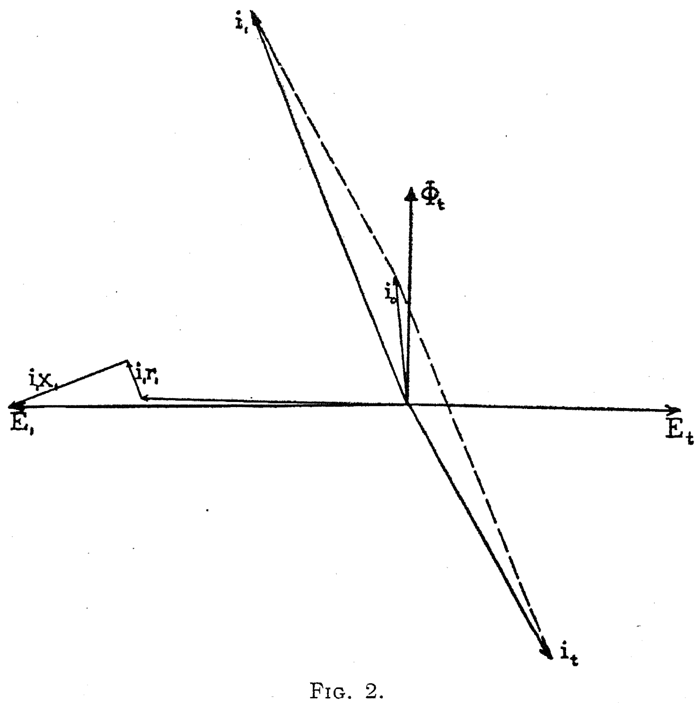


Fig. 2.

space, and coaxial with Φ_t; hence there will also be generated in the speed axis an electromotive force e_t', in time phase with i_t and proportional to ϕ_t and the speed. The resultant electro-

* The following system of nomenclature has been used: Main fluxes are represented by (capital) Φ, with appropriate subscripts; leakage fluxes by ϕ. Electromotive forces *induced* by a main flux are capitalized and bear the same subscript as the inducing flux; electromotive forces *generated* by a main flux are capitalized and primed; electromotive forces due to leakage fluxes are similarly designated, except that lower case is used.

motive force in the speed axis is then E_f, compounded vectorially of E_t' and e_t'.

Inasmuch as the conditions along the speed axis are those of a transformer on open circuit, the electromotive force E_f will set up a current i_s lagging heavily, and this will produce a magnetic "speed field," Φ_s, nearly in quadrature with E_f. The speed field, by its alternation through the rotor winding, will induce in the speed axis an electromotive force E_s lagging behind Φ_s by a quarter period, hence E_f must contain, as one component, an electromotive force equal and opposite to E_s; the other component of E_f drives the current i_s through the local impedance of the rotor. The relations between E_f, i_s,

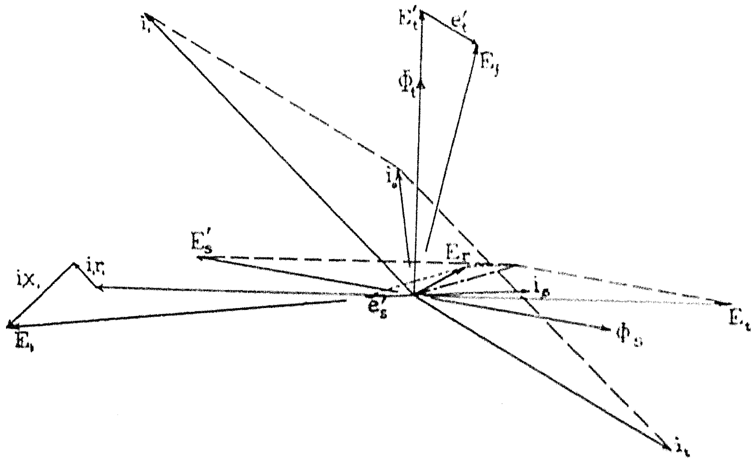


Fig. 3.

Φ_s and E_s are shown separately in Fig. 4 to avoid confusion in Fig. 3.

The motion of the rotor conductors through the speed field generates in the transformer axis an electromotive force E_s' which is in time-phase opposition to Φ_s, and, therefore, nearly opposed to E_t. E_s' is the equivalent of the counter electromotive force of the continuous-current motor. The speed-field current i_s also produces a small leakage flux (ϕ_s) coaxial with Φ_s and in time phase with i_s; hence there is a third electromotive force, e_s', in the transformer axis which is opposite in phase to i_s and proportional to ϕ_s and the speed. The electromotive force e_s' is small, but not necessarily neglible, especially near synchronism. The vector sum of E_t, E_s' and e_s', repre

sented in Fig. 3 by E_r, then serves to drive the current i_t through the local impedance of the rotor.

II. Mathematical Treatment.

Collecting symbols, let

E_1 = primary impressed electromotive force.
i_0 = primary exciting current—rotor on open circuit.
i_1 = total primary current.
Φ_t = transformer flux.
Φ_s = speed flux.
E_t = electromotive force induced by Φ_t.
E_t' = electromotive force generated by Φ_t.

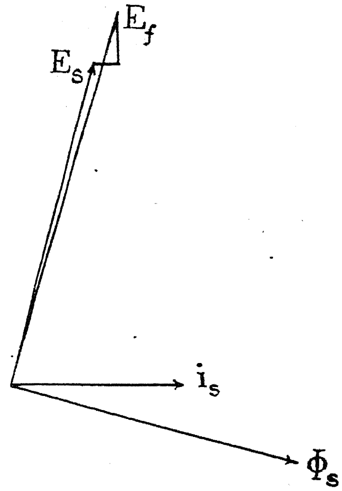


Fig. 4.

E_s = electromotive force induced by Φ_s.
E_s' = electromotive force generated by Φ_s.
i_t = rotor current in transformer axis.
i_s = rotor current in speed axis.
ϕ_t = leakage flux due to i_t.
ϕ_s = leakage flux due to i_s.
e_t' = electromotive force generated by rotation through ϕ_t.
e_s' = electromotive force generated by rotation through ϕ_s.
$z_1 = r_1 + j\,x_1$ = primary local impedance.
$z_0 = 1/y_1$ = primary exciting impedance.
$y_1 = g_1 - j\,b_1$ = primary exciting admittance.
$z_2 = r_2 + j\,x_2$ = secondary (rotor) local impedance.

$z_s = 1/y_2$ = secondary exciting impedance.
$y_2 = g_2 - j\, b_2$ = secondary exciting admittance (speed axis).
S = speed with synchronism as unity.

A little consideration will show that $e_t' = S\, x_2\, i_t$ and that $e_s' = S\, x_2\, i_s$ and that vectorially (see Fig. 3)

$$\underset{\cdot}{e_t'} = S\, x_2\, \underset{\cdot}{i_t} \tag{1}$$

$$\underset{\cdot}{e_s'} = -S\, x_2\, \underset{\cdot}{i_s} \tag{2}$$

From Figs. 3 and 4 it is seen that

$$\underset{\cdot}{E_f} = \underset{\cdot}{E_t'} + \underset{\cdot}{e_t'} = \underset{\cdot}{E_s} + \underset{\cdot}{i_s}\, z_2$$

or

$$\underset{\cdot}{E_s} = \underset{\cdot}{E_t'} + \underset{\cdot}{e_t'} - \underset{\cdot}{i_s}\, z_2 \tag{3}$$

But

$$\underset{\cdot}{i_s} = \underset{\cdot}{E_s}\, y_2,$$

hence from (1) and (3)

$$\underset{\cdot}{E_s} = \frac{\underset{\cdot}{E_t'} + \underset{\cdot}{i_t}\, S\, x_2}{1 + z_2\, y_2} \tag{4}$$

From the ordinary equations of the transformer and the generator it follows that

$$E_t = \kappa\, \Phi_t$$
$$E_t' = S\, \kappa\, \Phi_t = S\, E_t$$
$$E_s = \kappa\, \Phi_s$$
$$E_s' = S\, \kappa\, \Phi_s = S\, E_s$$

where κ is a constant, and vectorially

$$\underset{\cdot}{E_t'} = j\, S\, \underset{\cdot}{\dot{E}_t} \tag{5}$$

$$\underset{\cdot}{E_s'} = j\, S\, \underset{\cdot}{E_s} \tag{6}$$

From Fig. 3 there results

$$\underset{\cdot}{E_r} = \underset{\cdot}{E_t} + \underset{\cdot}{E_s'} + \underset{\cdot}{e_s'}$$

and upon substituting for E_s' and e_s' from (6), (5), (4) and (1) it follows that

$$\dot{E}r = \dot{E}_t \left[1 - \frac{S^2 (z_s + j\, x_2)}{z_2 + z_s} \right] + \dot{i}_t \left[\frac{j\, S^2 x_2 z_s - S^2 x_2}{z_2 + z_s} \right] \quad (7)$$

Substituting in (7) the relation $\dot{E}_r = \dot{i}_t z_2$,

$$\dot{i}_t \left[z_2 - \frac{S^2 (j\, x_2 z_s - x_2^2)}{z_2 + z_s} \right] = \dot{E}_t \left[1 - \frac{S^2 (z_s + j\, x_2)}{z_2 + z_s} \right] \quad (8)$$

The primary current is determined by the condition

$$\dot{i}_1 = \dot{i}_0 - \dot{i}_t = -\dot{E}_t y_1 - \dot{i}_t$$

and the primary impressed e.m.f. is given by

$$\dot{E}_1 = -\dot{E}_t + \dot{i}_1 z_1;$$

hence

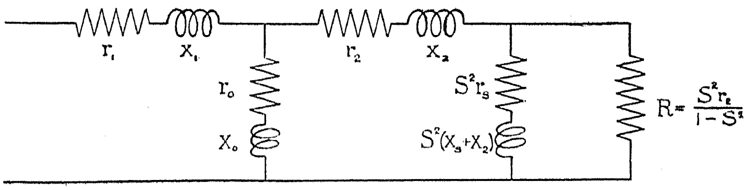


FIG. 5

$$\dot{E}_1 = \dot{i}_1 \left[z_1 + \cfrac{1}{y_1 + \cfrac{1}{z_2 + \cfrac{1}{\cfrac{1}{S^2 (z_s + j\, x_2)} + \cfrac{1 - S^2}{S^2 r_2}}}} \right] \quad (9)$$

The expression in the bracket in equation (9) is the equivalent impedance of the motor. Its form shows that the equivalent circuit is as sketched in Fig. 5, in which the exciting circuits are shown as impedances, or as in Fig. 6 where the exciting admittances are drawn.

It is to be noted that the equivalent circuit as here derived differs from the one ordinarily described in that the exciting admittance of the speed field is not constant, but a function of the speed. At standstill ($S = 0$) and at synchronism ($S = 1$)

the circuit of Fig. 6 becomes the same as the commonly accepted diagram, but at all other speeds it differs therefrom. Consequently it is not at once obvious that it is admissible to transfer the exciting circuits to the terminals a, b, as is usually done.

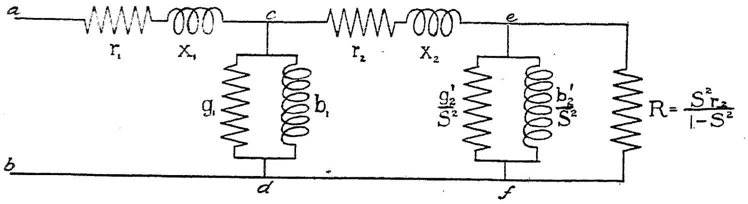


FIG. 6.

Under these circumstances it is of interest to determine the form of the primary current locus, assuming that the so-called "constants" of the motor are unaffected by variations in the load and speed. It is purposed to determine this locus for the equivalent circuit of Fig. 6. just as it stands.

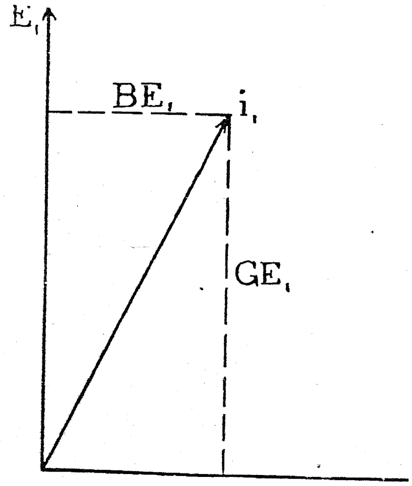


FIG. 7.

It then becomes convenient to transform equation (9) into

$$\dot{i}_1 = \dot{E}_1 (G - j B) \tag{10}$$

where G and B are, respectively, the equivalent conductance

and susceptance of the machine. The impressed e.m.f. E_1 being constant, the equation of the current locus will be given by the relation between G and B and the motor constants.

Referring to Fig. 6 it is clear that between points a and b the conductance is G and the susceptance B; at (c, d) the resistance is $\frac{G}{G^2+B^2} - r_1$ and the reactance is $\frac{B}{G^2+B^2} - x_1$; proceeding in this way through the diagram there results the following table:*

Points	Conductance or Resistance	Susceptance or Reactance	Unit
a, b	G	B	mho
c, d	$\frac{G}{G^2+B^2} - r_1 = \alpha$ (11)	$\frac{B}{G^2+B^2} - x_1 = \beta$ (12)	ohm
c, d	$\frac{\alpha}{\alpha^2+\beta^2} - g_1 = \gamma$ (13)	$\frac{\beta}{\alpha^2+\beta^2} - b_1 = \delta$ (14)	mho
e, f	$\frac{\gamma}{\gamma^2+\delta^2} - r_2 = \varepsilon$ (15)	$\frac{\delta}{\gamma^2+\delta^2} - x_2 = \zeta$ (16)	ohm
e, f	$\frac{\varepsilon}{\varepsilon^2+\zeta^2} = \frac{g_2'}{S^2} + \frac{1-S^2}{S^2\, r_2}$ (17)	$\frac{\zeta}{\varepsilon^2+\zeta^2} = \frac{b'_2}{S^2}$ (18)	mho

Starting first with equation (17) and then with (18), and eliminating successively ε, ζ, γ, δ, α and β, there result

$$\frac{K}{L_1}\left(\frac{g_2'}{S^2} + \frac{1-S^2}{S^2\, r_2}\right)\left[\left(G - \frac{M}{K}\right)^2 + \left(B - \frac{P}{K}\right)^2\right]$$

$$+\left[\left(G - \frac{N_1}{L_1}\right)^2 + \left(B - \frac{Q_1}{L_1}\right)^2 - \frac{1}{4\,L_1^2}\right] = 0 \qquad (19)$$

* The Exact Circular Current Locus of the Induction Motor, by K. J Laurell, *Electrical World*, Vol. 52, p. 78.

and

$$\frac{K}{L}\,\frac{b_2'}{S^2}\left[\left(G-\frac{M}{K}\right)^2+\left(B-\frac{P}{K}\right)^2\right]$$

$$+\left[\left(G-\frac{N}{L}\right)^2+\left(B-\frac{Q}{L}\right)^2-\frac{1}{4\,L^2}\right]=0 \qquad (20)$$

Where

$M = A\,r_1 + D$
$N = b_1\,r_1 + g_1\,x_2 + r_1\,x_2\,y_1^2$
$P = A\,x_1 + C$
$Q = b_1\,(x_1 + x_2) + x_1\,x_2\,y_1^2 + \frac{1}{2}$
$K = A\,z_1^2 + 2\,D\,r_1 + 2\,C\,x_1 + z_2^2$
$L = x_1 + x_2 + z_1^2\,(b_1 + x_2\,y_1^2) + 2\,x_2\,(g_1\,r_1 + b_1\,x_1)$
$L_1 = r_1 + r_2 + z_1^2\,(g_1 + r_2\,y_1^2) + 2\,r_2\,(g_1\,r_1 + b_1\,x_1)$
$N_1 = g_1\,(r_1 + r_2) + r_1\,r_2\,y_1^2 + \frac{1}{2}$
$Q_1 = g_1\,x_1 + b_1\,r_2 + r_2\,x_1\,y_1^2$
$A = 1 + 2\,b_1\,x_2 + 2\,g_1\,r_2 + y_1^2\,z_1^2$
$D = r_2 + g_1\,z_2^2$
$C = x_2 + b_1\,z_2^2$

Eliminating S^2 between (19) and (20) it is found that

$$\left[G-\frac{M\,\frac{b_2'}{r_2}+N\left(g_2'+\frac{1}{r_2}\right)-N_1\,b_2'}{K\,\frac{b_2'}{r_2}+L\left(g_2'+\frac{1}{r_2}\right)-L_1\,b_2'}\right]^2$$

$$+\left[B-\frac{P\,\frac{b_2'}{r_2}+Q\left(g_2'+\frac{1}{r_2}\right)-Q_1\,b_2'}{K\,\frac{b_2'}{r_2}+L\left(g_2'+\frac{1}{r_2}\right)-L_1\,b_2'}\right]^2$$

$$=\frac{b_2'^2+\left(g'_2+\frac{1}{r_2}\right)^2}{4\left[K\,\frac{b_2'}{r_2}+L\left(g_2'+\frac{1}{r_2}\right)-L_1\,b_2'\right]^2} \qquad (21)$$

which is the equation of a circle.

In any well-designed motor terms involving y_1^2 may be neglected; g_2' is likewise negligible with respect to $1/r_2$; but in general it does not appear that any considerable simplification of the final equation (21) is possible, several trials having been

made using the constants of different motors. It should be noted that the values of g_2' and b_2' are, respectively,

$$g_2' = \frac{r_s}{r_s^2 + (x_s + x_2)^2} \cong g_1$$

$$b_2' = \frac{x_s + x_2}{r_s^2 + (x_s + x_2)^2} \cong b_1$$

III. *Experimental results and conclusions.* In order to check the accuracy of equation (21), several test runs were made on a single-phase motor of standard make. The motor was rated at 5 h.p. at 104 volts and 60 cycles; its constants, determined in the usual way by open-circuit and short-circuit measurements, were found to be:

$$\begin{aligned} r_1 &= 0\,079 \\ r_2 &= 0.167 \\ x_1 + x_2 &= 0.449 \\ g_1 &= 0.029 \\ b_1 &= 0.078 \end{aligned}$$

Hence,

$A = 1.045$	$Q = 0.535$
$C = 0.231$	$K = 0.265$
$D = 0.169$	$L = 0.462$
$M = 0.252$	$L_1 = 0.254$
$N = 0.013$	$N_1 = 0.507$
$P = 0.466$	$Q_1 = 0.020$

The abscissa and ordinate of the center of the circle are, respectively,

$$\xi = E_1 \frac{P\dfrac{b_2'}{r_2} + Q\left(g_2' + \dfrac{1}{r_2}\right) - Q_1 b_2'}{K\dfrac{b_2'}{r_2} + L\left(g_2' + \dfrac{1}{r_2}\right) - L_1 b_2'} = 1.19\,E_1$$

$$\eta = E_1 \frac{M\dfrac{b_2'}{r_2} + N\left(g_2' + \dfrac{1}{r_2}\right) - N_1 b_2'}{K\dfrac{b_2'}{r_2} + L\left(g_2' + \dfrac{1}{r_2}\right) - L_1 b_2'} = 0.053\,E_1$$

and the radius is

$$\rho = E_1 \frac{\sqrt{b_2'^2 + \left(g_2' + \dfrac{1}{r_2}\right)^2}}{2\left[K\dfrac{b_2'}{r_2} + L\left(g_2' + \dfrac{1}{r_2}\right) - L_1 b_2'\right]} = 1.04\, E_1$$

The data of a run at a constant impressed electromotive force of 105 volts are plotted in Fig. 8. which shows a satisfactory agreement between the observed and the theoretical current locus. The coördinates of the center of the circle are

$\xi = 125$ amperes

$\eta = 5.56$ amperes,

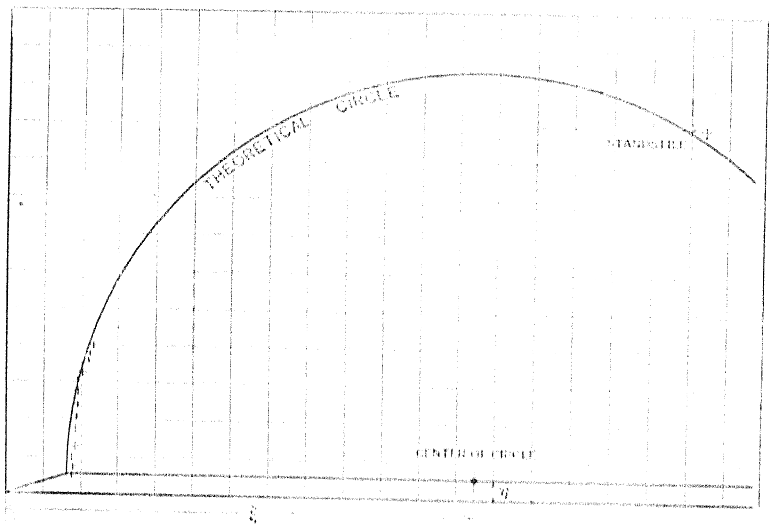


Fig. 8.

and the radius is

$\rho = 109$ amperes.

It thus appears that statements to the effect that the current locus is a curve of higher order are in error. The locus is a true circle in spite of the complicated nature of the equivalent circuit, and it may be readily determined for any given motor by two simple measurements—primary current and power-factor with the rotor open-circuited, and the impedance voltage and power-factor with rotor blocked and short-circuited.*

* The writer desires to express his indebtedness to Messrs. W. E. Beatty and I. A. Sims for assistance in obtaining experimental data.

Discussion on "The Current Locus of the Single-Phase Induction Motor." Frontenac, N. Y. June 28, 1909

V. Karapetoff: Professor Langsdorf says that he was induced to write this paper because of certain difficulties that he met with in teaching students. I should like to make one slight criticism from a pedagogical point of view. It appears that Professor Langsdorf wanted to get closer to the physical phenomena, and therefore he discarded Ferraris' assumption of two fields revolving in opposite directions. But he introduced another assumption which, while it may be clear to us, is by no means self-evident to the beginner. I refer to the diagram in Fig. 1. He replaces the squirrel-cage rotor by a direct-current armature, or rather by an armature such as is used in some single-phase commutator motors, with two sets of brushes at right angles to each other. These sets of brushes are intended to fix in space two fluxes, the transformer flux and the motor flux, as in the single-phase induction motor. The substitution is hardly justifiable from a pedagogical standpoint. A beginner would not see the equivalence of the two armatures directly. Moreover, this substitution is hardly necessary; for it could just as easily be explained to a beginner that the squirrel-cage rotor, when placed in a motor at rest, would act as the secondary of a transformer; that the same squirrel-cage rotor, if revolved in a constant field, would be equivalent to an apparatus whose field is stationary in space. In this case the flux is in a horizontal direction; that is, along the axis *ff*. For these reasons I think that the substitution of the armature with the commutator for the squirrel-cage rotor is unnecessary. Moreover, it is not quite correct. The squirrel-cage secondary adapts itself electromagnetically to the primary flux, because of the possibility of currents in various bars differing from one another: the equalization is accomplished through the end-rings. In the coil-wound armature, on the contrary, currents in all the coils must necessarily be the same, since the coils are connected in series. Therefore, the student may get a wrong impression, especially with regard to magnetic leakage.

The multistage formula (9) looks complicated because Professor Langsdorf uses reactances and resistances instead of admittances. As I understand it, the only reason is that he wishes to introduce the slip or speed, S. Could not the results be presented in a shape more comprehensive to a student by substituting admittances for impedances? In this case there would not be any stages, but an ordinary addition of terms.

A. S. Langsdorf: In reply to Professor Karapetoff's criticism of the form of Fig. 1, I may say that it has been my practice in class-room work to use both methods suggested by him, but it did not seem necessary to go into such detail in the present paper.

In deriving equation (9) there is little or nothing to be gained in the way of simplicity by using admittances exclusively; the form given in the paper lends itself readily to the determination of Fig. 6, from which the remainder of the work follows directly.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 28, 1909.

MULTISPEED INDUCTION MOTORS

BY H. G. REIST AND H. MAXWELL

The induction motor, as generally used, has a speed which varies slightly with the load but which is usually spoken of as being constant. For work where variable speed is required, a motor with collector rings is used, the speed variation being produced by varying the amount of resistance inserted in the secondary circuit. Such a motor is wasteful of power whenever resistance is used, and the speed varies greatly with the load when much reduction in speed is required.

It sometimes happens that the work is of such a character that two or three speeds will be satisfactory for the operation of the machinery. For such conditions multispeed motors can frequently be used. In these motors the different synchronous speeds are produced by changing the number of poles in the magnetic circuit. Each of these speeds is fixed if no resistance is used in the secondary circuit, so that a change in load will produce only a very slight variation in speed, and the motor will operate efficiently at each speed. With multispeed motors, as with ordinary motors, secondary resistance may be used to vary the speed below each of the synchronous speeds. In this way the complete variation of the speed over a wide range may be obtained more efficiently than with a simple collector-ring motor, but the motor and the control apparatus may be so complicated that the saving of power is not warranted.

A change in the number of poles may be produced in any one of the following ways:

a. By the use of single magnetic and electric circuits, changing the number of poles by re-grouping the coils.

b. By the use of a single magnetic circuit and inde]
electric circuits.

c. By means of separate magnetic and electric circu:

The selection and design of the multispeed motor is ɛ
principally by the number of speeds required, the ratic
desired speeds, and the ratio of the maximum outputs
different speeds.

a. Single magnetic and electric circuit. This type has pr
been used more extensively than any other type of mul
induction motor, due to its economical use of material
which is active at all speeds. Except in a special case, a
vantage of this type of motor is the large number o:
which must be brought out of the winding, and the comp
switching apparatus needed for interconnecting them.

For any ratio of two speeds other than 2 : 1, the or
three-phase winding requires 33 leads. For a two-phase
ing or a three-phase teaser winding, the winding may
ranged with 16 leads. If more than two speeds are d
the number of leads will be greatly increased and in g
will approximate the number made by bringing out the le
each coil, thus requiring nearly twice as many leads as
are coils in the machine. In this form of motor the sam
are used for the different arrangements of poles, the relative
of the winding depending on the number of poles. Sinc
economy of a winding rapidly decreases when the pitch
coil is less than one-half, it is disadvantageous to build m
of this type having a greater change of speed than 2 : 1.
account of the large number of leads required, it is not practi
to use this type of winding on the rotors of induction m
except with a ratio of 2 : 1 speed, with which arrangement
six collector rings are required. With all other speed-ratic
number of collector rings would become prohibitive, s
these motors a squirrel-cage winding is generally used or
secondary.

When the speed-ratio is 2 : 1, this type of motor can be ap
advantageously, and a great many have been built
have been in successful operation for several years. A m
wound in this way is practicable for either a wound type of r
since it requires only six collector rings, or for a squirrel-
rotor, which is more generally used.

There are a number of different methods that may be
in connecting the windings of such motors, the selection of

connection in any case depending on the relative maximum outputs required at the two speeds. The connection which has frequently been used where the material is worked to the best advantage, has a half-speed rating of from 60 per cent to 70 per cent of full-speed rating. The following table gives several different connections that may be used, with the approximate relative outputs of the different connections. The third combination shown on this table is the one generally used, as it best suits the average conditions of a load which requires constant torque at the two speeds; that is, when the output of the motor is proportional to the speed.

Speed	Connection	Approximate maximum output
(1) 100	2-Circuit Delta	100
50	Y-delta	11
(2) 100	2-circuit Y	100
50	1 " Y	22
(3) 100	2 " Y	100
50	1 " Delta	66
(4) 100	1 " Delta	100
50	2 " Y	117
(5) 100	1 " Y	100
50	2 " Y	350
(6) 100	Y Delta	100
50	2 Circuit Delta	700

These values are approximate and will vary slightly with the ratio of the reactance to the resistance and also with the ratio of the coil-end to the slot part of the reactance leakage.

From an inspection of the table it will be seen that there is a wide range of choice, from maximum output, a little over 10 per cent for the half-speed connection, to about seven times that of the full-speed connection. These various combinations are often valuable in extraordinary cases to maintain a good power-factor and efficiency at the desired loads. In most of these combinations the material is not worked to equal advantage at both speeds.

b. Motors with a single magnetic circuit and independent electric circuits. This type of multispeed motor has the advantage of using the same magnetic circuit at all speeds, and therefore requires only one stator and rotor structure, the same as an ordinary motor. These motors, having independent windings, have three leads for each speed in the stator for three-phase motors and four for two-phase motors.

The slots in this type of motor are usually made large enough to contain the two or more windings used in making the several windings for the different speeds. It is theoretically possible to obtain a number of synchronous speeds, but it is usually impracticable to use more than two windings. The relative maximum outputs are independent of each other, but to work the magnetic material to advantage it should be approximately proportional to the speed. The motors of this type have to be built large for their output in order to allow space for the increased amount of copper and insulation. The space occupied by insulation becomes excessive on high-voltage motors so that this type is applicable chiefly to low-voltage windings. It is possible in this form of motor to utilize one or more of the windings mentioned in type (*a*), thus increasing the possible number of speeds. Since there are two or more distinct windings, the construction is more complicated and the motors are somewhat more difficult to repair than when only one winding is used.

c. Separate magnetic and electric circuits. The concatenated motor is the only motor of this type in general use. This consists of two motors, the rotors of which are mounted on the same shaft arranged so that the secondary of the first motor is directly connected to the primary of the second motor. The two motors may have the same number of poles, but usually the number of poles is different. The effect produced by the second motor is that of adding to or subtracting its poles from the first motor. It will be seen that if the two motors have a different number of poles, there will be four synchronous speeds, two for the motors independently and two for the combination. If the speed corresponding to the difference in the number of poles is higher than the synchronous speed of the first motor, the set will not accelerate by itself to this speed; and although there might be ways of accelerating to this speed, the switching devices will be complicated so that from a practical standpoint this combination is not generally considered.

By using in concatenation two of the half-speed motors mentioned in class (*a*), the number of speed combinations is very large, and the number of leads and switching of the electric circuits, while complicated, may be warranted in some cases. If, for instance, we take a 4–8 pole and a 6–12-pole motor, we can obtain combinations to give motors having speeds equivalent to every possible number of poles from 4 to 20, except 18.

The ratio of speeds of the two motors is limited by the necessity of changing the poles of each motor, two at a time. A further limitation is the centrifugal stresses in the larger motor at high speeds. If the individual motors have the same synchronous speed and can be run in multiple at the high speed, the ratio of the maximum output of the high speed to the low speed is about 4 : 1. If the load requires this ratio of work with the different speeds, the material in the motor is worked to the best advantage.

The two conditions—of having a load as desired in the foregoing and having speeds with ratio of 2 : 1—rarely exist, so in general the material cannot be economically used in concatenated motors. The ratio of the outputs mentioned is usually unfavorable, since in general the output required at the lower speed is more than one-quarter that required at the higher speed. There

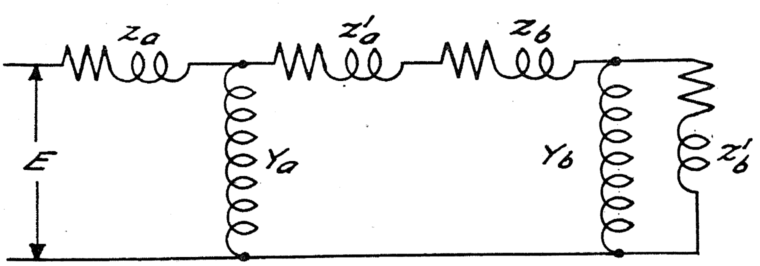


Fig. 1

are some classes of work, however, such as the driving of blowers, where approximately this load-ratio may exist.

On account of the large variation in output at the two speeds, the concatenated motors generally use material at the high speed to a very great disadvantage, so that such motors are usually large and expensive for the work to be done. The most favorable condition for the use of the concatenated motor is when the various speeds required are close together. In such a case the two motors would be very different in size and the large motor would be used for the highest speed; for example, if the large motor had 16 poles and the small one had 4, speeds corresponding to 12, 16, and 20 poles could be obtained.

Another disadvantage of the concatenated motor is its low power-factor. This is inherently low, even at the highest speed, owing to the fact that the normal load carried on the individual

motors is seldom as large in proportion as the necessary maximum output of these motors in order to obtain sufficient output when the motors are used in combination; in other words, the higher-speed motors will give a poor power-factor because they are underloaded.

To explain the characteristics of the concatenated motors

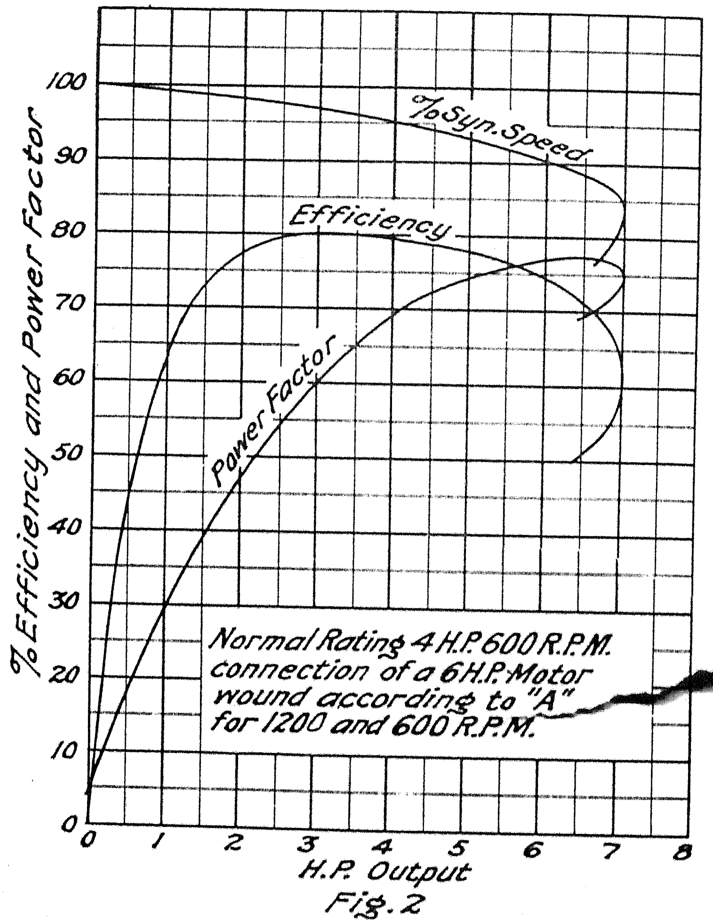


Fig. 2

more clearly, attention is called to Fig. 1* which represents circuits equivalently arranged;

In this diagram Z_a and Z_b represent the impedance of the primary circuit of the two motors A and B, and Z_a', Z_b' their

* For explanation of Fig. 1 see E. Arnold, "Die Wechselstromtechnik" Vol. 5, Part 1, Fig. 265.

secondary impedance, while Y_a and Y_b are their exciting admittances. The resistance component of the impedance is here understood to be corrected for each circuit according to the frequency of that circuit, so that all terms are given as their equivalent value for full frequency.

The load current travels around through Z_a, Z_a', Z_b, Z_b', in series, while the magnetizing circuits Y_a and Y_b are across the line with full voltage impressed upon them, except for the voltage drop in the primary circuit in the winding of A for Y_a,

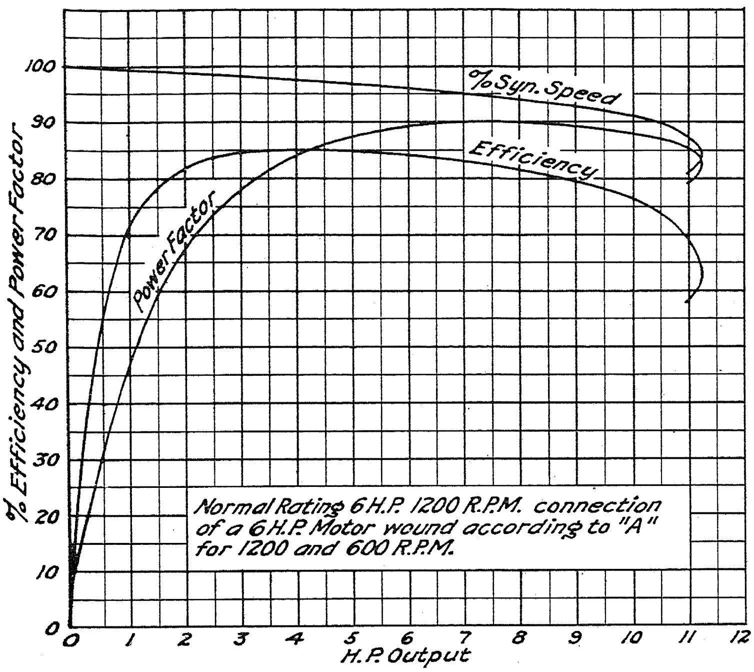


Fig. 3

and the additional drop of the secondary of A and the primary of B for Y_b. Since these voltage-drops are small at normal load, we can neglect them and say that the impedances of both motors are added, and, likewise, that the magnetizing currents of the two are added.

The maximum output of a motor is inversely proportional to its impedance at a given frequency and voltage, so that if the two single motors have equal impedance, the maximum output of the concatenated couple is approximately one-half

the maximum output of one motor alone, and its magnetizing current is approximately twice that of one motor—hence its inherent low power-factor in concatenation.

Two curves from actual tests of motors are reproduced to illustrate the points that we have tried to bring out regarding the various characteristics of two of the types of motor mentioned. The curves in Figs. 2 and 3 show the characteristics of a motor of type (*a*) wound for full speed and half speed ac-

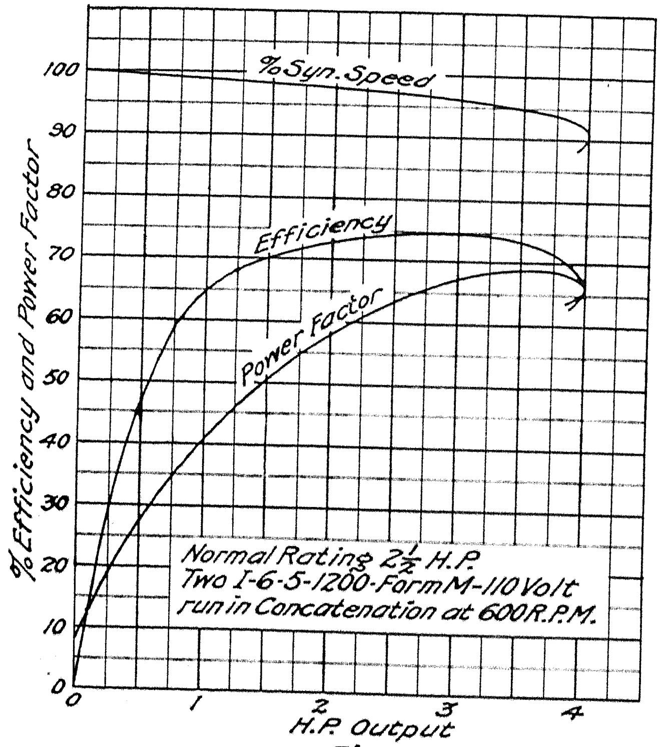


Fig. 4

cording to No. 3 connection of the table given. The curves in Fig. 4 show the characteristics of two motors of equal size run in concatenation. Fig. 5 shows the characteristics of one of the motors in Fig. 4 run at the high-speed connection, the other motor being idle. If both motors were run in multiple at the high speed, the output would be twice as great as indicated in Fig. 5.

As a suggestion in the selection of the best type for various

conditions, we have drawn up the following conclusions which must, however, be interpreted very broadly, as there are many modifying circumstances.

1. For two speeds having a ratio of 2 : 1, use motor type (*a*).

2. For two speeds having other ratios than 2 : 1, use motor type (*b*), except when the winding and insulation require too much space.

3. When three speeds are required, two having a ratio of 2 : 1, the other being intermediate, type (*b*) may be used with one winding connected as in (*a*).

4. If four speeds are necessary, one speed being one-half of

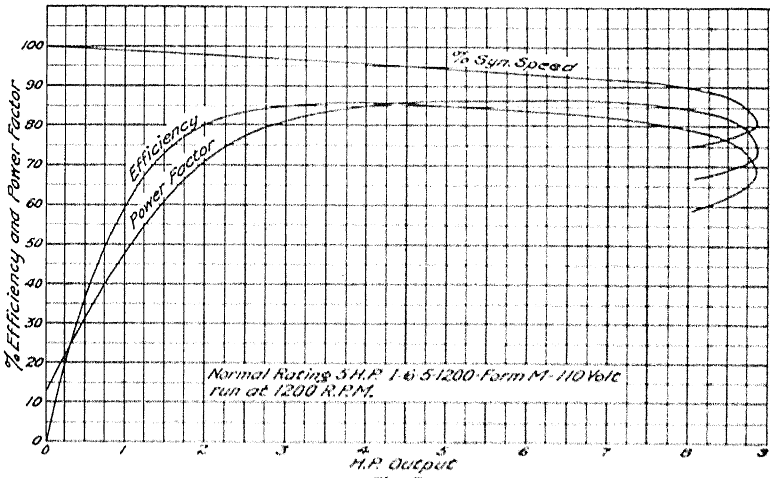


Fig. 5

the highest speed, and the lowest speed one-half of the next highest speed, type (*b*) may be used, with each winding such as is used in type (*a*).

5. Use type (*c*) concatenated motors, when two or more speeds close together are required.

6. Use concatenated motors when polar windings on the rotor are necessary with more than two speeds, or if the two speeds differ from 2 : 1.

7. Use concatenated motors for high-voltage machines except when type (*a*) can be used.

DISCUSSION ON "MULTISPEED INDUCTION MOTORS."
FRONTENAC, N. Y., JUNE 28, 1909

H. C. Specht: It is well known that the induction motor is the simplest motor to build and operate. It can be used with advantage for a variety of purposes, especially for constant-speed service. Where variable speeds are required, however, the induction motor is put at a disadvantage by the direct-current motor.

A great many methods for varying the speed of the induction motor have been employed. Some of these methods are discussed by Messrs. Reist and Maxwell, but treat chiefly of motors in concatenation and with multispeed apparatus consisting of a single motor. For a two-speed motor consisting of a single winding, and with a speed ratio other than 2:1, the authors say that 33 leads are required for a three-phase winding. It would be of interest to know whether this applies to any speed ratio other than 2:1 and to any number of poles, or what particular combination of two-speed connections the authors had in mind. For various schemes of connections the number of leads is generally dependent on the number of poles and the type of winding.

The authors give a table of approximate maximum outputs for the different combinations of series and multiple, star and delta, connections of a two-speed motor with a ratio of 2:1. I assume from the maximum output figures that the coils of the windings have a throw closer to the pitch of the double number of poles. This table would be of greater value if the current, the necessary amount of copper and iron, and the size of the mechanical parts were compared with some standard.

The maximum output depends on the limited density in the copper and iron, and the stresses in the mechanical parts. It is, therefore, clear that some of the outputs given in the table cannot be met without increasing or wasting some material.

The authors say, if the speed corresponding to the difference in the number of poles is higher than the synchronous speed of the first motor, the set will not accelerate by itself to this speed. This statement would be correct if the first motor, which has the greater number of poles, is connected to the line. If the synchronous speed of the second motor is higher than that of the differential concatenation, then the second motor, instead of the first motor, may be connected to the line. In the case where neither of the two motors has a higher synchronous speed than that of the differential concatenation, the speed for the latter cannot be obtained without auxiliary means.

In regard to a cascade set consisting of two two-speed motors each one having pole ratio of 2:1; I know that 12 different speeds can be obtained. For example, the first motor having 10 and 20 poles and the second motor 2 and 4 poles, speeds corresponding to the following number of poles can be obtained: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, and 24. In case only one of

the two motors is a two-speed motor, the maximum number of speeds is 7.

It may be well to mention, however, that the different speed combinations are not practicable in every case. Furthermore, sometimes it might not be possible to obtain the exact speeds for every one of the different connections. In such cases the two motors may be interconnected mechanically by means of gears, chains, etc. having a speed-ratio which will give the desired speeds. It is clear, then, that practically any possible combination of speeds can be obtained with a cascade set; whereas with multispeed motors consisting of a single motor, such variations in the number of speeds cannot be obtained.

The authors say that the number of speeds obtained by a cascade set is limited by the centrifugal stresses in the larger motor. This limitation is, however, much less than on a single multispeed motor for the same speeds. The individual motors of a cascade set can be designed so as to have greater width and smaller diameter, thus keeping the peripheral speed lower than would be the case with a single motor.

The electrical performances of a cascade set depend entirely on the design of the individual motors. The curves given by the writers have reference to a particular design only.

There is no doubt that the motors for concatenation are very simple and safe in regard to control and operation. Even though the cost of a cascade set is rather high, it can be recommended in various cases, especially where simplicity and safety of control and operation are of great importance. The application of one or the other method of the various multispeed motors depends on so many conditions that a general rule as to which method should be used can hardly be given.

A. M. Dudley: I would like to call attention to another method that has been used for getting multispeeds. Quite a little has been done with it in England. I refer to the so-called method of internal concatenation. This is accomplished by winding the stator and also the rotor of a motor of normal mechanical structure for two sets of poles. If the line current is led into one of the stator windings, the other being short-circuited, and the two rotor windings are connected in series, the motor will run with a synchronous speed corresponding to the sum of the two pole numbers for which the motor is wound. Resistance may then be inserted in the remaining stator winding for varying the speed, as in the case of a normal wound secondary motor. For example, if the stator is wound for 4 and 6 poles, and the rotor also, it is possible by means of such a connection, using a single magnetic and also a single electric circuit, to have a speed corresponding to 10 poles.*

Charles P. Steinmetz: There are two methods of getting different speeds from the induction motor. One is by changing the number of poles, the other by concatenation or cascade

* See article by Dunn, *Journal of the Institution of Electrical Engineers*, September, 1907.

connection. Anyone who has designed an induction motor realizes that the difficulty in getting good motor constants is to give sufficient peripheral speed without increasing excessively the size of the motor. A large pitch per pole, that is, many ampere-turns per pole, is necessary to get a reasonable exciting current and reasonable leakage flux. Changing the number of poles; for instance, doubling them to get half-speed, reduces the pole-pitch one-half. The motor constants are thereby made twice as bad. That is the disadvantage of changing the number of poles. It means that this type of multispeed motor is more difficult to design for low-speed operation. A motor well designed for the low speed produces an excellent machine for high-speed operation; but in any case the motor has to be more carefully designed than would be demanded by a motor intended merely for high-speed service. Unfortunately, concatenation does not overcome this difficulty; for in this case the second motor is fed from the first motor. While the motor always keeps the same number of poles, and thereby the full pole-pitch, the first motor carries not only its own, but also the second motor's exciting current; and its leakage reactance is the sum of the leakage reactances of both motors, as the second motor is an inductive load on the secondary of the first motor. Thus concatenation doubles the exciting current and the self-inductive impedance, making the constants of the motor twice as poor, just as is the case when doubling the number of poles in one motor. In short, the electrical characteristics of the concatenated couple of motors are the same as the electrical characteristics of one motor with a changeable number of poles.

That brings us to the question of the relative characteristics of the two types of motors. I have been interested in the subject for many years, but have been forced to the conclusion that, in general nothing is gained by concatenation over the method of changing the number of poles in a single motor; but something is lost by having two structures instead of a single structure. Concatenation is therefore applicable only to special purposes, as in the case of railway motors. For general stationary work, the motor with a changeable number of poles is generally preferable.

H. C. Specht: In regard to the statement made by Dr. Steinmetz that the rotor of the motors in a cascade set has to be greater in diameter, thus necessitating a higher peripheral speed; this is a question of design. If the motors of a cascade set are properly designed, the diameters can be kept smaller than on single multispeed motors. Consider, for example, a cascade set of which one motor has 12 poles and the other motor 4 poles, and speeds corresponding to 12 and 16 poles are required. In this case the motor with the 12 poles would be designed with a leakage, almost equal to that of an ordinary single-speed motor. Then the motor with 4 poles would have to be designed with very small leakage, so that the total leakage of the set would be very little increased over that of the 12-pole motor. With such a design it is clear that the 12-pole as well as the 4-pole

motor could be designed with a smaller diameter than a two-speed motor having a 12 and 16-pole winding.

Charles P. Steinmetz: It is entirely correct that a 4-pole motor can be designed with much smaller leakage flux than a 16-pole motor; and concatenated to the latter, the former would be less liable to spoil the constants of the couple than if it also had 16 poles. But the same applies to changing the number of poles: changing from 16 to 20 poles would spoil the motor constants less than doubling the number of poles. The two methods, concatenating two motors, or changing the number of poles of one motor, still give essentially the same change of motor constants with changes of speed.

E. F. Alexanderson (by letter): One of the speakers referred to the method of speed variation by means of internal concatenation, and to an article published on this subject in

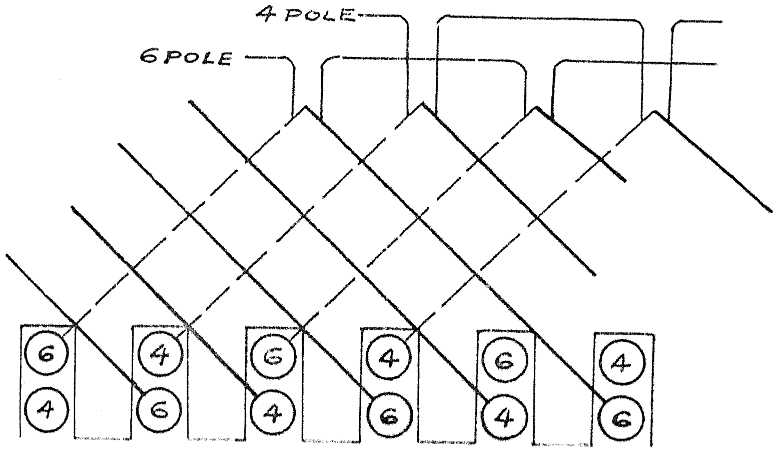


FIG. 1.

1907. I designed a motor of this kind in 1904. This experimental machine was built with the object of testing the internal concatenation as well as the connection of the same machine as a frequency-changer from 25 to 60 cycles. At the same time tests were obtained on a motor with two sets of windings; in other words, a motor which combined a four-pole and a six-pole winding on the same structure.

The field as well as the armature of this motor was made from the windings of a standard six-pole motor without changing anything but the connection of the coils. To my knowledge this was the first machine built with a double set of windings without introducing any special feature in the arrangement of the coils. Fig. 1 shows that each slot contains one member of the six-pole and one member of the four-pole winding, and that the two windings alternate in the upper and the lower positions

of the slot. The test confirmed the theory and demonstrated that the two windings were electrically independent and operated in the same way as if wound on separate cores connected to the same shaft. The machine could therefore be used as a concatenated set of motors or as a frequency-changer, in accordance with well-known methods. In the case of internal concatenation the collector rings were not used.

An interesting feature which was specially studied was the magnetic combination of the two fluxes. It had been foreseen in the design of the machine that two fluxes would combine in a resulting flux with an unbalanced distribution so as to cause a one-sided attraction of the armature, and a special reinforcement had been provided in the armature in order to avoid mechanical resonance. The test demonstrated, however, that no reinforcement could be applied to the shafts and bearings that would reduce the vibration sufficiently to make the machine commercially operative. This objection would of course be overcome if the number of poles were doubled, or in any combination where the difference in the number of poles was four or more, in which case the magnetic attraction would be balanced.

Endeavors were made after the test described to find a commercial application of the internal concatenation. However, it is difficult to find a justified application for ordinary concatenation and still more so for internal concatenation where a limitation is imposed on the relative number of poles. Although for the reasons mentioned the experiment did not give much encouragement to the use of internal concatenation, it proved of value as demonstrating a simplified method of winding a four-pole and a six-pole winding on the same core. At the same time experiments were made to obtain a continuous variation of speeds by controlling the four-pole and the six-pole windings simultaneously. The scheme proved perfectly feasible, although of course it gave a low efficiency. The most practical combination, after all, which was tried was simply a squirrel-cage rotor and a switch for changing the four-pole winding into an eight-pole by the well-known Dahlander-Lindstrom connection. In this way a good three-speed motor was produced with four, six, and eight poles.

A. E. Averett: In the concatenated motor the speed reduces somewhat in this way: with the collector-ring single-speed motor the reduction in speed is accomplished by reduction in efficiency, the power-factor remaining constant. In the concatenated motor the efficiency will remain approximately constant, but the power-factor goes down as the speed is decreased. The power-factor on the line is therefore high with secondary resistance, as against a low power-factor wheret he speed reduction is gained by concatenation.

H. G. Reist: The 33 leads mentioned are for a motor of the first class and will include any ratio of speed if the windings are Y-connected.

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OUTPUT AND REGULATION IN LONG-DISTANCE LINES*

BY PERCY H. THOMAS

The limitations of the power transmission line as to distance, power, and voltage, and its characteristics as to voltage-variation and energy-loss, have always excited a great deal of interest in the electrical art. The subject has been treated by a number of prominent engineers in its various phases, notable among these being Scott, Perrine, Steinmetz, and Mershon. These men and others have done pioneer work and have outlined the general limiting conditions of transmission. There are some of the relations and peculiarities of the long line, however, that are of importance and interest which have not, to the writer's knowledge, been clearly presented before electrical engineers. It is the object of the present paper to discuss some of these features, more especially the line-output, regulation or voltage-variation, and the line energy-loss.

In view of the uniform distribution of the resistance, inductance, and capacity in a transmission line, a proper study of the long line cannot be made without a suitable formula or equation for calculating the true effect of such a line on the electric current transmitted. As far as the author is aware, no such formula having a form sufficiently simple for convenient use and applicable to the ordinary transmission calculation has been offered. In the paper entitled " Calculation of the High-Tension Line," such a formula is derived, and is

* It is intended that this paper shall be considered in connection with a companion paper presented at the same time and entitled, "Calculation of the High-Tension Line."

recommended for numerical computations in practic where accurate results are desired.

In the analysis of the effects of the transmission o through the line which is to be given, reliance has bee upon the "wave" formula, as it is here called, thoug calculations have frequently been made from an appr formula based on the usual assumption in transmission tions—that the capacity of the lines is concentrated at t This second formula, while based on approximate assur is of very convenient form. It is called the " split ca formula, and is found in the companion paper.

With this introduction, the basis of the following di will be clear. While the principles stated apply in the lines of all sorts, they are of numerical importance and o interest chiefly in the longer and relatively high pow The determination of their bearing on practical trans must await the further development of the art.

Electrical action in the line. The critical electrical of a transmission system is the action of the line its the passing of energy over a line there is, first, the loss of en practical overhead work represented only by the resistanc But with power passing over a line at high voltage, a amount of leading energy flows to charge the line ca This energy in its travel causes a tendency for the p to rise along the line in the direction of the flow of the energy. Similarly there flows to the line a quantity of energy, in virtue of the inductance of the line and the current therein. This lagging energy tends to cause a potential in the line in the direction of its flow. The re effect is the difference of the leading and lagging energ superimposed, of course, upon the true power transmitt the resistance-drop.

This use of the terms " leading energy " and " lagging e is a little unusual; it is adopted only in the absence of suitable term for this situation. By " leading energy " is that flow of energy which is absorbed by the line capa rising voltage and is returned on falling voltage. By " energy ", is meant that energy which is absorbed by t inductance on rising current and returned on falling curr

When the lagging energy predominates over the energy, the tendency is for a drop in potential along t

in the direction of the lagging flow—the well-known inductive drop, which is combined with the ohmic drop.

When the leading energy predominates there is a tendency for a rise of potential in the direction of the leading energy-flow, which must be combined with the ohmic drop as before. The rise of potential effect of the leading energy will be opposed to the ohmic drop when the transmission of power and the flow of leading energy are in the same direction. If they flow in opposite directions in the line their effects will, however, be in the same sense.

Now consider the case when the leading and the lagging energies are equal and when they are opposite in phase; that is, when the load power-factor = 1. In this case there will evidently be no appearance in the line or demand on the generator for either lagging or leading energy, and no tendency either for a rise or a drop of voltage therefrom, and the ohmic drop is the only loss of voltage. This condition will be actually realized in practice if the leading and lagging energies of the line are equal and with a load of unity power-factor. If, however, the load current has a lag or a lead, the leading and lagging energies, while they may be equal, will not be opposite in phase and will not neutralize each other. The rather remarkable conclusion follows—that in this case of complete neutralization the loss of voltage is only the ohmic drop and is independent of frequency and is of the same value as with direct current. Thus if this condition can be realized, and there enters no secondary effect, there will be no further economy in direct-current transmission as far as energy-loss or voltage line-drop is concerned, the same effective line voltage being used. But the difficulty is that this condition of equality of the leading and the lagging energy in the line is usually not met with in practical transmissions of relatively short length, and often cannot be economically obtained.

The unfortunate condition exists that the leading energy, being the condenser energy, is practically constant, whatever the load on the line, since it depends only on the voltage of the line. On the other hand, the lagging energy depends only on the actual amount of power or current being transmitted, since it varies as the square of the line current. Thus with half load or no load, a line that was balanced for full load would have a large excess leading energy flow.

Suppose a line to be laid out so that there is an equality between the leading and the lagging energy at full load. At full load the line-drop in the direction of the power-flow will be the ohmic drop; but at half load the leading energy-flow will greatly predominate, and the actual line-drop in the original direction will be the ohmic drop less the effect of the excess leading energy in tending to produce a rising voltage in this direction. This assumes the leading energy to be supplied in the same direction as the power. Just what the resultant would be depends on the particular constants of the case. Now, when the load becomes zero, the line-drop will become a rise to the extent of the ability of the leading energy to bring about this result, since the ohmic drop will then be zero. Thus the range of delivered voltage from no load to full load will be the sum of the ohmic drop at full load and the leading energy rise at no load, which will in long lines be a very material amount.

In the case assumed suppose, however, that the power is transmitted in one direction while the leading energy is supplied from the other in any suitable manner. Then the full-load drop is the ohmic drop, as before; the no-load drop will be of the same numerical magnitude as before, but in the opposite direction. That is, the no-load drop will tend to make the end from which the true power is furnished the higher in voltage. Now, this is the same direction of potential-slope as the ohmic drop caused in the full-load condition of the line; thus the no-load and full-load cases show the same line voltages at both ends, provided, of course, the ohmic drop is equal numerically to the leading energy-rise at no load.

This relation of full-load and no-load drop may be often quite well approximated. When it is attained, the voltage variation of the line ceases to be a limiting feature of the transmission, and the amount of power that can be carried on a single line depends only on the amount of copper used. The effect of frequency, as already stated, is almost eliminated, within reasonable limits.

The practical problem, then, is to find some satisfactory way by which to realize the ideal adjustment of leading and lagging line energy, and also the approximate equality of the leading rise of voltage at no load with the ohmic drop of voltage at full load in any particular case.

It should be carefully borne in mind that the leading and lagging energy and energy-loss so far discussed are wholly confined to the line and have no necessary relation to the leading or lagging current in the load.

The factors by which the relative values of the leading and the lagging energy of the line are controlled are the line voltage, the actual load or current, and the ratio of the line inductance and capacity per unit length. The energy taken by the capacity of the line varies as the square of the line voltage, so that this factor is of extreme importance. The lagging energy of the load varies as the square of the current, as has already been mentioned. Neutralization of the leading and lagging line energies can be obtained only when the power-factor of the load current is unity, for only then are the leading and lagging energies opposite in phase.

The current of the line, of course, decreases directly with the increase of line voltage chosen, if the power transmitted be kept constant.

The energy flowing to the capacity of the line per unit length is proportional to $p\,C\,V^2$, where $p\,C$ is the capacity susceptance and V the voltage. The lagging energy will be proportional to $p\,L\,I^2$, where $p\,L$ is the inductive ohms and I the current. If these are equal then $\frac{V}{I} = \sqrt{\frac{L}{C}}$, which is the condition of balanced out-of-phase energy. It is clear that this can be reached for any line and voltage by properly choosing the current, and for any current by properly choosing the voltage. It is also clear that the condition is independent of frequency.

Equation (34) of the companion paper gives the line voltage equation when $R = 0$. Here if the current or voltage is chosen to fulfil the above condition, the generator voltage $= S \sin (p\,t + \theta + x\sqrt{p^2\,C\,L})$, showing that the voltage of the line is constant, as it should be. The resistance is chosen as zero, as otherwise the change of voltage along the line will destroy the exact balance of the leading and the lagging energy.

Thus as the net result, the ratio of the leading to the lagging energy varies as the fourth power of the voltage when the power transmitted remains the same. The capacity of the line can be increased by adding condensers in parallel across the line at regular and sufficiently frequent intervals; the inductance can similarly be increased by the addition of choke-coils in the same manner. But these expedients have never found much favor. As the power and voltage are often fixed by other considerations, some other method of altering this ratio than by change of power or voltage would be desirable.

The following method is suggested for consideration in favorable cases. The several line conductors may be divided into two, three, or more wires, which are separated somewhat from one another but mounted on the same insulator or adjacent insulators, the current dividing between them. The effect of this arrangement will be to increase the line capacity and to decrease the line inductance, both of which changes tend to alter the ratio of capacity to inductance in the same direction. With large power and long-distance lines, this increase of ratio is found in many cases to be in a favorable direction. This plan will be further discussed at a later point of this paper. Formulas for numerical calculations will be found in the companion paper.

It should be said that in this matter of adjustment of the leading and lagging line energies, a change of frequency is of no avail, since both elements, capacity and inductance, are affected in the same manner. Furthermore, while a certain voltage may be most favorable in some particular case for balancing the line effects and reducing the line-drop to the ohmic equivalent, it does not necessarily follow that this voltage will give the best transmission condition; for the advantage of a higher voltage may easily more than offset the favorable adjustment of the leading and lagging energy of the line. Furthermore, the most favorable line conditions will not always be the most favorable for other parts of the system, and any complete design will be a compromise, as usual.

Instead of feeding leading and lagging current from one end of the line or the other, it may be fed from both. In this case the result will be the resultant of the various tendencies already explained.

It is clear that the end from which the leading or lagging current is fed determines the direction in which the rise or fall of potential will be found. In other words, the same current that is leading, considering the direction from power house to the receiving end, will be lagging if taken in the other direction. A leading current fed into the line from the generator end tends to cause a higher potential at the receiving end, while the same current fed from the receiving end tends to cause a higher potential at the power-house end.

If the leading current be fed equally from both ends, the potential will tend to rise toward the middle and fall toward both ends of the line. Similarly with lagging currents; if the currents fed from the two ends are not equal, the point of maximum

potential will be thrown in one direction or the other, toward the end into which the smaller current flows. Thus in the control of the potential of a line, it may be advantageous to supply the leading current from the receiving end or from both ends, as will be pointed out later.

If the so-called synchronous form of generator be used there would be no difficulty in supplying the necessary amount of power and lagging or leading current to the line, as may be required in any case, but if either lagging or leading energy be demanded at the same time as useful power, the capacity of the generator must be greater in inverse proportion to the power-factor. This is well known.

If the lagging or leading energy be supplied from the receiving end of the line, it may be supplied either by additional generators at this point or by synchronous motors or converters. One disadvantage of this arrangement is the difficulty of making the two generators at the opposite ends of the line divide the line current between them in the desired ratio or of making the generator or the synchronous motor or converter at the receiving end supply all the leading current to the line to the exclusion of the power generator. Of course there would be no difficulty in making the power-house generator supply all the useful power, as this function is determined by the governor of the prime mover.

Where it is desired to supply the l ading current to the line from the receiving end, the expedient is suggested of using non-synchronous generators at the power-house end, and synchronous motors or auxiliary generators at the receiving end. In this case the non-synchronous machines, while generating the amount of power for which their governors may be set, will take only a certain definite amount of magnetizing current; that is, current suitable to supply leading energy to the line from the power-house end. The no-load leading current, except for the magnetizing current of the non-synchronous machines, will then be supplied from the synchronous machines at the receiving end, as is desired. The use of synchronous motors driven from non-synchronous generators has been fully discussed by Waters in the Institute TRANSACTIONS.* This use of non-synchronous machines will be again discussed later in this paper.

But the load as well as the line must be considered. Unfortunately, the load has its own particular characteristics, in-

* A.I.E.E. TRANS. Vol. XXVII, 1908, p. 157.

dependently of the other parts of the system. Its power-factor is not determined by the line current or the line conditions. It must take a certain current, usually lagging, and its requirements will vary as the load changes. Thus other conditions must be suited to the load conditions. But the line, when adjusted for best internal conditions of balanced leading and lagging energy, will deliver current without lag to the load, which is not usually suitable. In case the line conditions are not to be wholly sacrificed to the load condition—this is usually the case in most transmissions of moderate size—it is necessary to supply the load lagging current by synchronous apparatus, either motors or generators. Such an arrangement is perfectly feasible, though not often used at the present time. In case such synchronous apparatus at the load end has been already determined upon to supply leading current to the line, as already explained, these same machines may serve both functions. It is to be noticed that the lagging current of the load and the leading energy supplied by the synchronous machines into the line from the receiving end tend to neutralize each other; and that if a leading current enters the line from the receiving end equal to the lagging current of the load these will balance each other and there will be no current taken from the synchronous machine.

A compromise may be effected between the best line condition, requiring the delivery from the line of current at unity power-factor and the best condition from the point of view of the synchronous machine, in which the load lagging current at full load should be supplied wholly from the leading line current.

In the main the electrical action of the line has now been briefly outlined. The most notable features are:

1. That by properly balancing the leading or capacity energy and the lagging or inductive energy in the *line itself*, the full-load transmission will be as though the line had only ohmic resistance.

2. That a reasonable constancy of voltage conditions on the line *for all loads* may be obtained by supplying the leading energy in the condition when the load is off or light, partly or wholly from the receiving end.

In actual cases of transmission it is usually found that the lagging component so far overshadows the leading component as to render the former the dominating feature. In this case the only expedient for controlling line-drop would seem to be to

raise the voltage or to limit the maximum load. It will be possible, however, in such cases to apply the suggestion made above—to divide the conductor to reduce the line inductance and increase its capacity. In the case of large excess lagging current, the loss of capacity and good regulation in the generators due to the lagging component of the load added to that of the line is well understood. The lagging component of the load can be balanced by synchronous machines at the load end, but this adds too much complication for most plants, or at least seems to be so considered. Where such synchronous machines are not used at the receiving end, a large line capacity current may be desirable to balance the load lagging current.

But it is not the purpose of the present discussion to consider the smaller plants as distinguished from the large long-distance systems in which it is feasible to balance the leading and lagging line energy.

There is a remarkable effectiveness of the arrangements here suggested in rendering practicable, electrically speaking, the transmission of large amounts of power in a single line for long distances with an acceptable voltage regulation. This can best be shown by means of a few actual examples. The first case assumed (case *a* Table A) has been taken to represent the transmission distance of the well-known Victoria Falls transmission, but all attempts to parallel the other commercial or engineering conditions of this project have been carefully avoided.

The transmission is taken as having 700 miles of three conductors spaced 12 ft. apart and each having three times the section of No. 0000 wire. The frequency is taken as 25 cycles. Assume at the start that the power to be delivered at full load is 40,000 kw. at 150,000 volts and with a load power-factor of 0.85. Assuming that synchronous machines at the receiving end supply all of the load lagging current except enough to charge 300 miles of the line—giving a line power-factor of 0.918 at the receiving end—the drop in voltage will be 12.1 per cent and the generator power-factor will be 0.96, leading. The true energy-loss in the line will be 10.4 per cent., giving what seems to be at first not an unfavorable condition. The effect of throwing off the load (case *b* Table A), however, is to cause a rise in voltage of about 21 per cent, above generator voltage assuming that the synchronous machines at the load end supply no leading energy in the no-load condition or that they have dropped off the line. This gives too great a range of voltage at the receiving end for satisfactory direct operation,

TABLE A.

ILLUSTRATIVE LINE ADJUSTMENTS WITH CALCULATIONS.

	Distance in miles	Size of conductors	Spacing of conductors in feet	Transmitted power, three phase	Line voltage delivered	Line current load end, "-" means leading 1 φ – equivalent	Frequency	Line power-factor, load end	Voltage drop in % of generation voltage	Line energy loss in % gen. power	Line power-factor generator end, "-" means leading	Line current generator end, "-" means leading 1 φ – equivalent	$\frac{p\,x}{d}\,\frac{v}{2}\left(\frac{360}{2\pi}\right)$
a	700	3 #0000	12	40,000	150,000	145.3	25	0.918	12.1	10.4	-0.963	134.8	34° 48′
b	"	"	"	0	"	0	"	—	-21.4	—	—	—	"
c	"	"	"	60,000	"	200	"	1.00	11.5	13.8	-0.994	206.8	"
d	"	"	"	0	"	-63.8	"	0	-.2	—	—	—	"
e	"	"	"	60,000	"	202	60	0.99	7.2	13.2	-0.974	219.7	83° 6′
f	"	"	"	67,500	"	225	25	1.00	15.7	15.3	1.000	224	34° 48′
g	"	"	D* "	60,000	"	235	25	0.85	15.7	11.7	-0.96	199	35° 6′
h	"	$(3\frac{3}{4})\times$ #0000	D* "	75,000	"	278	"	0.99	19.6	13.7	-0.988	237†	"
i	"	"	D* "	50,000	"	231.6	"	0.72	17.5	11.5	-0.962	161.4‡	"
j	"	"	D* "	0	"	-175	"	0	10.4	2% 75,000 kw	-0.09	-53.3	"
k	300	3 #0000	"	60,000	"	235	"	0.85	17.1	7.8	0.876	205	Sp't Cap.
l	"	"	D* "	"	"	"	"	"	11.4	7.3	0.971	196.6	Sp't Cap.
m	"	$(\frac{8}{9})^2\times$ #0000	10.5	12,000	100,000	75	60	0.80	6.23	4.46	0.873	67.5	35° 36′
n	"	"	"	20,000	"	125	"	"	20.2	7.00	0.998	86.1	"
o	"	"	D* "	50,000	"	250	"	1.00	14.5	14.8	1.000	250.7	35° 48′

D* means divided conductor, each line wire divided into three separate conductors spaced 18″ apart. Inductance reduced to from 0.60 to 0.59 of normal.
† Leading component of line current at generator end = 36.5.
‡ Leading component of line current at generator end = 44.2, compare 53.3 amperes in case *j*.

but probably would be satisfactory for the regeneration of the power.

In this case it is evident that the leading energy in the line greatly overbalances the lagging energy in the line taken as a whole, but that at the load end the lagging component of the *load* causes a predominance of lagging at this end of the line, which is gradually overcome by the leading excess in the line until at the generator the leading energy predominates. This is in a way a favorable combination, for the load might easily be supplied with lagging current without the intervention of synchronous machines at the load, and yet the general excess of leading energy in the line tends to limit the line drop. The weak point of this arrangement is the rise in line voltage from generator to load on no load or light load, and the increased line energy-loss.

To balance the line leading and lagging energy components at full load, we may increase the total powert ransmitted, making it 60,000 kw. delivered (case *c* Table A). In this case to obtain unity power-factor at the load it may be assumed that there are synchronous generators to carry the load lagging current, as distinguished from the lagging line energy. The line drop will then be 11.5 per cent and the generator power-factor 0.994 leading, showing that the line leading and lagging energies are about equal. In this case the line current is approximately the same all the way through the circuit, and the line energy-loss calculated by the wave formula is 13.8 per cent, almost exactly the product of the resistance and the square of the delivered current, as it must be. The line drop is a little less than the product of the resistance and load current in this full-load condition, on account of the slight excess of the leading energy in the line.

Now if it be assumed that the leading current for the no-load condition be supplied from the load end, then the rise in voltage backward along the line from the load end to the generator end will be 21 per cent (see case *b* Table A). This is the same thing as a drop of the same amount in the direction of the power transmission, the net change in line voltage from full load to no load, and is only 10.5 per cent and this occurs at the power house, provided the voltage of the synchronous apparatus at the load end, which does the magnetizing, is kept constant by some regulator or other means. Thus this condition is a highly satisfactory one for the transmission, provided no indirect disadvantages appear.

If, on the other hand, enough charging current—about 64 amperes—be fed from the receiving end to charge about 300 miles of line, the rest being supplied from the power-house end, the no-load voltage at the power-house end will be 0.2 per cent lower than at the receiving end, showing the effect of feeding leading current from both ends (see case *d* Table A). The potential at the middle of the line, however, will be somewhat higher than at either end.

In the case just assumed the synchronous machines must carry the whole lagging component of the load at full load and leading energy for the line at no load.

But it is possible to improve the practical conditions a little by allowing a certain portion of the load lagging current to be carried by the line capacity, as the leading and lagging components are at right angles to the power component and produce little effect in the line as long as they are relatively small. Line charging current flowing from the line out to the load is there actually seen as lagging current. As far as carrying the lagging component of the load is concerned, the work to be done by the synchronous machines will be reduced by this expedient, thus permitting a larger power output for a peak load, though the no-load condition will be unchanged.

To accomplish this result, it will be necessary to increase the line capacity energy or to decrease the load. If the former alternative be chosen and the expedient suggested above of dividing the line conductors be used, assuming three conductors, each equal to one No. 0000 wire with the same average spacing but separated individually 18 in., the line inductance and capacity will each be altered in approximately the ratio 0.6. The line conditions will then be such as to give a drop of 15.7 per cent (see case *g*, Table A) taking the power-factor as 0.85 at the load at full load. This will give a power-factor at the generator of 0.96 leading. The drop has been thus increased as the lagging excess at the receiving end of the line dominates the leading excess at the power-house end. Here as far as the line itself is concerned, the leading energy far exceeds the lagging energy, with the consequent wide variation of current and power-factor through the length of the line. The lagging load component of the load entering the line at the load end is sufficient to dominate the line near this point, though the excess of leading energy in the line soon overpowers the lagging and leaves the generator current leading by a considerable factor.

These line conditions, while not the best for the line itself—the drop has increased to 15.7 per cent—is in some ways better than in case *c*, on the whole, as there is no call ont he synchronous apparatus at the load end at the full-load condition, and this apparatus is available as an auxiliary for supplementary power to carry peak loads. But it is evident that the leading component of the line is unnecessarily large, and a larger amount of power would more nearly balance the leading and lagging energies. Thus, assuming that the power be increased to 75,000 kw. and the power-factor at the load end be made 0.90 and that at the same time the section of the conductor be increased 25 per cent to keep down the true line energy loss, then the line-drop will be 19.6 per cent (see case *h*, Table A) and the generator power-factor will be 0.988 leading. The line loss is then reduced to 13.7 per cent. This is a favorable condition, for the energy loss is low, the synchronous apparatus is called on only for a relatively small amount of lagging current at full load, assuming that the *load* power-factor is as before 0.85, and at no load the synchronous apparatus is free to supply the leading energy to the line. On no load the rise in voltage toward the generator will be 10.4 per cent (see case *j*, table I), assuming 175 amperes charging current at the load end, which gives a net change at the power house of some 9 per cent, which is entirely satisfactory. This assumes constant voltage to be maintained at the load end by the synchronous machines.

Thus it is seen that by the principles here expounded, it was possible practically to double the output of this line over that obtained by the ordinary method of operation. This double output was obtained with a very much more satisfactory voltage regulation, and with a true energy loss that could be reduced to any desired value by the mere addition of copper.

It is interesting to note that the cost of copper per kilowatt, in the line with 75000 kw. transmitted and copper at 16c. is about $56, by no means a prohibitive value.

Similar examples might be worked out for higher frequencies, where these are desired, and though the service will be much more sensitive to the accuracy of proportioning, there is no theoretical reason why nearly as good regulation should not be gotten with 60 cycles as with 25.

Case *c*, Table A, above, taken for 60 cycles would give the results shown in case *e*, Table A. Here a slightly lower load power-factor has been chosen, since the leading energy is slightly in

excess. The results are very nearly the same for both frequencies, the drop being less with 60 cycles on account of the greater excess of leading current at the power house.

To show a more exact balance between the leading and lagging energy than is shown in case *c*, case *f* has been calculated for the same line with a somewhat larger load, 67,500 kw. The power-factor is here found to be unity at both ends of the line, the current the same within less than one half per cent, and the energy-loss nearly equal to the drop in per cent. The drop calculated as the product of the resistance and the current = 15.2 per cent for this case, which should be compared with the value by wave formula of 15.7.

The results cannot be in perfect accord, however, for in view of the drop in the line, the leading and lagging energy cannot be exactly balanced at all points. The leading energy varies from point to point, while the lagging energy is substantially constant. The leading and lagging energy are here equal at the middle of the line.

Another example will illustrate a 60-cycle line. Assume a 300-mile transmission, with conductors of a diameter $\frac{3}{5}$ that of No. 0000 wire, spaced 10.5 ft. With a delivered voltage of 100,000 volts and load power-factor of 0.80 and delivered power of 12,000 kw. three phase, the drop at 60 cycles will be 6.2 per cent and the true loss 4.5 per cent (see *m*, Table A). The power-factor of the generator is 0.87 leading. Here the leading energy greatly predominates and the heavy lagging load current is changed into a leading current at the generator. The drop is satisfactory but the power transmitted is small. If the power is increased to 20,000 kw. the drop becomes 20.2 per cent, which is excessive although the line energy loss is small (see *n*, Table A). The generator power-factor becomes 0.998.

Again in this case the leading energy predominates. The drop would here be improved if there were synchronous machines at the lowering end of the line to carry the lagging current of the load.

If, however, the divided conductor arrangement be used—three parts for each conductor spaced 18 in. apart—the inductance and capacity will each be changed in the ratio 0.59 (see formula in companion paper). Choosing now a power delivered such as to balance the leading and lagging energy at the center of the line, that is 50,000 kw., the drop will be 14.5 per cent and the generator power-factor unity as well as the

load power-factor (see case *o*, Table A). The line-loss is 14.8 per cent and the load current of 250 amperes becomes 250.7 at the generator. This is another excellent example of the effect of a close equality between leading and lagging energy.

The cost of line copper per kilowatt in this case with copper at 16c. is about $22, a reasonably small value.

In this connection it is interesting to compare the transmission curves given by Mershon in a paper before the Institute.* Here Mershon studies the economical conditions of power transmission as indicating the commercially feasible limits of distance. Mershon has tacitly assumed that the effect of the alternating character of the current on regulation can be ignored in his discussion by taking the line-drops as the product of the resistance by the current. Such an assumption will ordinarily be unjustifiable, for with the usual spacing of conductors the line-drop will be far from these ohmic values. With the actual values shown by Mershon's curves, however, the relations are such that for the long distances the leading and lagging energies are not very far apart. For example, the case *c*, Table A comes very closely to his curves. In other cases, however, especially the short distances, the lagging energy will greatly predominate, giving much larger line drops than those shown in Mershon's curves. It should be said, however, that in these cases the drop is relatively small on account of the short distances.

Mershon states that he uses synchronous machines to bring his average line power-factor to unity. What he does is to reduce by this means the power-factor of the energy at the middle or some other point of the line to unity, for this is the only power that the synchronous machine possesses. It cannot balance the leading and lagging energy of the line itself, as distinguished from the leading or lagging current of the load. As already explained, unity power-factor alone does not give the condition which will permit the transmission of power with a purely ohmic line drop, but the equality of leading and lagging energy within the line itself must also be obtained. Now, as has been shown above, this equality can in most cases of important transmissions be approximately obtained, by the proper choice of voltage and power transmitted and of relative line conductor arrangements; and the bad effects of the leading current building up a rise on light loads can be eliminated by supplying the leading

* Maximum Distance to which Power Can be Economically Transmitted, A. I. E. E. Transaction, Vol. XXIII, 1904, p. 759.

energy from the receiving end of the line, so that the Mershon condition of direct-current drop can be quite nearly attained by these expedients. It is a matter for comment that the Mershon curves chosen for reasons of economy come rather near, in the maximum cases, to the best conditions for internal line conditions. This equality is departed from in the shorter distances, however.

In the above discussion only the simple direct principles involved have been considered, since in this way they are most clearly pointed out. In most practical transmission systems these simple principles, while still perfectly true, are often dominated by naturally unfavorable proportions or by other features. Even in the long high-power lines where the line is the most important feature, many limiting conditions will often be found to prevent the realizing of ideal conditions. For example, the effect of leading current in the fields of synchronous generators is marked, and in the absence of automatic regulation will cause voltage variations. For this work in important cases possibly machines especially designed to hold the voltage constant might be desirable. The difficulty of obtaining the desired division of leading current among a number of stations may be troublesome, especially where the sub-stations are under independent control. Several lines of different lengths radiating from the same power house or those that are cross-connected may be troublesome. But it is the object of the present discussion to point out the principles governing the electrical action of the line, and not to discuss the secondary questions inevitably brought up by applications in special cases.

In calculations of lines in which the leading and lagging line energies are approximately equal, the split-capacity formula should not be used without great caution, for then it naturally has a large error especially in the value of line-drop. That this error will exist is evident from the fact that while in the line itself the effects of capacity and inductance are eliminated, the split-capacity formula shows an entirely different condition. Table I in the companion paper shows a number of cases calculated by both formulas. The close agreement in some instances with the wide divergence in others is notable.

Terminal apparatus. Much depends on the type and adjustment of the terminal apparatus, generators, motors, and synchronous converters, in controlling the line voltage, energy-loss and power-factor.

The object to be attained by the choice of type and method of control of the terminal apparatus, in addition to the generating and delivering of power, is the controlling and minimizing of line-drop and line energy-loss. As already pointed out, the terminal machines may control the line voltage by controlling the amount of charging current fed from the respective ends of the line. This is the same thing as controlling the power-factor at these points; for leading current to the line is the same as lagging current from the line to the apparatus.

It is important to bear in mind that one sort of terminal machines may be desirable when the power-factor adjustments are to be made by hand, another sort when it is desired to make the adjustments as far as possible automatic. The simplest case above was that in which the charging current is largely supplied from the receiving end.

The advantage of the non-synchronous generator at the power house in such a case is evident; for it will automatically throw the charging current; that is, the charging current over and above its own magnetizing current, on the synchronous machines at the load end. Since most large transmissions will have a steam auxiliary at the load end, this can to great advantage utilize synchronous type generators and supply the charging current of the line, except for a certain definite portion that will be supplied necessarily by the magnetizing current of the non-synchronous machine at the power-house end. This partial charging current from the power end is of advantage at full load, as it tends to reduce the ohmic drop. If desired, the non-synchronous machine can be made to take a relatively large magnetizing current for this purpose. But for the light load condition, the charging current should be fed largely from the load end, and this is to some extent automatically accomplished by the fact that at low load the non-synchronous machine takes less magnetizing current. But the dominating effect is the setting free at no load of the leading energy that was neutralized by the lagging energy of the line at full load, which causes the desired no-load rise in voltage toward the power house, and which is depended on to match the full-load drop toward the load end.

The use of non-synchronous machines has other advantages than this of allowing a definite automatic distribution of the charging current for the line, prominent among which are a number of mechanical and other advantages discussed in a paper by

Waters.* The non-synchronous machine at the power house gives another advantage, in that any tendency for excessive rise of potential at the power house will cause a greatly increased magnetizing current in the machine, that is, charging current to the line, which will help to keep down the rise of potential, since it will tend to cause a slope of potential along the line in the opposite direction. The limiting of the tremendous amount of short-circuit current that flows from large systems, through the well-known action of these non-synchronous machines, deserves special mention. Again, the fact that such machines need not be synchronized may be a very great advantage. While the paralleling of synchronous generators in a normal power house has no great difficulty or drawbacks, the separation of such machines by long distances is a great handicap, and the advantage of the non-synchronous machines here is noteworthy. While this is an operating advantage only, as such it is entitled to the fullest consideration.

Since even with a non-synchronous generator it is necessary to have accurate adjustment in the speed of the machine to avoid a sudden rush of current, and since in any case the throwing in of a large unit is always a disturbing factor, it is suggested that if a non-synchronous generator be used at the power house, load circuit-breakers be omitted, and that the step-up transformers and the generators be treated as a part of the line.

In some ways it is disadvantageous that the magnetizing current for the non-synchronous generator should come from the receiving end, as the power house would consequently be dependent on the distant station for excitation. In normal operation, however, the load will be continuous and there will normally always be magnetizing current available, while for emergencies a synchronous type of generator of the necessary capacity to handle one line may be installed at the power house. Such a generator could then be used for local power, if the power-house voltage should be found too variable.

In case *a*, Table A, the generator power-factor is 0.96 leading, which is suitable for a large non-synchronous generator under favorable speed conditions. Thus such a generator, with the assumed synchronous apparatus at the load end, could supply the line under the conditions assumed with the voltage drop and line-loss shown in this case. If the load were thrown off, the voltage would rise toward the power house (as the major

* Waters, TRANS. A. I. E. E., Vol. xxviii, p. 157, 1908.

portion of the line charging current would be supplied from the synchronous machines at the load end; that is, all but the magnetizing current of the non-synchronous machine) and to an amount not far from the full-load drop.

Cases *e* and *g* and *h*, Table A,--all have leading line power-factors at the power house end, which, by a slight increase or decrease, would be suitable for a non-synchronous machine.

In these cases the voltage on no load would be little different, as in case *a*. To show the effect of intermediate loads, case *i*, showing the effect of two-thirds load in case *h*, has been calculated, giving a drop from power house to load end of 17.5 per cent, the lagging current of the non-synchronous machine at the power house being assumed to be slightly reduced.

In many ways the plan of having the system controlled at the receiving end has great operating advantages. This arrangement is simplest with auxiliary prime movers at the receiving end. It is possible, however, to accomplish similar results with synchronous motors.

The natural location for the control of a system, from the point of view of the service, is the receiving end; the custom of control from the power house ordinarily results from the greater complication of the apparatus at this point. If the use of non-synchronous machines should render the transfer of control to the receiving end feasible, this would be a welcome change.

Again it is certainly advantageous to have the voltage controlled at the load end rather than at the generator end of the line. An automatic regulator may be used on the machines at the load end to keep the voltage constant on the receiving bus-bars for practically all load variations. What variations of potential do occur would then be at the power house, where they would ordinarily be of relatively little consequence.

It may be said in favor of using synchronous machines at both ends of the line, that they permit much greater flexibility of adjustment of the relative amounts of leading current furnished by each and thus greater flexibility in the line control. The synchronous machines can of course take either a leading or a lagging current at will. The very flexibility of this system is perhaps its weakness, for the attendants at all points must work intelligently and in harmony, while with the non-synchronous machines the control is automatically thrown to the synchronous machine.

In smaller transmissions it will usually not be worth while

to use machines of either type at the receiving end, as is shown by present practice, in which case only synchronous machines can be used at the power house.

General limitations. The limitations of a method of design or operation usually require more careful consideration than its merits, and the more important of the conditions limiting the methods here proposed will be considered It is a fact, however, that the weighing of advantages and disadvantages is very unsatisfactory in a general case, as the particular conditions of each system are a most important factor.

With such a close adjustment of proportions as is necessary for the most economical results theoretically, departures from the ideal condition may produce relatively large discrepancies. For example, in the full-load condition with leading and lagging energies balanced, overloads will add very rapidly to the drop; similarly, with a lowering of the power-factor required of the line at the receiving end. With the arrangement utilizing the auxiliary machines at the receiving end and non-synchronous machines at the power house, however, the variation of power-factor will be cared for automatically by the machines at the load end. If the line design be made approximately correct for the worst condition to be met; that is, full load at the lowest power-factor, so that the leading and lagging energies of the line are balanced, and if the drop is then not excessive and the non-synchronous machines or other generators at the power house take their correct amount of current, conditions should be stable.

A tendency to instability is inherent in any system in which the main generators do not assist in maintaining the voltage, such as systems using non-synchronous generators. Synchronous machines are notably at a disadvantage with lagging or leading current in the matter of voltage regulation, as such currents tend to demagnetize or strengthen the fields. If, then, the system is such that a relatively small machine must carry all the leading or lagging current for machines of much larger capacity, the tendency to instability is greatly increased. This will undoubtedly be one of the features requiring most careful consideration in such a system as here proposed. Special designs of synchronous machines would help somewhat. A quick-acting automatic regulator may be used to control voltage variations within reasonable limits in view of the sluggishness of large generator fields.

In the partial load or light load conditions the line-drop will be very sensitive to the relative amount of charging current supplied from the respective ends of the line, a greater portion from the power-house end causing a tendency toward a lessened drop toward the load end. With conditions tending to considerable variations, as they do in a large system, this fact will be a menace of line voltage variations. This again shows the value of automatic or semi-automatic control of the charging current distribution. On the other hand, it is clear that by properly proportioning the relative amounts of charging current supplied from the two ends of the line, an adjustment of the relative full-load and no-load voltage drops can be obtained and the two made equal. This adjustment can be much facilitated with non-synchronous generators by providing non-automatic means for adjusting the amount of lagging current at the power house. Separate adjustable-shunt lagging coils can be provided if desired.

On less than full load, the leading line energy and the lagging line energy no longer being equal, the most favorable conditions for economy of line loss no longer exist, and in the lighter loads the charging current or leading energy in the line materially increases the line energy loss. This condition can be reduced only by feeding charging current from both ends of the line, or even from three or more points along the line. The maximum charging current flowing at any point of the line will then be only that necessary to charge a fractional part of the whole.

In this matter of the added light-load line-energy loss, the divided conductor is at a disadvantage on account of its increased capacity and charging current. Thus in planning the line it is necessary to consider prominently the load at which the system is to run for the major portion of the time.

If the no-load voltage at the ends of an exceptionally long high-frequency line be adjusted to approximate equality by feeding charging current from both ends, and the voltage rise at the middle of the line is excessive, this can be corrected by feeding charging current also at the middle of the line either from synchronous or non-synchronous machines or from choke-coils.

Limitations will also be introduced by the layout of the system. A large plant of the type here under discussion would require at least two and probably three transmission lines, and with such large amounts of power as would be necessary to give economical financial conditions, two or three generators would be required for each line. In this case the interchanging of gen-

erators and loads between lines, if not carefully planned, might disturb the best adjustments of line and the regulation.

Again, when two separate sub-stations are fed from the same power house, changes of load and of field adjustment in synchronous generators at the two sub-stations would tend to cause overloading of one or the other of these machines and also a certain voltage variation.

Many special cases can be imagined in which will appear limitations and difficulties of varying importance in the use of the principles here discussed. Each case must be judged on its own merits, but the chief difficulties directly introduced have been pointed out.

The ratio of the leading and lagging line energies can be controlled, if desirable, by adding capacity in the form of condensers or synchronous types of machine across the line at sufficiently frequent intervals, or by similarly adding inductance by coils or machines. The coils might be added in series with the line, in which case they will tend to balance the leading energy for one load only, as with the normal line inductance, while with the shunt coils the leading energy is balanced for all loads and all voltages. These coils and condensers must be at sufficiently frequent intervals, the more frequent the more perfect their action. This use of distributed coils or condensers is analogous to that of the well-known Pupin load coils for telephone circuits.

The method of controlling the ratio of line inductance to line capacity by dividing conductors should receive fuller consideration. The electrical effect of dividing the conductor is not difficult to calculate approximately, and a suitable approximate formula is derived in the companion paper, formulas being given for the two-part and three-part divisions. The maximum practical numerical variation in favorable cases will be a reduction in the inductance and an increase in the capacity by the factor 2, obtained by the use of the three-part division. This gives an actual change in the ratio of capacity to inductance of 4. This is a little greater than will usually be easily realized. More or less serious mechanical difficulties are introduced by this construction, but these are by no means insuperable. It is probable that such an innovation will not be worth while, except in large, long, and important systems with relatively large conductors.

The divided parts of the conductors may be spaced equally and run straight and parallel to the line, or may be arranged spirally. They may be kept at the same spacing from one another at all points, or may be brought together at the insulators for mechanical support. They can be easily spaced by spreaders on the spans.

A certain mechanical reliability is introduced by the multiple conductors, in that if one is broken, through some defect in the wire, for example, the others will hold it up. With suspension-type insulators the supporting of multiple conductors will be mechanically relatively easy.

The wind and ice strains will be materially increased with the split conductors, but this fact will not necessarily be prohibitive: first, when the alternative will be an entirely new line on account of excessive drop, which will be still more expensive; secondly, with large conductors, since in such cases (where this method will be most useful) the wind strain, being relatively small in proportion to the weight and at right angles to the weight, will not add materially to the total, and since the ice strain will also be a much less important relative quantity than with the smaller wires, or may be absent entirely.

With two conductors the exposed area will of course be 1.4 times as great as with the same copper in one conductor. With three conductors the increase of surface will be 1.7.

The element of mechanical handling and stringing with very large conductors may work out in favor of divided conductors. In some cases it may be desirable to support each of the conductors on a separate insulator. This would give a distinct advantage in reliability, for if any one insulator should be shattered or the wire burn off, as in the case of a lightning ground, the others might support and insulate the line. This result, of course, assumes the automatic clearing of the line when the accident happens.

Again, the two or three divided conductors might be carried separately and insulated from one another all the way, being paralleled only at suitable points and mounted on separate insulators. In this case, under favorable conditions one bad conductor could be cut out and the others carry the current with a slightly increased loss. This arrangement has some advantages in maintaining continuity of service in a system exposed to lightning, for it is not likely that all three wires will be broken down simultaneously. In one sense, this arrangement has the flexi-

bility of three circuits, for separate loads can be carried on the three wires provided they are fed from the same bus-bar. They could also feed separate step-down transformers and be paralleled again on the low-tension distributing bus-bar. The tower for such an arrangement would be only slightly larger than that for the single circuit. Of course numerous objections to these various arrangements will appear. Several other arrangements are possible, though perhaps of doubtful practicability.

To show the electrical advantage of the change of ratio of capacity to inductance even where it is not practicable to balance leading and lagging line energy, cases *k* and *l*, Table A, have been calculated. In a 300-mile, 25-cycle transmission, the drop is reduced from 17.1 to 11.4 per cent and the generator power-factor is raised from 0.876 to 0.97.

A number of principles have been here considered, bearing directly on the allowable output and the regulation of long transmission lines. The subjects have been treated in an entirely general way, no effort being made to apply them to a particular installation. The writer wishes to point out that many of the possible arrangements mentioned have been added merely to illuminate the subject and not with the thought that they will be embodied in actual installations. Any particular plant must be a compromise, meeting in the best way the various more or less conflicting demands of full load and light load, voltage regulation, line energy-loss, output and cost of generators, and of line load-factor, operating simplicity, and climatic conditions.

The effectiveness of this proportioning of inductance and capacity in eliminating the effects of capacity and inductance in the transmission line, suggests its application in other work, as for example, submarine cable work. Here the conditions are different, for it is necessary to deal largely with the transient phenomena, and, further, instead of the line voltage being substantially a constant, it will vary from a maximum to zero, thus tending to give a constantly varying ratio between the leading energy and the lagging energy. Since the electrostatic capacity of the cable greatly exceeds the inductance, an increase of inductance will be required to obtain the necessary balance. This matter will not be treated further here.

Summary

The more important definite conclusions arrived at may be summarized as follows.

1. To obtain the most economical transmission of power over a long-distance line, it is necessary to have balanced the leading and lagging energies of the line itself, and at the same time to have unity power-factor, in which case the transmission is similar to direct-current both as to voltage-drop and energy-loss.

2. The power-factor at the load end may be made unity or otherwise controlled by synchronous machines, generators or motors, located at the receiving end of the line.

3. To obtain satisfactory regulation for high-power long-distance transmission, the leading current taken by the line at light loads should be fed mainly from the receiving end. By this means the voltage-rise from the receiving end; that is, the drop from the power house to the receiving end, can be kept nearly constant for all loads. Adjustment of the line-drop can be accomplished at any load by varying the amounts of the leading energy or charging current supplied to the respective ends of the line.

4. The voltage regulation and line-losses may be controlled for any commercial frequency and for any percentage load condition—as far as its electrical action is concerned—using a conductor of sufficient section, by making the leading and lagging line-energies equal at full load, and by feeding suitable proportions of the charging current at other loads from the respective ends of the line, the greater portion being 'rom the receiving end.

5. The adjustment of the leading and lagging energies taken by the line itself may be accomplished by the choice of voltage and load, or by adjusting the ratio of the capacity and inductance of the line. The latter adjustment can be accomplished effectively by dividing the conductors and spacing them apart a reasonable distance.

6. Calculations of these long high-voltage, high-power lines may be made approximately only by formulas founded on the assumption of localized capacity. The error in the voltage may be several per cent. The approximate formulas are especially unreliable when the leading and lagging energy in the line are equal.

7. Non-synchronous generators are suggested for the power house of a long high-power transmission where it is desired to supply the leading energy for the line from the receiving end. The latter then becomes the controlling and operating point of the system.

8. Constant voltage or suitable compounding should then be maintained at the receiving end and not at the generating end of the system, thus practically eliminating line voltage regulation as a disturbing factor.

9. While it is possible by a cut-and-try method for any given case to arrive at the best condition of capacity, resistance, inductance, and the power-factor at the load end of the line, yet the best result will be obtained most directly by planning the line in accordance with the principles here explained.

While comparatively few high-tension power-transmitting systems now installed are of sufficient magnitude to require as careful a treatment as that outlined here, the development of this art cannot proceed much further without some such method. It is the hope of the writer that here, either directly or, perhaps more likely, by suggestion, he has somewhat forwarded this development.

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CALCULATION OF THE HIGH-TENSION LINE *

BY PERCY H. THOMAS

Neglecting matters of cost, the electrical performance of the high-tension transmission line becomes the critical factor as the quantity of power and distance of transmission become large. The loss and variation of voltage and, to a lesser extent, perhaps, the loss of energy in the line tend to become the dominating features. A careful study of these characteristics is thus most important in planning such a transmission. It is the purpose of the present paper to discuss the calculation of the high-tension line as relating to the important matters of regulation and energy line loss, and with especial reference to the unusually long and high-power lines. For this purpose certain new formulas suited for practical calculations are derived.

While it has been freely recognized that, strictly speaking, calculations of line quantities should, where appropriate, be made in such a manner as to take cognizance of the fact that the capacity, inductance, and capacity of the line are distributed uniformly, the difficulty of such calculations and the lack of suitable formulas have led to the use of approximations. It may be said that many of these approximations are sufficiently accurate for all ordinary commercial systems. However, in large and very long-distance transmissions the error introduced by the approximations becomes quite noticeable, and while not great enough to threaten seriously the usefulness of the system is too great for a conscientious engineer to neglect.

In the following pages is derived a formula for calculating

* It is intended that this paper shall be considered in connection with a companion paper presented at the same time and entitled "Output and Regulation in Long-Distance Lines."

the true current, voltage, and power-factor at the generator end of a transmission line when the current, voltage, and power-factor at the receiving end are known. This formula is especially adapted to slide-rule work and when once familiar requires relatively little labor.

This formula reduces to the usual well-known forms when the resistance, inductance, or the capacity is put equal to 0.

For purposes of comparison an approximate formula is given, used by the writer and based on the much used assumption that the line capacity is concentrated at the ends of the line. This is called the "split-capacity" formula in distinction from the "wave" formula with the distributed capacity first mentioned. The split-capacity formula is exceptionally simple and readily used with a table of cosines and tangents and is believed to be as accurate as any available approximate formula; it would be perfectly accurate were the capacity actually located at the ends of the line as assumed.

To show the relation numerically of the results given by the split-capacity and the wave formulas, Table I is added showing the actual numerical values for a number of selected cases.

The conditions which tend to make a large discrepancy between the two formulas are length, high frequency and low resistance, as well as the particular relation of capacity and inductance energy in the line.

The last column of the table gives the actual phase change due to the time consumed in the passage of the wave through the length of the line as is represented by the quantity $\frac{px}{v}$.

Derivation of Wave Formula

Current flow in a uniform conductor containing inductance, capacity, and resistance, distributed, obeys the following laws:

1. Current flowing through resistance produces a drop equal to the product of the current and the resistance at any instant.

2. A varying current produces in an inductance an electromotive force proportional to the rate of change of the current and to the inductance and opposing the current variation.

3. The current to a condenser is proportional to the rate of increase of potential and to the capacity.

From these laws it follows that at any instant

$$\frac{d^2 V}{d x^2} = C R \frac{d V}{d t} + C L \frac{d^2 V}{d t^2} \tag{1}$$

TABLE I.
COMPARISON OF FORMULAS.

	Distance in miles	Size of conductors	Spacing of conductors	Transmitted power, three-phase	Line voltage delivered	Line current load end, "–" means leading 1 ϕ equivalent	Frequency	Line power-factor, load end	Voltage drop in % of gen. voltage	Line energy loss in % of generator power	Line power-factor gen. end, "–" means leading	Line current, generator end 1 ϕ equivalent	$\frac{p x}{v}\left(\frac{360}{2\pi}\right)$	Type of Formula
A	150	#000	7	10000	80000	73.5	25	0.85	10.2	8.6	0.922	66.6	8° 9′	Wave
B	"	"	"	"	"	"	"	"	10.0	8.6	0.920	67.0	—	Sp't Cap.
C	"	"	"	"	"	"	"	"	12.6	7.6	0.985	60.1	18° 13′	Wave
D	"	"	"	"	"	"	60	"	12.9	7.7	0.984	60.1	—	Sp't Cap.
†*E*	700	2 #00	10	40000	150000	142.5	25	0.935	20.5	20.4	–0.950	140.0†	35° 29′	Wave
F	"	"	"	"	"	"	"	"	22.7	20.1	–0.948	136.4	—	Sp't Cap.
G	"	"	"	0	"	–63.8	"	—	1.1	—	—	—	35° 29′	Wave
H	"	"	"	"	"	"	"	—	1.0	—	—	—	—	Sp't Cap.
I	"	3 #0000	12	40000	150000	145.3	25	0.918	12.1	10.4	–0.963	134.8	34° 48′	Wave
K	"	"	"	"	"	"	"	"	13.5	9.5	–0.968	131.5	—	Sp't Cap.
L	"	"	"	60000	"	200	"	1.00	11.5	13.8	–0.994	206.8	34° 48′	Wave
M	"	"	"	"	"	"	"	"	14.1	14.7	–0.996	202.7	—	Sp't Cap.
N	"	"	"	"	"	202	60	0.99	7.2	13.2	–0.974	219.7	83° 6′	Wave
O	"	"	"	"	"	"	"	"	34.0	17.8	–0.941	170.7	—	Sp't Cap.
P	"	$(3\frac{3}{4}) \times$ #0000	*D** "	75000	"	278	25	0.90	19.6	13.7	–0.988	237	35° 6′	Wave
Q	"	"	*D** "	"	"	"	"	"	25.6	13.8	–0.998	215.7	—	Sp't Cap.
R	75	#0000 lead cable	‡	12000	40000	187.5	"	0.80	15.5	15.7	0.973	154.3	13° 34′	Wave
S	"	"	"	"	"	"	"	"	15.3	15.5	0.973	154.3	—	Sp't Cap.
T	300	$(\frac{3}{2})^2 \times$ #0000	10.5	20000	100000	125	60	"	20.2	7.0	0.998	86.1	35° 36′	Wave
U	"	"	"	"	"	"	"	"	21.9	6.9	0.996	84.3	—	Sp't Cap.
V	"	"	"	12000	"	75	"	"	6.2	4.46	–0.873	67.5	35° 36′	Wave
W	"	"	"	"	"	"	"	"	7.4	3.9	–0.89	64.8	—	Sp't Cap.
† Condition at middle of line.						142.5	25	93.5	13.9	11.0	1.000	129.0	17° 45′	Wave

*D** means divided conductor—each line wire divided into three separate conductors spaced 18″ apart.
‡ Assumed constants, $R = 0.515$, $p\,L = 0.264$, $p\,C = 0.00002358$, all per mile

where

V is the potential at any point of the conductor,
R, L, and C are its resistance, inductance, and capacity, respectively, per unit length,
x is the distance of any given point from the end.
t is the time, measured from any starting instant.

This equation may be arrived at as follows: The current flowing to any small element of capacity is $(C\,d\,x)\left(\frac{d\,V}{d\,t}\right)$ where $C\,d\,x$ is the capacity of the short length $d\,x$, see Fig. 1.

This represents the total current subtracted from the current flowing along the conductor to increase the charge in the small condenser $C\,d\,x$, corresponding to the increase in voltage $d\,V$ accomplished in the short time $d\,t$. This small current

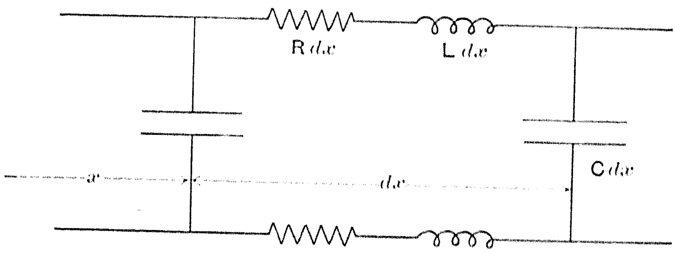


FIG. 1—Elementary section of transmission line.

$C\,d\,x\,\frac{d\,V}{d\,t}$ which is subtracted from the current already flowing, causes a resistance drop of

$$(R\,d\,x)\left(C\,d\,x\;\frac{d\,V}{d\,t}\right),$$

since the resistance of the short length $d\,x$ will be $R\;d\,x$. Similarly since the condenser current also flows through the inductance $L\,d\,x$ it will produce a counter voltage

$$(L\,d\,x)\,\frac{d}{d\,t}\left(C\,d\,x\;\frac{d\,V}{d\,t}\right)=C\,L\,d\,x^2\,\frac{d^2\,V}{d\,t^2}$$

Thus

$$R\,C\,d\,x^2\,\frac{d\,V}{d\,t}+L\,C\,d\,x^2\,\frac{d^2\,V}{d\,t^2}$$

gives the voltage caused in the length $d\,x$ by the current flowing to the capacity $C\,d\,x$.

But the voltage difference between the points $x+d\,x$ and x is $(d\,x)\left(\frac{d\,V}{d\,x}\right)$

This voltage difference, however, includes the drop due to the total current flowing in the conductor and not the *variation* of the total drop which is caused by the charging current $C\,d\,x\,\frac{d\,V}{d\,t}$, alone.

Therefore, this *increase* of the total drop due to the current $C\,d\,x\,\frac{d\,V}{d\,t}$ in the length $d\,x$ will be proportional to the rate of increase of

$$d\,x\,\frac{d\,V}{d\,x}, \text{ that is, } d\left[d\,x\,\frac{d\,V}{d\,x}\right] = d\,x^2\,\frac{d^2\,V}{d\,x^2}$$

Therefore we have the equation

$$d\,x^2\,\frac{d^2\,V}{d\,x^2} = R\,C\,d\,x^2\,\frac{d\,V}{d\,t} + L\,C\,d\,x^2\,\frac{d^2\,V}{d\,t^2}$$

or dividing by $(d\,x)^2$

$$\frac{d^2\,V}{d\,x^2} = R\,C\,\frac{d\,V}{d\,t} + L\,C\,\frac{d^2\,V}{d\,t^2} \qquad (1)$$

This is the usual differential equation for such a circuit. It has been derived, integrated and discussed by many authors, during the last twenty years or more—including Vaschy, Heaviside, Bedell, Pupin, Leblanc and Steinmetz, the latter three in the Institute Transactions.* All these authors have given solutions in such a general form as to be very awkward for use in connection with actual circuits for numerical computation. Stein-

* Pupin, Vol. xvii, A. I. E. E. 1900, p. 445.
Leblanc, Vol. xix, A. I. E. E. TRANSACTIONS, p. 759.
Steinmetz, Vol. xvii, 1908, p. 1231.

metz and Leblanc, and presumably the others, have given the necessary equations by which the specific conditions for a given problem can be worked out, but this only at great labor.

To return to the derivation of the formula. Having given the above differential equation, it is necessary to find a form of equation without differential quantities, which when differentiated will satisfy this equation (1); that is, to find its primitive. This will be a matter of trial. The above writers have pointed out some suitable forms and of these the most convenient for this case is that of LeBlanc* and Vaschy as follows:

$$V = \varepsilon^{\alpha(l-x)} \left[A \sin\left(p\,t - \frac{p\,x}{v}\right) + B \cos\left(p\,t - \frac{p\,x}{v}\right)\right] \qquad (2)$$
$$+ \varepsilon^{-\alpha(l-x)} \left[D \sin\left(p\,t + \frac{p\,x}{v}\right) + F \cos\left(p\,t + \frac{p\,x}{v}\right)\right]$$

where

l = length of the line.

x the distance from the start

$p = 2\,\pi\,n$ (n = frequency), and where v and α are constants which must be determined to make equation (2) consistent with the differential equation (1). This equation giving the voltage at any point of the circuit will always satisfy the differential equation (1) with such suitable values for the constants α and v. The constants A, B, D, and F, are the constants of integration and can be determined when the terminal voltages or terminal currents are known sufficiently to make the problem actually determinate. In other words equation (2) will be consistent with the differential equation with any values of A, B, D, and F.

This equation (2) is not the most general possible form of primitive equation for the differential equation, since no factors occur having an exponential time function. This form is chosen purposely for the condition here to be studied; that is, the steady condition in which the voltage and current have been flowing long enough to reach a permanent condition, and all the starting disturbances have disappeared.

The value of α and v in equation (2) will next be determined to satisfy the differential equation (1).

The values of

$$\frac{d^2\,V}{d\,x^2},\ R\,C\,\frac{d\,V}{d\,t},\ \text{and}\ L\,C\,\frac{d^2\,V}{d\,t^2}$$

*Le Blanc, A, I. E. E. TRANSACTIONS, Vol. xix, 1902, p. 759.

can be determined from equation (2) by differentiating

$$\frac{d\,V}{d\,t} = \varepsilon^{\alpha(l-x)}\left[p\,A\cos\left(p\,t - \frac{p\,x}{v}\right) - p\,B\sin\left(p\,t - \frac{p\,x}{v}\right)\right]$$

$$+\varepsilon^{-\alpha(l-x)}\left[p\,D\cos\left(p\,t + \frac{p\,x}{v}\right) - p\,F\sin\left(p\,t + \frac{p\,x}{v}\right)\right] \quad (3)$$

$$\frac{d^2\,V}{d\,t^2} = \varepsilon^{\alpha(l-x)}\left[-p^2 A\sin\left(p\,t - \frac{p\,x}{v}\right) - p^2 B\cos\left(p\,t - \frac{p\,x}{v}\right)\right]$$

$$+\varepsilon^{-\alpha(l-x)}\left[-p^2 D\sin\left(p\,t + \frac{p\,x}{v}\right) - p^2\,F\cos\left(p\,t + \frac{p\,x}{v}\right)\right] \quad (4)$$

$$\frac{d\,V}{d\,x} = \varepsilon^{\alpha(l-x)}\left[\left(-\alpha\,A + \frac{p}{v}\,B\right)\sin\left(p\,t - \frac{p\,x}{v}\right) + \left(-\alpha\,B - \frac{p}{v}\,A\right)\right.$$

$$\left.\times\cos\left(p\;t - \frac{p\,x}{v}\right)\right] + \varepsilon^{-\alpha(l-x)}\left[\left(\alpha\,D - \frac{p}{v}\,F\right)\sin\left(p\,t + \frac{p\,x}{v}\right)\right.$$

$$\left.+\left(\alpha\,F + \frac{p}{v}\,D\right)\cos\left(p\,t + \frac{p\,x}{v}\right)\right] \quad (5)$$

$$\frac{d^2\,V}{d\,x^2} = \varepsilon^{\alpha(l-x)}\left[\left(\frac{\alpha^2 v^2 - p^2}{v^2}\,A - \frac{2\,\alpha\,p}{v}\,B\right)\sin\left(p\,t - \frac{p\,x}{v}\right)\right.$$

$$\left.+\left(\frac{2\,\alpha\,p}{v}\,A + \frac{\alpha^2 v^2 - p^2}{v^2}\,B\right)\cos\left(p\,t - \frac{p\,x}{v}\right)\right]$$

$$+\varepsilon^{-\alpha(l-x)}\left[\left(\frac{\alpha^2 v^2 - p^2}{v^2}\,D - \frac{2\,\alpha\,p}{v}\,F\right)\sin\left(p\,t + \frac{p\,x}{v}\right)\right.$$

$$\left.+\left(\frac{2\,\alpha\,p}{v}\,D + \frac{\alpha^2 v^2 - p^2}{v^2}\,F\right)\cos\left(p\,t + \frac{p\,x}{v}\right)\right] \quad (6)$$

The differential equation (1) must be satisfied by these values so that substituting in equation (1) and rearranging terms, we have

$$\varepsilon^{\alpha(l-x)}\Big[\left\{\frac{\alpha^2v^2-p^2}{v^2}A-\frac{2\,\alpha\,p}{v}B-RC\,(-p\,B)-LC\,(-p^2 A)\right\}$$

$$\times\sin\left(p\,t-\frac{px}{v}\right)+\left\{\frac{2\alpha\,p}{v}A+\frac{\alpha^2v^2-p^2}{v^2}B-RC(p\,A)-LC(-p^2B)\right\}$$

$$\times\cos\left(p\,t-\frac{px}{v}\right)\Big]$$

$$+\,\varepsilon^{-\alpha(l-x)}\Big[\left\{\frac{\alpha^2v^2-p^2}{v^2}D-\frac{2\,\alpha\,p}{v}F-RC\,(-p\,F)-LC\,(-p^2\,D)\right\}$$

$$\times\sin\left(p\,t+\frac{px}{v}\right)+\left\{\frac{2\,\alpha\,p}{v}D+\frac{\alpha^2\,v^2-p^2}{v^2}F-RC(pD)-LC(-p^2F)\right\}$$

$$\times\cos\left(p\,t+\frac{px}{v}\right)\Big]$$

$$=0$$

This equation must be true for all values of t and for all values of x and for all values of A and B and D and F. This can be true only when each of the four coefficients of the sine and cosine terms is 0.

Therefore

$$\left(\frac{\alpha^2v^2-p^2}{v^2}+p^2\,L\,C\right)A+\left(-\frac{2\,\alpha\,p}{v}+p\,R\,C\right)B=0$$

$$\left(\frac{2\,\alpha\,p}{v}-p\,R\,C\right)A+\left(\frac{\alpha^2\,v^2-p^2}{v^2}+p^2\,L\,C\right)B=0$$

$$\left(\frac{\alpha^2\,v^2-p^2}{v^2}+p^2\,L\,C\right)D+\left(-\frac{2\,\alpha\,p}{v}+p\,R\,C\right)F=0$$

$$\left(\frac{2\,\alpha\,p}{v}-p\,R\,C\right)D+\left(\frac{\alpha^2\,v^2-p^2}{v^2}+p^2\,L\,C\right)F=0$$

But these equations can be zero for all values of A, B, D and F, only if the coefficients of each of these later quantities $=0$.

That is

$$\frac{\alpha^2\,v^2-p^2}{v^2}+p^2\,L\,C=0 \tag{7}$$

$$\text{and}\ \frac{2\,\alpha\,p}{v}-p\,R\,C=0 \tag{8}$$

From equation (8),

$$\alpha = \frac{R\,C\,v}{2} \tag{9}$$

Substituting this value of α in equation (7) and multiplying both sides of the equation by $4\,v^2$

$$R^2\,C^2\,v^4 + 4\,p^2\,L\,C\,v^2 = 4\,p^2$$

$$v^4 + \frac{4\,p^2\,L}{C\,R^2}\,v^2 = \frac{4\,p^2}{C^2\,R^2}$$

$$v^4 + \frac{4\,p^2\,L}{C\,R^2}\,v^2 + \frac{4\,p^4\,L^2}{C^2\,R^4} = \frac{4\,p^2}{C^2\,R^2} + \frac{4\,p^4\,L^2}{C^2\,R^4}$$

$$v^2 = -\frac{2\,p^2\,L}{C\,R^2} \pm \sqrt{\frac{4\,p^2\,R^2 + 4\,p^4\,L^2}{C^2\,R^4}}$$

$$v = \pm\frac{1}{\sqrt{L\,C}}\sqrt{2\,p\,L\left(\frac{-p\,L \pm \sqrt{R^2 + p^2\,L^2}}{R^2}\right)}$$

multiplying numerator and denominator under the radical by

$$p\,L \pm \sqrt{R^2 + p^2\,L^2}$$

$$v = \pm\frac{1}{\sqrt{L\,C}}\sqrt{\frac{2\,p\,L}{p\,L \pm \sqrt{R^2 + p^2\,L^2}}} \tag{10}$$

Of the four possible values of v represented by the alternative signs and all satisfying the differential equation, only that involving the positive sign will satisfy the physical conditions of the transmission of current.

Equation (2), with equations (9) and (10), thus constitutes a formula, giving a variation of potential along a conductor such that current flow follows the laws of resistance, inductance and capacity, as laid down at the beginning of this discussion and as is shown by its satisfying the differential equation (1). This equation is perfectly determinate except for the four constants A, B, D, and F. If these are chosen in such a way that the equation represents the correct potential of the conductor at any one point, it will then represent it at all points.

Before determining these constants for a particular case, however, the equation for the current flow at any point of the

same conductor will be determined. Referring to Fig. 1, it is clear that the decrease in the current between the points x and $(x+d\,x)$ is the capacity $C\,d\,x$ times the rate of change in voltage at this point, namely

$$(C\,d\,x)\left(\frac{d\,V}{d\,t}\right)$$

Thus, the total current will be

$$-\int\left(\frac{d\,V}{d\,t}\right)(C)\,d\,x+K=-C\int\frac{d\,V}{d\,t}\,d\,x$$

For the constant K will be zero, since it may be assumed that there is no current flow existing in the conductor, which is independent of time and space.

Equation (3) gives the value of $\dfrac{d\,V}{d\,t}$.

It is evident by inspection that this equation when integrated with respect to x will have the same form as equation (2) but with different coefficients for the sine and cosine terms. It will be simpler to differentiate such a general expression for the current with respect to x and equate the coefficients to those of the expression for

$$-C\frac{d\,V}{d\,t}$$

and from these determine the general coefficients assumed in the general equation of current, than to integrate

$$-C\int\frac{d\,V}{d\,t}\,d\,x \quad \text{directly.}$$

Assume, then, that the letters a, b, d and f represent the coefficients of the integrated curve of current, which is taken to have the same form as the equation of potential (2). Here the coefficient a of the current equation corresponds to A of the potential equation, b to B, etc. That is, we may consider that equation (2) is the new integrated current equation assumed, if the quantities A, B, D and F become a, b, d and f with new values. We have then to determine a, b, d and f of the current

curve in terms of the quantities in the potential equation. If I is the current, then

$\frac{d I}{d x}$ will be the same form as equation (5),

but with the quantities a, b, d and f substituted for A, B, D and F. But

$$\frac{d I}{d x} = -C \frac{d V}{d t} = -\frac{d}{d x} \int C \frac{d V}{d t} d x, \text{ as above.}$$

Therefore the coefficients of the like sine and cosine terms must be equal in this latter equation, see equation (3), and in the modified equation (5).

Therefore

$$p C A = \frac{p a}{v} + \alpha b \tag{11}$$

$$p C B = \frac{p b}{v} - \alpha a \tag{12}$$

$$p C D = -\frac{p d}{v} - \alpha f \tag{13}$$

$$p C F = -\frac{p f}{v} + \alpha d \tag{14}$$

From equations (11) and (12),

$$a = p v C \frac{p A - \alpha v B}{p^2 + \alpha^2 v^2}$$

$$b = p V C \frac{\alpha v A + p B}{p^2 + \alpha^2 v^2}$$

From equations (13) and (14),

$$d = p v C \frac{-p D + \alpha v F}{p^2 + \alpha^2 v^2}$$

$$f = p v C \frac{-\alpha v D - p F}{p^2 + \alpha^2 v^2}$$

substituting these values of a, b, d and f in the equation for current we have

$$I = \frac{p\,v\,C}{p^2+\alpha^2 v^2}\Big[\varepsilon^{\alpha(l-x)}\Big\{\Big(p\,A-\alpha\,v\,B\Big)\sin\Big(p\,t-\frac{px}{v}\Big)+(\alpha\,v\,A+p\,B)\times\cos\Big(p\,t-\frac{px}{v}\Big)\Big\}+\varepsilon^{-\alpha(l-x)}\Big\{\Big(-p\,D+\alpha\,v\,F\Big)\sin\Big(p\,t+\frac{px}{v}\Big)-(\alpha\,v\,D+p\,F)\cos\Big(p\,t+\frac{px}{v}\Big)\Big\}\Big] \quad (15)$$

the general current equation.

We may now determine the values of A, B, C, and D for the particular case here under discussion; namely, where we have given the current, and voltage, and angle of lag between them, at the receiving end of a conductor. When these constants are then determined, the calculation of the voltage and current and lag angle can be made by the voltage and current equations for any other point of the conductor.

Assume that the received voltage is

$$S\sin(p\,t+\theta)$$

and the received current is

$$Q\sin p\,t$$

Then if we assume (for convenience) that the receiving end is at the point $x=0$, we have at the instant when $t=0$ from equation (2) and equation (15)

$$V_0 = \varepsilon^{\alpha l}B+\varepsilon^{-\alpha l}F = \mathrm{S}\sin\theta \quad (16)$$

$$I_0 = \frac{p\,v\,C}{p^2+\alpha^2 v^2}\Big\{\varepsilon^{\alpha l}(\alpha\,v\,A+p\,B)-\varepsilon^{-\alpha l}(\alpha\,v\,D+p\,F)\Big\}=0 \quad (17)$$

And when $pt=\frac{\pi}{2}$ we have

$$V_{\frac{\pi}{2}} = \varepsilon^{\alpha l}A+\varepsilon^{-\alpha l}D = S\cos\theta \quad (18)$$

$$I_{\frac{\pi}{2}} = \frac{p\,v\,C}{p^2+\alpha^2 v^2}\Big\{\varepsilon^{\alpha l}(p\,A-\alpha\,v\,B)-\varepsilon^{-\alpha l}(p\,D-\alpha\,v\,F)\Big\}=-Q \quad (19)$$

Q is negative, since normal current to the receiving circuit is in the negative direction in the line, as the receiving circuit is at the point $x=0$.

From equations (17) and (19),

$$\varepsilon^{\alpha l} A - \varepsilon^{-\alpha l} D = -\frac{p\,Q}{p\,v\,C} \tag{20}$$

$$\varepsilon^{\alpha l} B - \varepsilon^{-\alpha l} F = \frac{\alpha\,v\,Q}{p\,v\,C} \tag{21}$$

From equations (18) and (20).

$$A = \frac{1}{2\,\varepsilon^{\alpha l}} \left(-\frac{p\,Q}{p\,v\,C} + S \cos\theta \right) \tag{22}$$

$$D = \frac{1}{2\,\varepsilon^{-\alpha l}} \left(\frac{p\,Q}{p\,v\,C} + S \cos\theta \right) \tag{23}$$

From equations (16) and (21),

$$B = \frac{1}{2\,\varepsilon^{\alpha l}} \left(\frac{\alpha\,v\,Q}{p\,v\,C} + S \sin\theta \right) \tag{24}$$

$$F = \frac{1}{2\,\varepsilon^{-\alpha l}} \left(\frac{-\alpha\,v\,Q}{p\,v\,C} + S \sin\theta \right) \tag{25}$$

and by substituting in equation (2), we have

$$\begin{aligned} 2\,V = {} & \varepsilon^{-\alpha x}\left[\left(\frac{-p\,Q}{p\,v\,C} + S\cos\theta\right)\sin\left(p\,t - \frac{px}{v}\right) + \left(\frac{\alpha\,v\,Q}{p\,v\,C} + S\sin\theta\right)\right. \\ & \left.\times \cos\left(p\,t - \frac{px}{v}\right)\right] + \varepsilon^{\alpha x}\left[\left(\frac{p\,Q}{p\,v\,C} + S\cos\theta\right)\sin\left(p\,t + \frac{px}{v}\right)\right. \\ & \left. - \left(\frac{\alpha\,v\,Q}{p\,v\,C} - S\sin\theta\right)\cos\left(p\,t + \frac{px}{v}\right)\right] \end{aligned}$$

$$\begin{aligned} 2\,V = {} & \varepsilon^{-\alpha x}\left[\frac{Q}{p\,v\,C}\left\{\left(-p\sin\left(p\,t - \frac{px}{v}\right) + \alpha\,v\cos\left(p\,t - \frac{px}{v}\right)\right\}\right. \\ & \left. + S\left\{\cos\theta\sin\left(p\,t - \frac{px}{v}\right) + \sin\theta\cos\left(p\,t - \frac{px}{v}\right)\right\}\right] \\ & + \varepsilon^{\alpha x}\left[\frac{Q}{p\,v\,C}\left\{p\sin\left(p\,t + \frac{px}{v}\right) - \alpha\,v\cos\left(p\,t + \frac{px}{v}\right)\right\}\right. \end{aligned}$$

$$+S\left\{\cos\theta\sin\left(p\,t+\frac{px}{v}\right)+\sin\theta\cos\left(p\,t+\frac{px}{v}\right)\right\}\Bigg]$$

$$2\,V=\varepsilon^{-\alpha x}\left[\frac{Q\sqrt{p^2+\alpha^2 v^2}}{p\,v\,C}\left\{\cos\left(p\,t-\frac{px}{v}+\tan^{-1}\frac{p}{\alpha v}\right)\right\}\right.$$

$$\left.+S\sin\left(p\,t-\frac{px}{v}+\theta\right)\right]$$

$$+\varepsilon^{\alpha x}\left[\frac{Q\sqrt{p^2+\alpha^2 v^2}}{p\,v\,C}\left\{-\cos\left(p\,t+\frac{px}{v}+\tan^{-1}\frac{p}{\alpha v}\right)\right\}\right.$$

$$\left.+S\sin\left(p\,t+\frac{px}{v}+\theta\right)\right]$$

but

$$\frac{\sqrt{p^2+\alpha^2 v^2}}{p\,v\,C}=\sqrt{\frac{\dfrac{(p\,L+\sqrt{R^2+p^2 L^2})^2+R^2}{(p\,L+\sqrt{R^2+p^2 L^2})^2}}{\dfrac{p\,C}{p\,L+\sqrt{R^2+p^2 L^2}}}}$$

from equations (9) and (10),

$$=\sqrt{\frac{\sqrt{R^2+p^2\,L^2}}{p\,C}}$$

Therefore

$$2\,V=\varepsilon^{-\alpha x}\left[Q\sqrt{\frac{\sqrt{R^2+p^2\,L^2}}{p\,C}}\left\{\cos\left(p\,t-\frac{px}{v}+\tan^{-1}\frac{p}{\alpha v}\right)\right\}\right.$$

$$\left.+S\sin\left(p\,t-\frac{px}{v}+\theta\right)\right\}\Bigg]$$

$$+\varepsilon^{\alpha x}\left[Q\sqrt{\frac{\sqrt{R^2+p^2\,L^2}}{p\,C}}\left\{-\cos\left(p\,t+\frac{px}{v}+\tan^{-1}\frac{p}{\alpha v}\right)\right\}\right.$$

$$\left.+S\sin\left(p\,t+\frac{px}{v}+\theta\right)\right\}\Bigg] \tag{26}$$

This equation shows the voltage at any point of a line of known resistance, capacity, and inductance, at any instant of time, assuming only that the steady condition has been reached and all starting oscillations have disappeared, and taking the current and voltage at the end of the line as, $Q\sin p\,t$, and, $S\sin(p\,t+\theta)$, respectively.

For the purposes of later discussion and for facilitating

numerical computation it is convenient to put this equation into a somewhat different form.

Expanding the expressions $\sin\left(p\,t - \frac{px}{v} + \theta\right)$ and the similar terms, in the form $\sin\left\{\left(p\,t+\theta\right) \pm \frac{px}{v}\right\}$, etc.,

$$V = \frac{\varepsilon^{\alpha x}+\varepsilon^{-\alpha x}}{2}\left[\left\{Q\sqrt{\frac{\sqrt{R^2+p^2L^2}}{p\,C}}\sin\left(p\,t+\tan^{-1}\frac{p}{\alpha\,v}\right)\right\}\right.$$

$$\left.\times \sin\frac{p\,x}{v} + \left\{S\sin\left(p\,t+\theta\right)\right\}\cos\frac{p\,x}{v}\right]$$

$$+\ \frac{\varepsilon^{\alpha x}-\varepsilon^{-\alpha x}}{2}\left[\left\{-Q\sqrt{\frac{\sqrt{R^2+p^2L^2}}{p\,C}}\cos\left(p\,t+\tan^{-1}\frac{p}{\alpha\,v}\right)\right\}\right.$$

$$\left.\times\cos\frac{p\,x}{v} + \left\{S\cos\left(p\,t+\theta\right)\right\}\sin\frac{p\,x}{v}\right] \qquad (27)$$

It is interesting to notice that the quantities $\frac{\varepsilon^{\alpha x}+\varepsilon^{-\alpha x}}{2}$ and $\frac{\varepsilon^{\alpha x}-\varepsilon^{-\alpha x}}{2}$ are respectively the hyperbolic cosine and hyperbolic sine of $\alpha\,x$ respectively; that is, are $\cosh\alpha\,x$ and $\sinh\alpha\,x$, respectively.

Returning now to the current equation and substituting in equation (15), the values of the constants A, B, D and F as found in equation (22), (23), (24) and (25).

$$2\,I = \frac{p\,v\,C}{p^2+\alpha^2v^2}\left[\varepsilon^{-\alpha x}\left\{\left[p\left(-\frac{p\,Q}{p\,v\,C}+S\cos\theta\right)\right.\right.\right.$$

$$\left.-\ \alpha\,v\left(\frac{\alpha\,v\,Q}{p\,v\,C}+S\sin\theta\right)\right]\sin\left(p\,t-\frac{p\,x}{v}\right)$$

$$+\left[\alpha\,v\left(-\frac{p\,Q}{p\,v\,C}+S\cos\theta\right)+p\left(\frac{\alpha\,v\,Q}{p\,v\,C}+S\sin\theta\right)\right]$$

$$\left.\times\cos\left(p\,t-\frac{p\,x}{v}\right)\right\}$$

$$+\varepsilon^{\alpha x}\left\{\left[-p\left(\frac{p\,Q}{p\,v\,C}+S\cos\theta\right)+\alpha\,v\left(\frac{-\alpha\,v\,Q}{p\,v\,C}\right.\right.\right.$$

$$\left.\left.+S\sin\theta\right)\right]\sin\left(p\,t+\frac{p\,x}{v}\right)-\left[\alpha\,v\left(\frac{p\,Q}{p\,v\,C}+S\cos\theta\right)\right.$$

$$\left.\left.+p\left(\frac{-\alpha\,v\,Q}{p\,v\,C}+S\sin\theta\right)\right]\cos\left(p\,t+\frac{p\,x}{v}\right)\right]\Big\}$$

rearranging and simplifying terms:

$$2\,I=\varepsilon^{-\alpha x}\left\{-Q\sin\left(p\,t-\frac{p\,x}{v}\right)+S\,\frac{p\,v\,C}{p^2+\alpha^2v^2}\Big[(p\cos\theta\right.$$

$$\left.-\alpha\,v\sin\theta)\sin\left(p\,t-\frac{p\,x}{v}\right)+(\alpha\,v\cos\theta+p\sin\theta)\Big]\right\}$$

$$\times\cos\left(p\,t-\frac{p\,x}{v}\right)$$

$$+\varepsilon^{\alpha x}\left\{-Q\sin\left(p\,t+\frac{p\,x}{v}\right)+S\,\frac{p\,v\,C}{p^2+\alpha^2\,v^2}\Big[(-p\cos\theta\right.$$

$$\left.+\alpha\,v\sin\theta)\;\sin p\,t+\frac{p\,x}{v}\Big)-(\alpha v\cos\theta+p\sin\theta)\cos\left(p\,t+\frac{p\,x}{v}\right)\Big]\right\}$$

$$2\,I=\varepsilon^{-\alpha x}\left\{-Q\sin\left(p\,t-\frac{p\,x}{v}\right)+S\,\frac{p\,v\,C}{\sqrt{p^2+\alpha^2\,v^2}}\right.$$

$$\times\Big[-\sin\left(\theta-\tan^{-1}\frac{p}{\alpha v}\right)\sin\left(p\,t-\frac{p\,x}{v}\right)$$

$$\left.+\cos\left(\theta-\tan^{-1}\frac{p}{\alpha\,v}\right)\cos\left(p\,t-\frac{p\,x}{v}\right)\Big]\right\}$$

$$+\varepsilon^{\alpha x}\left\{-Q\sin\left(p\,t+\frac{p\,x}{v}\right)+S\,\frac{p\,v\,C}{\sqrt{p^2+\alpha^2\,v^2}}\right.$$

$$\times \left[\sin\left(\theta - \tan^{-1}\frac{p}{\alpha v}\right) \sin\left(p\,t + \frac{p\,x}{v}\right) - \cos\left(\theta - \tan^{-1}\frac{p}{\alpha v}\right)\right.$$

$$\left.\cos\left(p\,t + \frac{p\,x}{v}\right)\right]\Bigg\}$$

$$-2\,I = \varepsilon^{-\alpha x}\Bigg\{ Q \sin\left(p\,t - \frac{p\,x}{v}\right) - S\sqrt{\frac{p\,C}{\sqrt{R^2 + p^2\,L^2}}}$$

$$\times \cos\left(p\,t + \theta - \frac{p\,x}{v} - \tan^{-1}\frac{p}{\alpha v}\right)\Bigg\}$$

$$+ \varepsilon^{\alpha x}\Bigg\{ Q \sin\left(p\,t + \frac{p\,x}{v}\right) + S\sqrt{\frac{p\,C}{\sqrt{R^2 + p^2\,L^2}}}$$

$$\times \cos\left(p\,t + \theta + \frac{p\,x}{v} - \tan^{-1}\frac{p}{\alpha v}\right)\Bigg\} \qquad (28)$$

since as already shown

$$\frac{p\,v\,C}{\sqrt{p^2 + \alpha^2\,v^2}} = \sqrt{\frac{p\,C}{\sqrt{R^2 + p^2\,L^2}}}$$

This equation shows the current at any point of the line for which equation (26) shows the voltage.

This equation can be put into a form similar to equation (27) by expanding the sine and cosine terms as before.

Thus,

$$-I = \frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2}\left[Q \sin p\,t \cos\frac{p\,x}{v} - S\sqrt{\frac{p\,C}{\sqrt{R^2 + p^2\,L^2}}}\right.$$

$$\left.\times \sin\left(p\,t + \theta - \tan^{-1}\frac{p}{\alpha v}\right) \sin\frac{p\,x}{v}\right]$$

$$+ \frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2}\left[Q \cos p\,t \sin\frac{p\,x}{v} + S\sqrt{\frac{p\,C}{\sqrt{R^2 + p^2\,L^2}}}\right.$$

$$\left.\times \cos\left(p\,t + \theta - \tan^{-1}\frac{p}{\alpha v}\right) \cos\frac{p\,x}{v}\right] \qquad (29)$$

The current is negative in equations (28) and (29), since when the potential rises in the positive direction from the origin, the current is in the opposite direction toward the origin.

The equations (27) and (29) give a second form of the exact equations (26) and (28) for the voltage and current at any point along a line when the current and voltage delivered by the line at the origin are

$Q \sin p\,t$ and $S \sin (p\,t + \theta)$ respectively.

Special Cases

These values should hold for the special cases where C, or L or $R = O$.

First case.

If $C = 0$,

$$\alpha = 0, \quad \frac{p\,x}{v} = x\sqrt{\frac{pC}{2}(p\,L + \sqrt{R^2 + p^2\,L^2})} = 0,$$

$$\frac{p}{\alpha\,v} = \frac{p\,L + \sqrt{R^2 + p^2\,L^2}}{R}$$

and equation (27) becomes.

$$V = Q\sqrt{\sqrt{R^2 + p^2\,L^2}}\left[\sin\left(p\,t + \tan^{-1}\frac{p}{\alpha\,v}\right)\left\{\frac{\sin\frac{px}{v}}{\sqrt{pC}}\right\}\right.$$

$$\left.- \cos\left(p\,t + \tan^{-1}\frac{p}{\alpha\,v}\right)\left(\frac{\frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2}}{\sqrt{p\,C}}\right)\right] + S \sin\,(p\,t + \theta)$$

But the expressions $\dfrac{\sin\frac{p\,x}{v}}{\sqrt{p\,C}}$ and $\dfrac{\frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2}}{\sqrt{p\,C}}$ both become $\frac{0}{0}$ when $C = 0$ and hence are indeterminate. As $\sin\frac{p\,x}{v}$ becomes 0, it equals $\frac{p\,x}{v}$. Thus substituting for, $\sin\frac{p\,x}{v}$, its value in the limit, $\frac{p\,x}{v}$,

$$\frac{\sin\frac{p\,x}{v}}{\sqrt{p\,C}} = \frac{\frac{p\,x}{v}}{\sqrt{p\,C}} = x\sqrt{\frac{p\,L + \sqrt{R^2 + p^2\,L^2}}{2}}$$ which is determinate.

The expression

$$\frac{\frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2}}{\sqrt{p\, C}}$$

may be evaluated by differentiating numerator and denominator with regard to C and again putting $C = 0$.

Then

$$\frac{\frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2}}{\sqrt{p\, C}} = \frac{\frac{R\, x}{2}\sqrt{\frac{2\, p}{p\, L + \sqrt{R^2 + p^2\, L^2}}}\left(\frac{1}{2}\sqrt{\frac{1}{C}}\right)\left(\frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2}\right)}{\frac{1}{2}\sqrt{p}\sqrt{\frac{1}{C}}}$$

$$= Rx\sqrt{\frac{1}{2\,(p\, L + \sqrt{R^2 + p^2\, L^2}}}$$

Therefore

$$V = x\, Q\sqrt{\sqrt{R^2 + p^2\, L^2}}\left[\sqrt{\frac{p\, L + \sqrt{R^2 + p^2\, L^2}}{2}}\sin\left(p\, t + \tan^{-1}\frac{p}{\alpha v}\right)\right.$$

$$\left. - R\sqrt{\frac{1}{2\,(p\, L + \sqrt{R^2 + p^2 L^2})}}\cos\left(p\, t + \tan^{-1}\frac{p}{\alpha\, v}\right)\right]$$

$$+ S\sin\,(p\, t + \theta)$$

combining sine and cosine terms

$$V = x\, Q\sqrt{\sqrt{R^2 + p^2\, L^2}}$$

$$\times\sqrt{\left(\sqrt{\frac{p\, L + \sqrt{R^2 + p^2\, L^2}}{2}}\right)^2 + \left(R\sqrt{\frac{1}{2\,(p\, L + \sqrt{R^2 + p^2\, L^2}}}\right)^2}$$

$$\sin\left(p\, t + \tan^{-1}\frac{p}{\alpha\, v} - \tan^{-1}\frac{R}{p\, L + \sqrt{R^2 + p^2\, L^2}}\right) + S\sin\,(p\, t + \theta)$$

The two radicals forming the coefficient of the *sine* combined

$$= \sqrt{\sqrt{R^2 + p^2\, L^2}}\sqrt{\frac{p\, L + \sqrt{R^2 + p^2\, L^2}}{2} + \frac{R^2}{2(p\, L + \sqrt{R^2 + p^2\, L^2})}}$$

$$= \sqrt{\sqrt{R^2 + p^2\, L^2}}\sqrt{\frac{2\,(p^2\, L^2 + R^2 + p\, L\sqrt{R^2 + p^2\, L^2})}{2\,(p\, L + \sqrt{R^2 + p^2\, L^2})}}$$

$$= \sqrt{\sqrt{R^2 + p^2\, L^2}}\sqrt{\sqrt{R^2 + p^2\, L^2}} = \sqrt{R^2 + p^2\, L^2}$$

The tangent of the difference of the two angles $\tan^{-1} p/\alpha v$ and

$$\tan^{-1} \frac{R}{pL+\sqrt{R^2+p^2L^2}}$$

$$= \tan^{-1} \frac{\dfrac{pL+\sqrt{R^2+p^2L^2}}{R} - \dfrac{R}{pL+\sqrt{R^2+p^2L^2}}}{1+\dfrac{pL+\sqrt{R^2+p^2L^2}}{R}\;\dfrac{R}{pL+\sqrt{R^2+p^2L^2}}}$$

$$= \tan^{-1} \frac{\dfrac{2p^2L^2+2pL\sqrt{R^2+p^2L^2}}{R(pL+\sqrt{R^2+p^2L^2})}}{2} = \tan^{-1}\frac{pL}{R}$$

Therefore,

$$V = xQ\sqrt{R^2+p^2L^2}\sin\left(pt+\tan^{-1}\frac{pL}{R}\right)+S\sin(pt+\theta) \qquad (30)$$

where $x\sqrt{R^2+p^2L^2}$ is the total impedance of the line.

This is seen to be the usual voltage formula for an inductive current.

With $C = 0$, from equation (29),

$$I = -Q\sin pt, \qquad (31)$$

as must be the case since, in the absence of capacity, no loss or gain of current can occur along the conductor.

Second case.

If $L = 0$, $\dfrac{px}{v} = x\sqrt{\dfrac{pRC}{2}}$, $\alpha = \sqrt{\dfrac{pRC}{2}}$, $\dfrac{p}{\alpha v} = 1$

Equation (27) becomes

$$V = \frac{\varepsilon^{x\sqrt{(pRC)\div 2}}+\varepsilon^{-x\sqrt{(pRC)\div 2}}}{2}\left[Q\sqrt{\frac{R}{pC}}\sin\left(pt+\frac{\pi}{4}\right)\sin x\sqrt{\frac{pRC}{2}}\right.$$

$$\left.+S\sin(pt+\theta)\cos x\sqrt{\frac{pRC}{2}}\right]$$

$$+\frac{\varepsilon^{x\sqrt{(pRC)\div 2}}-\varepsilon^{-x\sqrt{(pRC)\div 2}}}{2}\left[-Q\sqrt{\frac{R}{pC}}\sin\left(pt+\frac{\pi}{4}\right)\right.$$

$$\left.\times\cos x\sqrt{\frac{pRC}{2}}+S\cos(pt+\theta)\sin x\sqrt{\frac{pRC}{2}}\right] \qquad (32)$$

Equation (29) becomes

$$-I = \frac{\varepsilon^{x\sqrt{(pRC) \div 2}} + \varepsilon^{-x\sqrt{(pRC) \div 2}}}{2} \left[Q \sin p\, t \cos x \sqrt{\frac{p\,R\,C}{2}} \right.$$

$$\left. - S \sqrt{\frac{p\,C}{R}} \sin \left(p\,t + \theta - \frac{\pi}{4} \right) \sin x \sqrt{\frac{p\,R\,C}{2}} \right]$$

$$+ \frac{\varepsilon^{x\sqrt{(pRC) \div 2}} - \varepsilon^{-x\sqrt{(pRC) \div 2}}}{2} \left[Q \cos p\, t \sin x \sqrt{\frac{p\,R\,C}{2}} \right.$$

$$\left. + S \sqrt{\frac{p\,C}{R}} \cos \left(p\,t + \theta - \frac{\pi}{4} \right) \cos x \sqrt{\frac{p\,R\,C}{2}} \right] \qquad (33)$$

Third case.

If $R = 0$,

$$\frac{p\,x}{v} = x\sqrt{p^2\,L\,C}, \quad \frac{p}{\alpha\,v} = \infty, \quad \alpha = 0$$

Equation (27) becomes

$$V = Q\sqrt{\frac{L}{C}} \cos p\,t \sin x\sqrt{p^2 L C} + S \sin (p\,t + \theta) \cos x \sqrt{p^2\,L\,C} \quad (34)$$

Equation (29) becomes

$$-I = Q \sin p\,t \cos x\sqrt{p^2\,L\,C} + S\sqrt{\frac{C}{L}} \cos (p\,t + \theta) \sin x \sqrt{p^2 L C} \quad (35)$$

Other cases. If $R = 0$ as well as C from equation (30) [or if $C = 0$ as well as R in equation (34)]

$$V = p\,L\,x\,Q \cos p\,t + S \sin (p\,t + \theta)$$

where $x\,p\,l$ = total inductance of the line.

If $L = 0$ as well as C from equation (30)

$$V = x\,R\,Q \sin p\,t + S \sin (p\,t + \theta)$$

where $x\,R$ is the total resistance of the line.

If $L = 0$ as well as R in equation (34),

$$V = S \sin (p\,t + \theta)$$

and from equation (35)

$$-I = Q \sin p\,t + S\sqrt{C} \cos (p\,t + \theta) \left(\frac{\sin x\sqrt{p^2\,L\,C}}{\sqrt{L}}\right)$$

The expression $\frac{\sin x\sqrt{p^2\,L\,C}}{\sqrt{L}} = \frac{0}{0}$. As before in the limit

$\sin x\sqrt{p^2\,L\,C} = x\sqrt{p^2\,L\,C}$

therefore

$$-I = +Q \sin p\,t + p\,C\,x\,S \cos (p\,t + \theta)$$

This is the well-known formula for a shunt condenser, where $C\,x$ = total capacity of the line.

Thus in the various special cases where the quantities, voltage and current cease to be wave functions, the general equations reduce to the well-known forms, but in each case the change from the wave function to the simple function is through the evaluation of an indeterminate form.

As long as the line contains capacity, and either resistance or inductance, these functions are in wave form, as they include trigonometrical functions of angles including the distance x from the origin. In other cases the formulas represent the algebraic form of function.

Physical Meaning of Equations

Equations (26) and (28) give perhaps the best idea of the physical nature of the phenomenon of voltage and current variation. As a matter of fact there exists only a single resultant succession of waves passing in the line. It is helpful sometimes, however, to consider that the resultant wave is made up of a number of component waves.

Suppose, for example, that a perfectly uncharged line be connected to a source of sine electromotive force. A wave will pass into the line and will lose a little of its energy or amplitude as it passes along, and at the end will be reflected and pass backward along the line in the same manner, getting gradually less all the way. This wave reaching the generator will be absorbed, while the generator continues to send out similar waves.

Considering now the receiving end where the potential waves are reflected and the neighborhood of this point, there are at all times two component potential waves at each point in the line, one going toward the reflecting point and the other coming from it. These waves remain the same as they travel, except for decrease in amplitude due to the resistance losses, and for their phase which is, of course, further advanced in the time angle and less advanced by the space angle in the leading wave. The reflected potential wave is represented by the term

$$S \sin \left(p\,t - \frac{p\,x}{v} + \theta\right)$$

and the direct wave by the expression

$$S \sin \left(p\,t + \frac{p\,x}{v} + \theta\right)$$

the angle $\frac{p\,x}{v}$ being the part of a cycle $(2\,\pi)$ occupied by the wave in passing over the distance x to or from the point of reflection. But the current produces a drop in the line, proportional to the current and square root of the impedance, and inversely proportional to the square root of the capacity susceptance, as is shown by the equation (26). The current wave passes along together with the potential wave toward the reflecting point, being represented by the expression

$$Q \sin \left(p\,t + \frac{p\,x}{v}\right)$$

Similarly with the reflected current wave

$$Q \sin \left(p\,t - \frac{p\,x}{v}\right)$$

which passes back through the line from the reflecting point. But the potential due to the current wave is not in phase with the current but occurs earlier than the current by the angle

$$\tan^{-1} \frac{p}{\alpha\,v} = \tan^{-1} \frac{p\,L + \sqrt{R^2 + p^2\,L^2}}{R}$$

Again the direction of the drop in line, produced by the current wave after reflection, is opposite with regard to the voltage from that due to the direct current wave. This appears in the equation by the positive sign of the quantity

$$+Q\sqrt{\frac{\sqrt{R^2+p^2 L^2}}{p\,C}}\cos\left(p\,t-\frac{p\,x}{v}+\tan^{-1}\frac{p}{\alpha\,v}\right),$$ representing

the reflected wave.

Thus the total resultant voltage is the combination of these four quantities, the direct and reflected potential waves, which decrease as they travel, as indicated by the two factors: first,

$$\epsilon^{\alpha x}.$$

indicating that the direct wave is greater than the value at the reflecting point, since it will decrease somewhat by travelling along the line to the reflecting point; and, second, by the factor

$$\epsilon^{-\alpha x},$$

indicating that the reflected wave is less than the value at the reflecting point, since it has passed along the line some distance from this point, and the direct and reflected current wave drops.

Since Q and S, however, should be the total delivered current and voltage but momentarily taken for the maximum value of each wave, the sum of the four waves will be twice the actual voltage, as is shown by the equation (26).

Similarly with the current formula (28) where the resultant current is made up of four elements, the direct and reflected current waves which diminish as they travel at the rate $\epsilon^{\alpha x}$, and a direct and reflected curve due to the effect of the voltage wave and consisting of charging current. The direct-current wave, due to the direct-voltage wave, is represented by the expression

$$S\sqrt{\frac{p\,C}{\sqrt{R^2+p^2 L^2}}}\sin\left(p\,t+\frac{p\,x}{v}+\theta-\tan^{-1}\frac{p}{\alpha\,v}\right)$$

The phase angle here shown is due partly to the ordinary time variation ($p\,t$), partly to the distance it has to pass to reach the reflecting point $\frac{p\,x}{v}$, partly to the lead of the delivered voltage

over the current (θ) and partly due to the lag between the charging current wave and the voltage wave causing it, $\tan^{-1}\frac{p}{\alpha v}$

Similarly with the phase of the corresponding reflected wave, in which, however, the wave has passed the reflecting point and is thus earlier than the waves at that point by the angle $\frac{p\,x}{v}$.*

Again it will be noticed that while the component current waves are sine quantities, the charging-current waves produced by the voltage waves appear as cosine quantities. This is because the charging current is primarily proportional to the rate of change of voltage and not directly to the voltage. The cosine, of course, gives the rate of change of the sine.

Numerical Calculations by Wave Formula

Numerical calculations. For purposes of numerical calculation these equations can be considerably simplified. Starting with equation (27) the voltage is seen to be the summation of four terms, sines and cosines, having varying phase relations. The simplest way to determine the maximum value of the resultant wave, numerically in a given case, is to substitute in the equation separately two values of $p\,t$, 90 degrees apart, and the square root of the sum of the squares of the results of these substitutions will be the resultant maximum. In this case $p\,t$ will be put $= 0$ and then $= \pi/2$.

Then equation (27) gives for $p\,t = 0$

$$V_0 = \left[S \sin\theta \cos\frac{p\,x}{v} + Q\sqrt{\frac{\sqrt{R^2+p^2L^2}}{p\,C}}\sin\tan^{-1}\frac{p}{\alpha v}\right.$$

$$\left.\times \sin\frac{p\,x}{v}\right]\frac{\varepsilon^{\alpha x}+\varepsilon^{-\alpha x}}{2} + \left[S\cos\theta\sin\frac{p\,x}{v} - Q\sqrt{\frac{\sqrt{R^2+p^2L^2}}{p\,C}}\right.$$

$$\left.\times\cos\tan^{-1}\frac{p}{\alpha v}\cos\frac{p\,x}{v}\right]\frac{\varepsilon^{\alpha x}-\varepsilon^{-\alpha x}}{2} \qquad (36)$$

* In this connection, attention is called to the writer's paper before the A. I. E. E. of February, 1902, TRANSACTIONS, Vol. xiv, p. 213, where a somewhat similar, non-mathematical, description of electric waves is given. The equations here derived conform exactly to the principles there laid down.

For $p\,t = \frac{\pi}{2}$

$$V_{\frac{\pi}{2}} = \left[S \cos \theta \cos \frac{p\,x}{v} + Q \sqrt{\frac{\sqrt{R^2 + p^2 L^2}}{p\,C}} \cos \tan^{-1} \frac{p}{\alpha\,v} \right.$$

$$\left. \times \sin \frac{p\,x}{v} \right] \frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2} + \left[- S \sin \theta \sin \frac{p\,x}{v} \right.$$

$$\left. + Q \sqrt{\frac{\sqrt{R^2 + p^2 L^2}}{p\,C}} \sin \tan^{-1} \frac{p}{\alpha\,v} \cos \frac{p\,x}{v} \right] \frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2} \tag{37}$$

Similarly, with equation (29) putting $p\,t = 0$

$$- I_0 = \left[0 + S \sqrt{\frac{p\,C}{\sqrt{R^2 + p^2 L^2}}} \left(- \sin \left(\theta - \tan^{-1} \frac{p}{\alpha\,v} \right) \right. \right.$$

$$\left. \left. \times \sin \frac{p\,x}{v} \right) \right] \frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2} + \left[Q \sin \frac{p\,x}{v} + S \sqrt{\frac{p\,C}{\sqrt{R^2 + p^2 L^2}}} \right.$$

$$\left. \times \left(\cos \left(\theta - \tan^{-1} \frac{p}{\alpha\,v} \right) \cos \frac{p\,x}{v} \right) \right] \frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2} \tag{38}$$

putting $p\,t = \frac{\pi}{2}$

$$- I_{\frac{\pi}{2}} = \left[Q \sin \frac{p\,x}{v} + S \sqrt{\frac{p\,C}{\sqrt{R^2 + p^2 L^2}}} \left(- \cos \left(\theta - \tan^{-1} \frac{p}{\alpha\,v} \right) \right. \right.$$

$$\left. \left. \times \sin \frac{p\,x}{v} \right) \right] \frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2} + \left[0 + S \sqrt{\frac{p\,C}{\sqrt{R^2 + p^2 L^2}}} \right.$$

$$\left. \times \left(- \sin \left(\theta - \tan \frac{p}{\alpha\,v} \right) \cos \frac{p\,x}{v} \right) \right] \frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2} \tag{39}$$

Rearranging terms and putting $\frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2} = k_1$ and $\frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2} = k_2$, and dropping the negative signs before I_0 and $I_{\frac{\pi}{2}}$ as having no significance in the numerical computations, we have the following :

FORMULAS FOR NUMERICAL CALCULATIONS.

$$V_0 = S\left(\left(k_1 \cos\frac{px}{v}\right)\sin\theta + \left(k_2 \sin\frac{px}{v}\right)\cos\theta\right) + Q\sqrt{\frac{\sqrt{R^2+p^2L^2}}{pC}}\left(\left(k_1 \sin\frac{px}{v}\right)\sin\,\tan^{-1}\frac{p}{\alpha v} - \left(k_2 \cos\frac{px}{v}\right)\cos\tan^{-1}\frac{p}{\alpha v}\right) \quad (40)$$

$$V_{\frac{\pi}{2}} = S\left(\left(\text{"} \quad \text{"}\right)\cos\theta - \left(\text{"} \quad \text{"}\right)\sin\theta\right) + Q\sqrt{\frac{\sqrt{R^2+p^2L^2}}{pC}}\left(\left(\text{"} \quad \text{"}\right)\cos\,\tan^{-1}\frac{p}{\alpha v} + \left(\text{"} \quad \text{"}\right)\sin\tan^{-1}\frac{p}{\alpha v}\right) \quad (41)$$

$$I_0 = \qquad + \left(\text{"} \quad \text{"}\right)Q + S\sqrt{\frac{pC}{\sqrt{R^2+p^2L^2}}}\left(\left(\text{"} \quad \text{"}\right)\sin\left(\tan^{-1}\frac{p}{\alpha v}-\theta\right) + \left(\text{"} \quad \text{"}\right)\cos\left(\tan^{-1}\frac{p}{\alpha v}-\theta\right)\right) \quad (42)$$

$$I_{\frac{\pi}{2}} = \left(\text{"} \quad \text{"}\right)Q \qquad - S\sqrt{\frac{pC}{\sqrt{R^2+p^2L^2}}}\left(\left(\text{"} \quad \text{"}\right)\cos\left(\tan^{-1}\frac{p}{\alpha v}-\theta\right) - \left(\text{"} \quad \text{"}\right)\sin\left(\tan^{-1}\frac{p}{\alpha v}-\theta\right)\right) \quad (43)$$

Note, as determined below :

$$V = \sqrt{V_0^2 + V_{\frac{\pi}{2}}^2} = V_0 \div \sin\,\tan^{-1}\frac{V_0}{V_{\frac{\pi}{2}}} = V_{\frac{\pi}{2}} \div \cos\,\tan^{-1}\frac{V_0}{V_{\frac{\pi}{2}}};\; I = \sqrt{I_0^2 + I_{\frac{\pi}{2}}^2} = I_0 \div \sin\,\tan^{-1}\frac{I_0}{I_{\frac{\pi}{2}}} = I_{\frac{\pi}{2}} \div \cos\,\tan^{-1}\frac{I_0}{I_{\frac{\pi}{2}}}$$

Generator power-factor $= \cos\left(\tan^{-1}\frac{V_0}{V_{\frac{\pi}{2}}} - \tan^{-1}\frac{I_0}{I_{\frac{\pi}{2}}}\right)$, line energy loss $= V\,I$ (gen. P. F.) $- S\,Q\cos\theta$

$k_1 = 1 + \frac{\alpha^2 x^2}{2}$, $k_2 = \alpha x\left(1 + \frac{\alpha^2 x^2}{6}\right)$, load voltage $= S$, load current $= Q$, load power-factor $= \cos\theta$

Then

$$V = \sqrt{V_0^2 + V_{\frac{\pi}{2}}^2}, \quad I = \sqrt{I_0^2 + I_{\frac{\pi}{2}}^2}$$

These equations serve to give the calculation of generator current and voltage in actual cases. It is more convenient instead of using the expressions $\sqrt{V_0^2 + V_{\frac{\pi}{2}}^2}$ and $\sqrt{I_0^2 + I_{\frac{\pi}{2}}^2}$ to consider the trigonometrical relations of the quantities V_0, $V_{\frac{\pi}{2}}$, I_0, and $I_{\frac{\pi}{2}}$, graphically in their triangles, in this case

$$V = V_0 \div \sin \tan^{-1} \frac{V_0}{V_{\frac{\pi}{2}}} \text{ and } I = I_0 \div \sin \tan^{-1} \frac{I_0}{I_{\frac{\pi}{2}}} \qquad (44) \quad (45)$$

The phase angle by which the current lags behind the voltage at the generator is the difference in the phase angles of the two resultant quantities just determined, namely, the angles, $\tan^{-1} \frac{V_0}{V_{\frac{\pi}{2}}}$ and $\tan^{-1} \frac{I_0}{I_{\frac{\pi}{2}}}$. The power-factor of the generator power is the cosine of this angle

$$\cos \left(\tan^{-1} \frac{V_0}{V_{\frac{\pi}{2}}} - \tan^{-1} \frac{I_0}{I_{\frac{\pi}{2}}} \right)$$

The generator conditions are thus completely determined.

In making calculations it should be noted that

$$\frac{\varepsilon^{\alpha x} + \varepsilon^{-\alpha x}}{2} = k_1 = 1 + \frac{\alpha^2 x^2}{1.2.3} + \frac{\alpha^4 x^4}{1.2.3.4} + \dots$$

and that

$$\frac{\varepsilon^{\alpha x} - \varepsilon^{-\alpha x}}{2} = k_2 = \alpha x + \frac{\alpha^3 x^3}{1.2.3} + \frac{\alpha^5 x^5}{1.2.3.4.5} + \dots$$

Two terms of these series are always accurate enough for numerical calculations.

$\sqrt{R^2 + p^2 L^2}$ is of course the impedance per unit length of line. Any convenient unit of length may be taken provided it is used throughout.

In actual calculations the following procedure is recommended:

Having given:

Length line, 700 miles.
Size conductors, 3 No. 0000.
Spacing wires, 12 foot triangle.
Power delivered, 40,000 three-phase, 20,000 in single phase for equivalent drop.
Voltage delivered 150,000.
Power-factor of delivered power, .918.
Current delivered, 145, single phase.

Take from tables:

Resistance of line per loop unit length $= R = 0.17$ ohms.
Inductive ohms $= p\,L$ (L in henrys) $= 2\,\pi\,n\,L = 0.619$ ohms.
Impedance ohms $= \sqrt{R^2 + p^2\,L^2} =$ .... 0.643 ohms.
Capacity susceptance $= p\,C = 2\,\pi\,n\,C$
= charging current per volt (capacity must be in farads,
= capacity in microfarads $\div 10^6$) = ...000001193

By calculation,

$\cos\theta = 0.918$, $\theta = 23°, 20'$, $\sin\theta = 0.400$.

$$\frac{p\,x}{v} = x\sqrt{\frac{p\,C\,(p\,L + \sqrt{R^2 + p^2 L^2})}{2}}$$

$$= x\sqrt{\frac{\text{cap. suscept. (induct. + imped.)}}{2}}$$

$$700\sqrt{\frac{0.000001193}{2}\,(0.619 + 0.643)} = 0.607$$

To get 0.607 into degrees multiply by $\frac{360}{2\,\pi}$

$$0.607 \times \frac{360}{2\,\pi} = 34°, 48'$$

$$\sin\frac{p\,x}{v} = 0.571,\ \cos\frac{p\,x}{v} = 0.821$$

$$\frac{p}{a\,v} = \frac{p\,L + \sqrt{R^2 + p^2\,L^2}}{R} = \frac{0.619 + 0.643}{0.17} = 7.43$$

$$\tan^{-1} 7.43 = 82^\circ\ 20', \ \sin \tan^{-1} \frac{p}{\alpha v} = 0.991, \cos \tan^{-1} \frac{p}{\alpha v} = 0.133$$

$$\left(\tan^{-1} \frac{p}{\alpha v} - \theta\right) = 59^\circ.$$

$$\operatorname{Sin}\left(\tan^{-1} \frac{p}{\alpha v} - \theta\right) = 0.857, \cos\left(\tan^{-1} \frac{p}{\alpha v} - \theta\right) = 0.515$$

$$\alpha x = \frac{R\,x}{2} \sqrt{\frac{2\,p\,C}{p\,L + \sqrt{R^2 + p^2\,L^2}}}$$

$$= \frac{700 \times 0.17}{2} \sqrt{\frac{2 \times .000001193}{0.619 + 0.643}} = 0.0819$$

$$\sqrt{\frac{p\,C}{\sqrt{R^2 + p^2\,L^2}}} = \sqrt{\frac{.000001193}{0.643}} = 0.001364$$

$$k_1 = 1 + \frac{\alpha^2 x^2}{2} = 1.0033$$

$$k_2 = 0.082\left(1 + \frac{\alpha^2\,x^2}{6}\right) = 0.0819\ (1 + .0011) = 0.0820$$

$$k_1 \sin \frac{p\,x}{v} = 0.574 \qquad k_1 \cos \frac{p\,x}{v} = 0.825$$

$$k_2 \sin \frac{p\,x}{v} = 0.0468 \qquad k_2 \cos \frac{p\,x}{v} = 0.0673$$

These quantities can now be substituted in the equations 40-43, giving

$$V_0 = 150000\,[(0.825)\ (0.400) + (0.0468)\ (0.918)]$$

$$+ \frac{145}{0.001364}\,[(0.5735)\ (0.991) - (0.0673)\ (0.133)]$$

$$V_{\frac{\pi}{2}} = 150000\,[(0.825)\ (0.918) - (0.0468)\ (0.400)]$$

$$+ \frac{145}{0.001364}\,[(0.5735)\ (0.133) + (0.0673)\ (0.991)]$$

$$I_0 = 145\ (0.0468)$$

$$+150000 \times .001364\,[(0.5735)\ (0.857) + (0.0673)\ (0.515)]$$

$$I_{\frac{\pi}{2}} = 145\ (0.825)$$

$$-150000 \times 0.001364\,[(0.5735)\ (0.515) - (0.0673)\ (0.857)]$$

Then

$$V_0 = 150000\ (0.370) + 145\ (409) = 114{,}900$$

$$V_{\frac{\pi}{2}} = 150000\ (0.738) + 145\ (104.7) = 125{,}900$$

$$I_0 = 145\ (0.0468) + 150000\ (0.000719) = 114.6$$

$$I_{\frac{\pi}{2}} = 145\ (0.825) - 150000\ (0.000325) = 71.2$$

$$V = 125{,}900 \div \cos \tan^{-\prime} \frac{114{,}900}{125{,}900}$$

$$= 125{,}900 \div \cos\ (42^\circ,\ 25') = 170{,}600$$

$$I = 114.6 \div \sin \tan^{-\prime} \frac{114.6}{71.2}$$

$$= 114.6 \div \sin\ (58^\circ,\ 10') = 135.$$

$$\text{Generator power-factor} = \cos\ (42^\circ,\ 25' - 58^\circ,\ 10')$$

$$= \cos\ (-15^\circ,\ 45') = 0.963.$$

Since the lag angle is negative the current is leading.
The true power at the generator is $V\,I \times 0.963 = 22{,}320$ kw.
The line energy loss is 2320 kw. = 10.7%

$$\text{The voltage drop is } \frac{170{,}600 - 150{,}000}{170{,}600} = 12.1\%.$$

Thus the conditions of the generator and line are all determined.

The sign of the various quantities making up V_0, $V_{\frac{\pi}{2}}$, I_0, $I_{\frac{\pi}{2}}$ must be carefully preserved to determine the generator power-factor properly; it should be noted that $\tan^{-\prime}(-x/y) = 180^\circ - \tan^{-\prime} x/y$ and that $\cos(-x) = \cos x$, while $\sin(-x) = -\sin x$.

Once the calculation has been made on one particular line for one load, the recalculation for other loads and other voltages is very easy. If the power-factor of the load is the same, only the quantities S and Q need be changed in equations (40) (41) (42) and (43).

If the power-factor of the delivered load is changed, the angles containing θ are of course correspondingly changed, but this does not greatly lengthen the recalculations.

Split-capacity formula

For purposes of comparison with the wave formula as here derived, the writer ventures to introduce an approximate

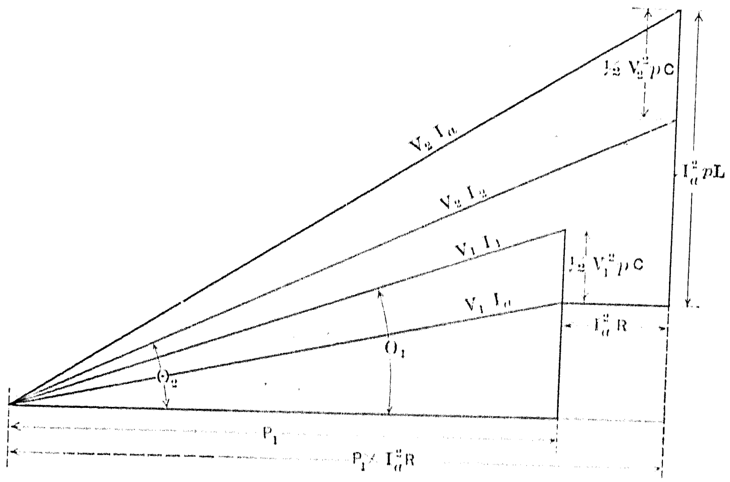


Fig. 2—Relation of power quantities in transmission line.

formula for calculation of the line quantities, based on the assumption that one-half of the line capacity is concentrated at each end of the line.

Figure 2 gives the graphical representation of the various power quantities in connection with such a line. Here P_1, V_1, $\cos\theta_1$ are the power, voltage, and power-factor of the delivered load. Let I_a be the current in the receiving end in the line itself, and

Let P_2, V_2, I_2, and $\cos\theta_2$ be the power, voltage, current and power-factor at the generator. The relations of the *power* quantities are shown in Fig. 2, and the following equations can be written down by inspection:—

$$V_1 I_a = P_1 \div \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2}}{P_1}$$

$$V_2 I_a = (P_1 + I_a^2 R) \div \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2} + I_a^2\, pL}{P_1 + I_a^2 R}$$

$$V_2 I_2 = (P_1 + I_a^2 R) \div \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2} + I_a^2\, pL - \frac{p\, C\, V_2^2}{2}}{P_1 + I_a^2 R}$$

These can be more conveniently written

$$I_a = \frac{P_1}{V_1} \div \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2}}{P_1} \tag{46}$$

$$V_2 = \frac{P_1 + I_a^2 R}{I_a} \div \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2} + I_a^2\, pL}{P^1 + I_a^2 R} \tag{47}$$

$$I_2 = \frac{P_1 + I_a^2 R}{V_2} \div \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2} + I_a^2\, pL - \frac{p\, C\, V_2^2}{2}}{P_1 + I_a^2 R}$$

$$= \frac{P_1 + I_a^2 R}{V_2} \div \cos \theta_2 \tag{48}$$

Generator power factor =

$$\cos \theta_2 = \cos \tan^{-1} \frac{P_1 \tan \theta_1 - \frac{p\, C\, V_1^2}{2} + I_a^2\, pL - \frac{p\, C\, V_2^2}{2}}{P_1 + I_a^2 R}$$

These equations are well suited to numerical calculations, especially when several sections of line carrying different loads and having different constants are fed in series.

The difference between the mathematical forms of the wave formula and the split-capacity formula is marked, and it becomes important to know how much difference there may be in the numerical results shown by the two formulas. For purposes of comparison, a considerable number of cases, including some of those calculated by the wave formula in the companion paper, have been calculated by both formulas and the results tabulated in Table I.

Here it is seen that for ordinary commercial cases up to perhaps 150 miles (as cases A, B, C, D, R, and S in Table I) there is only a few tenths of one per cent difference in the results of the two formulas, but that with very long lines the error may reach several per cent in voltage drop, as in (I, K, L, M, P, Q, T, U, V, and W of table I).

The error may occur in voltage, energy loss, current or power-factor and may be either positive or negative.

Cases R and S, Table I are of a lead-covered cable of assumed constants; here the discrepancy between the formulas is very small, even though this is an extreme case for a cable transmission. Strictly speaking in this case the leakage of current through the insulation should be considered. The effect of this leakage would here be negligible, however.

It is interesting to note that the line may act to a limited extent as a raising transformer in those cases where the capacity is relatively important. That is, energy given to the line at one voltage may be delivered to the load at a lower voltage and larger current. For example, in cases C, P, and T, the total generator current is less than the power component of the load current. In view, however, of the higher generator voltage the *energy* represented by the generator current is greater than that represented by the load current, the excess being the energy line loss.

In applying the split-capacity formula to the case where the load is 0 and only charging current flows, the expression for I_a becomes indeterminate. In this case, however, I_a = the charging current of one-half the line capacity and the formula for I_a need not be used. The rest of the formula does not become indeterminate.

In applying the split-capacity formula to cases where leading current is taken by a synchronous machine at the load end of the line, it must be noted that while this current is leading in the machine it is lagging in the line,—measuring toward the load—and thus has the same effect as lagging load current. With such machines taking leading current, their apparent power is introduced in the formula as part or the whole of the quantity, $P_1 \tan \theta_1$, with the positive sign.

These formulas are all adapted to single-phase calculations. Three-phase lines may be calculated by the well-known rule, that the energy line loss, voltage drop, and power-factor expressed *in percentage* are the same, including effect of capacity

current, for a three-phase system as for a single-phase system transmitting one-half the power over any pair of the three-phase wires, which are assumed to be arranged in the usual equilateral triangle grouping. The correctness of this rule is easily seen. With two parallel wires carrying currents, the ohmic drop is the resultant of the drops in the two wires; the inductive e.m.f. depends upon the total number of lines of force between the wires, which will be due to the resultant of the magnetism due to the currents in the two wires individually and the capacity depends upon the potential between wires necessary to produce a given charge on the wires. This potential may be measured by carrying a unit positive charge of electricity from one wire to the other and determining the work done in virtue of the two charges on the two wires. It should be noted that the third wire of a three-phase system, in virtue of its symmetrical position, has no effect either on the lines of force passing between the other two wires, nor upon the total work done in moving a charge between them.

In each of the above cases when the wires carry single-phase currents the effects of the two wires are 180 degrees apart in phase, but are additive in the summation giving the total ohmic and inductive electromotive force and the work done by a positive charge is equal to double that from one wire only.

With three-phase, on the other hand, the two wires have their effects 120 degrees apart instead of 180 degrees so that the resultant effects are not twice but $\sqrt{3}$ times the effect of one wire. So the ratio $\frac{2}{\sqrt{3}}$ exists between these quantities when the wires carry the same current three-phase and single-phase. To have the same ohmic and inductive e.m.f., which depend upon the current, whether the wires carry single-phase or three-phase current, the single-phase current must be reduced in the ratio $\frac{2}{\sqrt{3}}$, reducing the single-phase power by this amount. But in virtue of the third wire of the three-phase system, it carries $\sqrt{3}$ times as much power as the single-phase having the same current. Therefore, the ratio $\frac{2}{\sqrt{3}} \times \sqrt{3} = 2$ gives the ratio of the power in a single-phase circuit and a three-phase circuit, when in the single-phase circuit and in any two wires of the three-phase cir-

cuit, there exists the same resultant ohmic drop total and the same resultant inductive e.m.f. In this case the actual line current in the case of the three-phase will be $\frac{2}{\sqrt{3}}$ times that of the single-phase.

Since the work done by the positive charge in passing between the two wires of the three-phase is only $\frac{\sqrt{3}}{2}$ times that in the single-phase case, the capacity and charging current must be $\frac{2}{\sqrt{3}}$ greater in the three-phase case. Thus the ratio of charging currents in the two circuits is the same as that of the load cur-

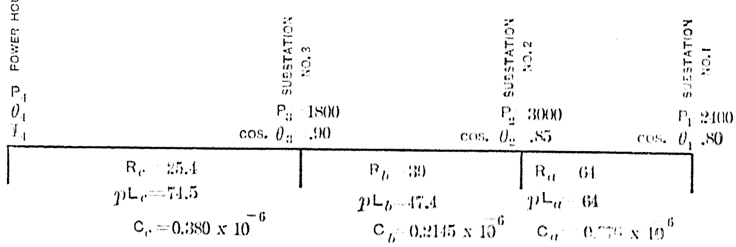


FIG. 3—Distribution of loads and line constants in assumed transmission line

rents, and the power-factor relations, as well as the voltage relations, must remain the same in the two circuits. Thus the true energy loss in per cent must be the same in the two cases, as already stated.

This conclusion can be reached by algebraic and trigonometrical methods, but the above demonstration is much simpler and its logic is believed to be sound.

The split-capacity formula is especially convenient where on account of intermediate substations several sections of the line have different constants and different loads; in this case if only the generator quantities are required the calculation can be made as follows: Let there be three stations along a line each receiving power, and let the different sections of the line have different constants all as indicated in Fig. 3. Calculations are made for single-phase, assuming half the power transmitted.

	True power quantities	Wattless quantities
Delivered voltage 60,000, 60 cycles		
Power at substation 1	1,200,000	
Wattless component $P_1 \tan \theta_1$		+ 905,000
Capacity energy at 1, $\frac{p\,C_a\,V_1^2}{2}$		− 187,000
	1,200,000	718,000
$I_a = \frac{P_1}{V_1} \div \cos\tan^{-1}\frac{718{,}000}{1{,}200{,}000} = 23.3$		
$I_a^2\,R_a =$	35,000	
$I_a^2\,pL_a =$		35,000
	1,235,000	753,000
$V_2 = \frac{1{,}235{,}000}{23.3} \div \cos\tan^{-1}\frac{753{,}000}{1{,}235{,}000} = 62{,}000$		
Capacity energy at 2, $V_2^2\left(\frac{p\,C_a}{2} + \frac{p\,C_b}{2}\right) =$		− 356,000
Power at substation 2	1,500,000	
Wattless at substation 2, $(P_2 \tan \theta_2)$		+ 930,000
	2,735,000	1,327,000
$I_b = \frac{2{,}735{,}000}{62{,}000} \div \cos\tan^{-1}\frac{1{,}327{,}000}{2{,}735{,}000} = 49$		
$I_b^2\,R_b =$	72,000	
$I_b^2\,pL_b =$		114,000
	2,807,000	1,441,000
$V_3 = \frac{2{,}807{,}000}{49} \div \cos\tan^{-1}\frac{1{,}441{,}000}{2{,}807{,}000} = 64{,}400$		
Capacity energy at substation 3 $= V_3^2\left(\frac{p\,C_b}{2} + \frac{p\,C_a}{2}\right) =$		− 451,000
Power at substation $3 = P_3$	900,000	
Wattless at substation $3 = P_3 \tan \theta_3$....		+ 435,000
	3,707,000	1,425,000

$$I_c = \frac{3{,}707{,}000}{64{,}400} \div \cos \tan^{-1} \frac{1{,}425}{3{,}707} = 61.7$$

$I_c^2 R_c =$	 96,500	
$I_c^2 pL_c =$	....	+283,000
	3,804,000	1,708,000

V_4 (at generator)

$$= \frac{3{,}804{,}000}{61.7} \div \cos \tan^{-1} \frac{1{,}708{,}000}{3{,}804{,}000}$$

$$= 67{,}600$$

Capacity energy at generator

$\dfrac{p\, C_c\, V_4^2}{2}$	....	$-$327,000
	3,804,000	1,381,000

$$I_4 = \frac{3{,}804{,}000}{67{,}600} \div \cos \tan^{-1} \frac{1{,}381{,}000}{3{,}804{,}000}$$

$$= \frac{3{,}804{,}000}{67{,}600} \div \cos 20° \quad = \frac{3{,}804{,}000}{67{,}600} \div 0.94 = 59.8$$

Cos θ_4 = power factor = 0.94

For 3-phase at generator.

Power = 7610 kw., $K\,V\,A$ = 8100, power factor = 0.94

Drop = 7,600 volts. Energy line loss = 410 kw.

Induction Formulas for Split Conductors

The use of split conductors to vary the ratio of capacity and inductance in a transmission line is discussed in the companion paper. An approximate formula for calculating the inductance of a line with split conductors will be derived. Assume a three-phase line of the usual form, but with each conductor split into three parts as shown in Fig. 4; the total inductance between one group of conductors and the neutral point will be determined and then compared with the usual formula for such total inductance, with a single conductor, placed at the center of gravity of the three, at the point O_1 Fig. 4.

The total inductance sought will be the resultant of the inductances caused by the various currents of the system separately. It will first be proved that the total amount of induction between two points in space, perpendicular to the line joining these points, due to a long straight wire carrying a current I, is equal to $2 \log_e \frac{M}{N}$, where M and N are, respectively, the distances to

the more remote and the nearer points. In Fig. 5 let I be the point where the current I passes through the plane of the paper and let S and U be the two points between which induction is to be measured. Let T be any point in the line between the points U and S at a distance x from the base O of a normal to the line $U\,S$ from I. The magnetic field at T is then $\dfrac{2\,I}{\sqrt{q^2+x^2}}$ when q is the length of the normal, and $\sqrt{q^2+x^2}$ is the distance

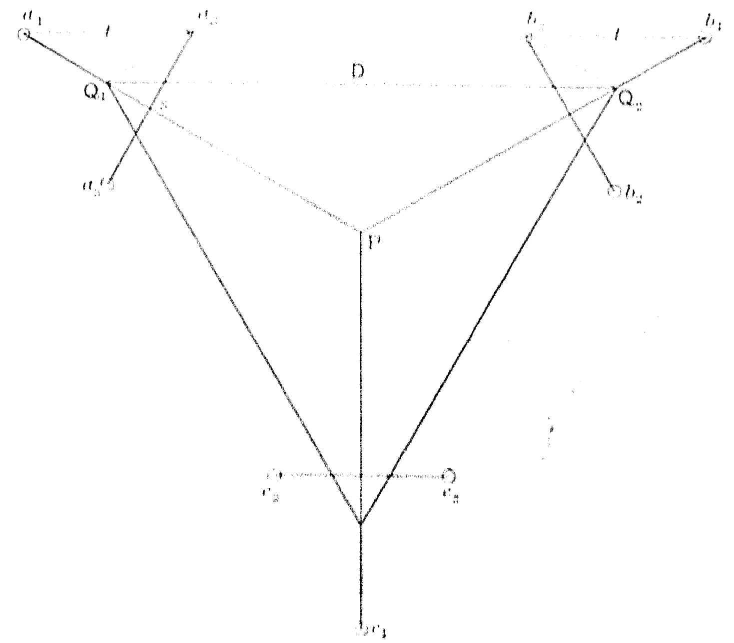


FIG. 4—Arrangement of three-part split conductors for three-phase line

from I to T. The factor 2 represents the effect of the fact that the wire is very long. The component of this magnetism perpendicular to the line $U\,S$ is

$$\frac{2\,I}{\sqrt{q^2+x^2}} \cdot \frac{x}{\sqrt{q^2+x^2}} = \frac{2\,I\,x}{q^2+x^2}.$$

The total flux of magnetism resolved perpendicular to $U\,S$ between points U and S will be

$$I\int\frac{2\,x}{q^2+x^2} = I\log\left(q^2+x^2\right)\Big|_{O\,S}^{O\,U} = 2\,I\log\frac{M}{N}.$$

From considerations of symmetry it is evident that at times when current in the a group is a maximum, at which times the current in the b and c groups are equal, the resultant magnetism at all points along the line a_1, P, Fig. 4, due to a_1, a_2, a_3 will be perpendicular to this line. It is further evident that the maximum magnetic field between the points s and the point P will be a maximum, when the current in the a group of conductors is a maximum. The value of I in the a group will then be $I \sin 90°$. At this time the current in the b group will be $I \sin (90° + 120°) = -\sin 30° = -\frac{1}{2}$ and the current in the c group will be $I \sin (90° + 240°) = \sin - 30° = -\frac{1}{2}$. Furthermore the current is rising in the b group at this instant and falling in the c group. Thus the resultant effect of the b and c group on the magnetism between S and P is not changing at

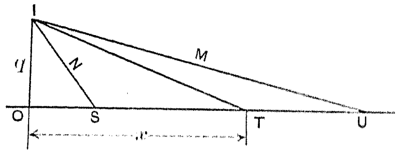


FIG. 5—Diagram of summation of magnetic flux

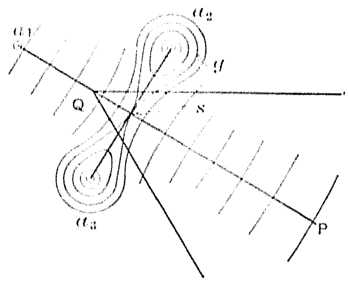


FIG. 6.—Distribution of magnetism due to the currents in conductors a_2 and a_3 of Fig. 4

this time, showing that the resultant of the three groups is at a maximum as already stated.

Again since the currents in the b and c groups are negative when the current in the a group is a maximum, their effect on the magnetism cutting the line $S\,P$ is added to that of the a group.

The form of the lines of force due to the currents in a_2 and a_3 only is shown in Fig. 6. The lines of force included between a_2 and P may be considered in two parts, that outside the line of force marked g, which can easily be measured along the line $s\,P$, and the part inside this line of force which can easily be measured along the line $s\,a_2$.

The total magnetism linked between conductor a_2 and P is the same as between either a_3 or a_1 and P. This is evident for a_3

from conditions of symmetry, but is strictly true of a_1 only when its ohmic resistance is 0. Any greater number of lines between a_1 and P than between a_2 and P will cause in the outer wire a greater counter electromotive force, which will reduce the current in a_1, since a_1 and a_2 are in parallel at both ends, and will increase the current in a_2 and a_3 until the resultant magnetism between a_1 and a_2 is O, the effects of current in all groups of conductors being considered. Any resistance in the conductors a_1, a_2, and a_3 will, however, tend to resist this division of current. The resistance effect will be small in large conductors such as would be considered in actual transmission problems. The conclusion is that the current in a_2 or a_3 is greater than one third of the total current. Let k be this corrective increment, that is, the current in $a_2 = I/3\ (1+k)$, I being the total current for one group of conductors.

The various components of the maximum resultant magnetism due to the currents in the various conductors can now be determined.

The portion of the magnetism due to the current in a_2 cutting the line $s\ P$ perpendicular thereto is

$$= \frac{2\,I\,(1+k)}{3} \log \frac{M}{N}, \text{ as above.}$$

$$= \frac{2\,I\,(1+k)}{3} \log \frac{\sqrt{\left(\frac{D}{\sqrt{3}} - \frac{t}{2\sqrt{3}}\right)^2 + \left(\frac{t}{2}\right)^2}}{\frac{t}{2}}$$

$$= \frac{2\,I\,(1+k)}{3} \log \frac{\sqrt{\frac{D^2 - Dt + t^2}{3}}}{\frac{t}{2}}$$

$$= \frac{I\,(1+k)}{3} \log \left(\frac{4}{3}\,\frac{D^2 - Dt}{t^2}\right) \text{ approx.}$$

But the current in a_4 causes this same amount of magnetism on the line $s\ P$, and that due to both a_2 and a_4 between s and P is

$$\frac{2\,I\,(1+k)}{3} \log \left(\frac{4}{3}\,\frac{D^2 - Dt}{t^2}\right) \text{ approx.}$$

The magnetism inside of the line of force g due to the current in a_2 will cut the line $a_2\,s$ perpendicularly and equals

$$\frac{2\,I\,(1+k)}{3}\left(\log\frac{\frac{t}{2}}{r}+\frac{1}{4}\right)$$

where the term

$$\frac{2\,I\,(1+k)}{3}\cdot\frac{1}{4}$$

is the inductance within the wire itself and where r = the radius of the conductor. Similarly the magnetism due to the current in a_3 between a_2 and s is

$$\frac{2\,I\,(1+k)}{3}\log\frac{t}{\frac{t}{2}}=\frac{2\,I\,(1+k)}{3}\log 2$$

Therefore the magnetism due to a_2 and a_3 between a_2 and s =

$$\frac{2\,I\,(1+k)}{3}\left(\log\frac{\frac{t}{2}}{r}-\log 2+\frac{1}{4}\right)=\frac{2\,I\,(1+k)}{3}\left(\log\frac{t}{4\,r}+\frac{1}{4}\right)$$

since the effect of a_3 along the line $a_2\,s$ is opposite to that of a_3.

The magnetism produced by the current in a_1 between a_2 and P will be

$$\frac{2\,I\,(1-2k)}{3}\log\frac{\frac{D}{\sqrt{3}}+\frac{t}{\sqrt{3}}}{t}=\frac{2\,I\,(1-2\,k)}{3}\log\frac{D+t}{\sqrt{3}\,t}$$

since the lines of force due to a_1 are circles about a_1.

The effect of the group b may be taken to be the same as though the three conductors were concentrated at the point Q_2. The magnetism between s and P due to the c group will be the same as the b group at this instant of time when the current in the a group is a maximum. The effect of the two groups between a_2 and P is

$$2\,I\log\frac{\sqrt{\left(\frac{D}{2}\right)^2+\left(\frac{\sqrt{3}\,D}{2}-\frac{t}{2\sqrt{3}}\right)^2}}{\frac{D}{\sqrt{3}}}\ \text{approx.}$$

$$=I\log 3\ \text{approx.}$$

Summing all these components of the magnetism:

Total magnetism =

$$\frac{2\,I\,(1+k)}{3}\left[\log\left(\frac{4}{3}\cdot\frac{D^2-D\,t}{t^2}\right)+\log\frac{t}{4r}+\frac{1}{4}\right]$$

$$+\frac{2\,I\,(1-2\,k)}{3}\cdot\log\frac{D+t}{\sqrt{3}\,t}+I\log 3,\ \text{approx.}$$

$$=\frac{2\,I}{3}\left[(1+k)\left\{\log\frac{4}{3}\,\frac{D^2}{t^2}+\log\frac{t}{4\,r}\right\}\right.$$

$$\left.+\,(1-2\,k)\log\frac{D}{\sqrt{3}\,t}+3\log\sqrt{3}+\frac{1+k}{4}\right]\ \text{approx.}$$

$$=\frac{2\,I}{3}\left[(1+k)\log\frac{D^2}{r\,t}+(1-2\,k)\log\frac{D}{t}+\frac{1+k}{4}\right]\text{approx.}$$

$$=\frac{2\,I}{3}\left[(1+k)\left(\log\frac{D}{r}+\frac{1}{4}\right)+2\log\frac{D}{t}\right]\ \text{approx.}$$

Then the inductance $= L$

$$=\frac{2}{3}\left[(1+k)\left(\log\frac{D}{r}+\frac{1}{4}\right)+2\log\frac{D}{t}\right] \tag{49}$$

These logarithms are natural logs to the base ε.

For a three-phase line without the split conductors, the inductance from one wire to the point P would be

$$2\log\frac{D}{r}+\frac{1}{2} \tag{50}$$

So that the ratio of these two quantities (49) and (50) represents the change in the inductance due to the splitting of the conductors.

The capacity of the system with split conductors will be increased in substantially the same ratio that the inductance is decreased. The capacity can be determined by finding the work done against static charges on the several wires in moving a unit positive charge, from a_2 to P, for example:

As this work will integrate in exactly the same form as the inductance above (neglecting the inductance within the wire itself) the potential for a given charge between wires will be reduced by splitting the conductors in the same ratio that the inductance was reduced. But the capacity of the circuit is inversely proportional to the potential existing between wires for a given charge, so that the capacity is increased as the inductance is decreased.

There remains the determination of the quantity k. Referring to Fig. 4 the total flux between a_1 and a_2 or between a_1 and a_3 will be the same. This can be calculated by the methods already used.

The magnetism due to a_2 and a_3 between a_1 and s will be

$$\frac{4\,I\,(1+k)}{3} \log \frac{t}{\frac{t}{2}} = \frac{4\,I\,(1+k)}{3} \log 2$$

$$= \frac{2\,I\,(1+k)}{3} \log 2^2$$

The magnetism between s and a_2 due to a_2 and a_3 as before =

$$\frac{2\,I\,(1+k)}{3} \log \frac{t}{4\,r} + \frac{I\,(1+k)}{6}$$

Therefore the total magnetism between a_1 and a_2 due to a_2 and a_3 will be

$$\frac{2\,I\,(1+k)}{3} \log \frac{t}{r} + \frac{I\,(1+k)}{6}$$

The magnetism between a_1 and a_2 due to a_1 =

$$-\frac{2\,I\,(1-2\,k)}{3} \log \frac{t}{r} - \frac{I\,(1-2\,k)}{6}$$

The sum of these values

$$= 2\,I\,k \log \frac{t}{r} + \frac{k\,I}{2}$$

$$= 2\,I\,k\,(\log \frac{t}{r} + \frac{1}{4})$$

The magnetism between a_1 and a_2 due to the b and c groups which this must balance will be

$$4\,I \log \frac{\sqrt{\left(\frac{D}{2}\right)^2+\left(\frac{\sqrt{3}\,D}{2}+\frac{t}{\sqrt{3}}\right)^2}}{\sqrt{\left(\frac{D}{2}\right)^2+\left(\frac{\sqrt{3}\,D}{2}-\frac{t}{2\sqrt{3}}\right)^2}}$$

$$= 2\,I \log \frac{D^2+Dt+\frac{t^2}{3}}{D^2-\frac{D\,t}{2}+\frac{t^2}{12}}$$

$$= 2\,I \log \left(1+\frac{3}{2}\ \frac{t}{D}\right) \quad \text{approx.}$$

That is

$$2\,I\,k \left(\log \frac{t}{r} + \frac{1}{4}\right) = 2\,I \log \left(1+\frac{3}{2}\ \frac{t}{D}\right)$$

and

$$k = \frac{\log\left(1+\frac{3}{2}\frac{t}{D}\right)}{\log \frac{t}{r} + \frac{1}{4}} \quad \text{approx.} \qquad (51)$$

which completes the formula for the inductances of the split conductor, having three parts arranged as shown in Fig. 4.

In calculations of three-phase circuits with split conductors by the equivalent single-phase circuit, the ratio of inductances and capacities determined above should be used.

In case the conductor be split into two parts instead of three as above, the following formula can be deduced by the same methods.

Inductance between a_2 and $P = \log \frac{D}{\sqrt{3}\,t} \cdot \frac{D}{r} + \frac{1}{4}$, assuming that the two parts of the split conductor lie symmetrically on opposite sides of the line $s\,P$.

Other formulas can be easily derived for other arrangement of the wires of the two or three-part combinations or for single-

phase circuits. The spiral arrangement of the split conductors will have little effect on the inductance as long as the distances between the centers of gravity of the groups are the same.

This completes the new formulas utilized in the companion paper. It is believed that these, especially the wave and split-capacity formulas will be found especially convenient and serviceable by transmission engineers.

DISCUSSION ON "OUTPUT AND REGULATION IN LONG-DISTANCE LINES," AND "CALCULATION OF THE HIGH-TENSION LINE." FRONTENAC, N. Y., JUNE 29, 1909

T. R. Rosebrugh (by letter): Mr. Thomas refers to the time required to work out formulas given by previous writers. The time occupied in making a calculation, however, consists of two parts: first, the verification of the formulas to be used, by a critical examination, for they may contain typographical errors, or be misunderstood; secondly, the actual computation. The longer and more unsymmetrical a formula is the more time one would in general take in checking it.

The following are simple, convenient, and flexible, yet give the general solution for the steady state:

$$E_1 = E_0 + z\, I_0 + \tfrac{1}{2}\, z\, y\, E_0 + \tfrac{1}{6}\, z^2\, y\, I_0 +$$
$$I_1 = I_0 + y\, E_0 + \tfrac{1}{2}\, y\, z\, I_0 + \tfrac{1}{6}\, y^2\, z\, E_0 +$$

two series converging in all practical cases quite rapidly, in which each symbol denotes a complex quantity according to Dr. Steinmetz' notation, E_0 and I_0 being for the receiving end E_1 and I_1 for the transmitting, and z and y are the quantities $r+x\,j$ and $g+b\,j$, respectively, for the whole line.

Whether it has occurred to anyone to publish this form of the solution I do not know, but it seems not to be so well known as it deserves.

In addition to these equations, those obtained by using a minus sign before each even numbered term may be employed in preference when the known end of the line (E_0, I_0) is transmitting power instead; this makes the power component of I_0 in both cases positive when E_0 is denoted by a positive real quantity.

Suppose in the first place that one condition only of the line is to be calculated, the least labor will then be expended by proceeding as follows:

Starting with I_0 (complex quantity) multiply it by z and this again by $y/2$. Similarly starting with E_0, multiply by y, and this product by $z/2$.

On looking at the two series it will be seen that following two zigzag lines the first three terms of voltage and current are now known, and simple addition will now usually determine both components of each with sufficient accuracy.

If the line be so long that the last term calculated above in either series is not small enough, proceed further along either or both zigzags until satisfied. It will be noticed that the successive factors for one zigzag are $\frac{z}{1}$, $\frac{y}{2}$, $\frac{z}{3}$, $\frac{y}{4}$, etc. and $\frac{y}{1}$, $\frac{z}{2}$, $\frac{y}{3}$, $\frac{z}{4}$ for the other.

Thus no labor is wasted, each multiplication giving one term, and the error is known to be much less than the last term calculated in the given series.

Secondly, if the calculation is to be made for several different conditions (which may be given for either end) it is easiest to find first the coefficients of E_0 and I_0 in the two series, and these are readily found numerically also, three multiplications only serving to determine what is required to the end of the fourth term in both series. In this simple manner A, B, C are found such that

$$E_1 = A\,E_0 + B\,I_0$$

$$I_1 = A\,I_0 + C\,E_0$$

hold for given conditions at receiving end and

$$E_1 = A\,E_0 - B\,I_0$$

$$I_1 = A\,I_0 - C\,E_0$$

for given power house conditions. These forms of the series are now conveniently ready to deal with any values of E_0 and I_0 which may be inserted.

To prove that the first series given are correct it is only necessary to remember that a Taylor's expansion requires merely the successive derivatives of the variable at one given point. Now since $\frac{d\,E}{d\,s} = z\,I$ and $\frac{d\,I}{d\,s} = y\,E$ everywhere on the line, (s being the distance from 0 in terms of the line as unit of length), these successive derivatives may be written down by inspection, and their values at the zero end are seen to be such that multiplication by $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{24}$, etc. produce the terms of each series as stated, at the other end of the line where $s = 1$.

Intermediate conditions on the line may be found by retaining the proper powers of s. The two methods of calculation which take account of capacity, either all at the middle or divided equally between the ends, amount to taking three terms both of voltage and current, the former, however, includes also 3/2 of the fourth term of voltage, the latter also 3/2 of the fourth term of current.

Mr. Thomas' paper contains a recommendation to calculate three-phase transmissions by reference to a single-phase circuit in which all the quantities concerned are different from those of the actual three-phase circuit or its parts.

Would it not be better to calculate the actual circuit as a star arrangement; that is, each of the three wires with its own actual resistance, reactance, capacity to neutral, and leakage conductance as the line portion of a circuit carrying its share, namely one third of the power and at the star voltage?

It may perhaps be said that the recommendation is harmless, but the result of such propositions is apt to be, as has already

occurred, that tables are prepared giving fictitious quantities for three-phase lines instead of the real ones. This leads to a state of affairs in which handbooks become unsafe to use, for such tables are apt to be given without adequate warning as to the special fiction they favor.

The paper deals with three distributed line constants but omits the fourth—leakage conductance. This may be taken into account and his formulas thus given greater generality as follows. Let r, x, g and b have their usual value as real numerics, and in this case z and y also the real values $\sqrt{r^2+x^2}$, $\sqrt{g^2+b^2}$ instead of complex quantities as above, and all six for the whole line.

For	Write
$\alpha x = \frac{R x}{2} \sqrt{\frac{2 p C}{p L + \sqrt{R^2+p^2 L^2}}}$	$\sqrt{\frac{z y + r g - b x}{2}}$
$\frac{p x}{v} = x \sqrt{\frac{p C (p L + \sqrt{R^2+p^2 L^2})}{2}}$	$\sqrt{\frac{z y - r g + b x}{2}}$
$\sqrt{\frac{\sqrt{R^2+p^2 L^2}}{p C}}$	$\sqrt{\frac{z}{y}}$
$\frac{p}{\alpha v} = \frac{p L + \sqrt{R^2+p^2 L^2}}{R}$	$\sqrt{\frac{z y + r g + b x}{z y - r g - b x}}$

One might gather from the reference to the perfectly uncharged line that the scope of the paper is sufficient to deal with transient states of the system; this, however, is elsewhere stated not to be the case.

The statement that the current produces a drop in the line proportional to the current and the square root of the impedance and inversely proportional to the square root of the capacity susceptance seems inadequate and misleading, for the factor omitted is also a function of the line constants for both terms involving the current.

In the investigation of the so-called split conductor the currents in the strands are taken to be different as indicated by the factor k, but there appears to be no reason given for neglecting the slight phase difference one would naturally expect between the currents a_2 and a_3, due to the revolving field in which they are situated.

However this may be, if the three strands are transposed so that each has an equal length in each position, then k becomes zero, and this phase difference vanishes also.

Checking by a different method the final result for L, and on the assumption that the strands are given a cyclical transposi-

tion, I obtain the same expression on the understanding that t is small in comparison with D.

It would then be a question whether for slight separation of the strands (if the construction should be adopted) the extra losses would be sufficient to justify the expense of the slight insulation necessary with transposition to avoid them.

V. Karapetoff: I think the first paper contains a good deal of what could be called prophetic vision. At the present stage of the development of exceedingly long transmission lines such prophetic pictures are as important to us as detailed investigations of the performance of such lines. It is necessary for us, and particularly for our leaders of industry, to have such clear mental pictures, before we can see these transmission lines realized in practice.

I would suggest that Professor McMahon's book of hyperbolic functions is a good book for engineers who wish to use hyperbolic functions in addition to plain trigonometry. It is a book which is easily readable, and what makes it particularly valuable to electrical engineers is the fact that the same problem on transmission line which is solved in Dr. Kennelly's article, in the 1895 TRANSACTIONS of the Institute, is worked out and considerably simplified in this book. In place of Dr. Kennelly's charts, Professor McMahon has calculated more accurate tables of hyperbolic functions of complex quantities, with double entry, so that the calculations are easily performed.

However, I think that all the possible mathematical methods should be open for the use of engineers. Some of us may prefer hyperbolic functions, others may prefer Dr. Steinmetz's method. Others again may prefer the treatment within the limits of plain trigonometry, as given in Mr. Thomas' paper. I do not think that we should limit ourselves to any specific method, because the temperaments and natural inclinations of engineers differ very materially. This may be likened to the case of two mechanics building similar devices. One would spend most of his time in making special tools with which to construct the device; the other would prefer not to make special tools, but to devote all the available time to building the device itself.

But even granting that Mr. Thomas's method of treatment is legitimate and useful, I do not believe that he did full justice to the method: the deductions can be considerably simplified. The paper will undoubtedly be referred to by many of us later on. Therefore, I would suggest that Mr. Thomas give the paper to some professional mathematician and ask him to go over it and simplify the equations.

John B. Taylor: In transmitting power with direct current, we are all familiar with the fact that the only limitation as to distance is the size and price of copper conductors. With alternating-current work we find other limitations. After obtaining the right of way and paying, say, a million dollars for this with steel towers, insulators, etc., we find that we can

effectively put up only a certain size of copper wire. This is because the reactance of the circuit is reduced practically not at all as the wire is increased in size, and the reactance may be the limiting factor in the amount of power to be got over the line. Of course the reactance can be reduced to a certain extent, either by bringing the wires closer together, or by putting up a multiple circuit. The multiple circuit in its most efficient form means a distinct pole line, with perhaps another million dollars for duplicating right of way, steel towers, etc. Reduction of inductance by splitting the conductor, as suggested in the paper, may be effective; but this involves more dollars, and special construction of insulators and supports which have not been tried out.

The paper disappoints me from the fact that after reading it I find myself with no clearer phy ical conception of the interactions and reactions of distributed resistance, inductance, and capacity in the transmission line. This I think is due to the use of the term "energy" for something which is not energy, and the lack of any fixed direction of energy transfer as a reference, which allows Mr. Thomas to call the same current leading or lagging according to which way he looks at it. Receiving apparatus is also spoken of as *supplying* current and energy. This special use of terms and shifting point of view make it difficult to follow the exposition. I hoped that Mr. Thomas would give a clear mental picture of the changing values and phases of current and potential at numerous points of the long-distance, alternating-current line, which from one point of view is merely an indefinite number of condensers, reactances, and resistances connected together. If we can solve the more familiar case of reactive coil, tin-foil condenser, and resistance box on alternating current, by extension we should understand the long-distance transmission line. Determination of actual values may involve mathematics too difficult for many of us, but a rational understanding of what we are dealing with seems desirable before passing to the long equations.

Charles P. Steinmetz: I consider the extensive use of the terms leading and lagging power as one of the most valuable features in this important paper. For a number of years I have been trying to introduce these terms instead of the frequently employed terms "lagging current" and "leading current." Lagging power and leading power, or combined in one expression, *reactive power*, is a real, existing quantity, but not so reactive current, as lagging or leading current. There is no such thing as lagging current, absolutely; it is only relative. Referring to an induction motor under a given condition of operation, it may be said that the motor takes so many volt-amperes reactive power, but it cannot be said that it takes so many amperes reactive current for magnetization. Magnetizing current is wattless or reactive only with reference to the induced electromotove force, but it is very far from wattless with

regard to the impressed electromotive force. Hence, applied to current and voltage, " reactive ", " leading ", and " lagging " are relative terms. This is particularly true in the case of a transmission line. For instance, in a 700-mile transmission line operating at 60-cycles, if the current and voltage are in phase at the generator, then at the receiving end the current will be approximately 90 degrees out of phase with the generator voltage; that is, the current from the line would be wattless in regard to the generator voltage. But it is not, obviously, wattless or reactive power; it is practically in phase with its own voltage at that place, the receiving end of the line; it merely means that the entire power has shifted 90 degrees, or, in other words, that the storage capacity of the line is sufficient to store the entire energy, and then send it out again at the receiving end, producing 90 degrees phase displacement. This feature makes the use of lagging current and leading current misleading and unsatisfactory in dealing with complex circuits.

Mr. Thomas' paper concludes that it is necessary, for economical transmission, to balance the leading and lagging energies of the line, and that balancing is done by adjusting the relations of voltage and current to the relations of inductance and capacity, so that the value of volts over amperes must be equal to the square root of inductance over capacity. This relation has a very simple physical meaning, which we must realize. It is the condition of free flow of electric power, the relation which exists for electric power in the transmission line as a circuit, which adjusts itself to change of conditions. For instance, if the electrostatic charge impressed upon the line by a lightning stroke readjusts itself by dissipation, or if the line discharges to ground, or an impulse or wave travels along the line, or if a sudden oscillation makes the line surge—in all these cases voltage and current are related as the square root of inductance and capacity. It will be seen, therefore, that the condition of maximum economy of power transmission is the relation of free flow of power.

It means, however, that for economic transmission, and more particularly for voltage regulation, we must have in these very long lines such methods of controlling the power-factor and the voltage at the receiving end that, at the economical load, the receiving circuit shall be non-inductive, and at light load the leading power of the line shall be supplied from the receiving circuit; that is, the receiving circuit feed leading current into the line, or, as it is usually expressed, the line sends lagging current into the receiving circuit. This necessitates a method of adjusting the power-factor by varying the phase relation of the load, so as to have the phase relation of the load change from lagging at light load, over non-inductive condition at the economical load, to leading at overload; in short, the well-known method of phase control is required to get maximum efficiency of transmission.

In long-distance transmission lines, as a general thing, this is not yet done to any extent, but it is common practice in those long-distance transmissions in which the receiving apparatus is specially suited for the automatic variation of phase relation; that is, in railway systems with converter sub-stations. There this method of phase control of the line is generally used as it gives the best regulation and best economy.

As a corollary, therefore, in conclusion 7, the non-synchronous machine, or, as commonly called, the induction generator, is referred to. This is a type of machine especially adapted to a load in which the synchronous apparatus controls the phase relation with the load. The induction generator naturally adapts itself to the synchronous phase controlling receiver.

As regards the computing formulas, there are a number of formulas that give fair approximations. One of the simplest is the following: first consider the entire line capacity as a condenser at the generator; this gives a simple, quick computation. Then consider the entire line capacity as a single condenser at the receiving end; this is also a simple and quick computation. By averaging these two approximations we come close to the actual condition. These combined computations are far simpler than a single computation with the condenser at the middle of the line; at the same time they give a closer approximation. To find out whether they are really close enough, the complete calculation should be made for a few points. However, these calculations may be checked by estimating the capacity (the charging current of the line) as a percentage of the full-load current. So long as the charging current is less than a certain percentage, perhaps 20 or 30 per cent of the full-load current of the line, these localized computations are fairly satisfactory. Beyond that the complete line equations must be used, as stated by Mr. Thomas.

These complete equations are not so formidable as they look at first sight, but can be modified to very simple expressions. The disadvantage of the general equations for computation is the existence of the exponential function, which must be evaluated. Some months ago Mr. E. J. Berg and I had occasion to look into the question of calculating long power transmission lines quickly, so as to get within a half-day complete curves for different loads and different power-factors of the load. We found that for practically any commercial line, up to 300 or 400 miles, or even more, almost a perfect approximation could be got by using the complete equation, but substituting for the exponential functions the first terms of their series, which eliminates the stumbling blocks of the computation. The simple equations resulting therefrom will be found later in this discussion.

Paul M. Lincoln: The practical engineer approaches these two papers with an interrogation something like this—what is there in them that can be of assistance to the electrical engineer who is about to design a long-distance, power transmission line?

The answer to that question, as I see it, is that for present conditions, and for those which will arise in the immediate future, as far as we can foretell them, there is little or nothing contained therein that will be of assistance to him. But for conditions which are apt to arise in the distant future the discussion and the analysis made in papers may be of considerable value. Mr. Thomas has made a close and careful study of the conditions which will obtain when a transmission line contains a certain type of resonance. This resonance depends upon at least three conditions, all of which must be satisfied before resonance can obtain: first, there must be a given load transmitted over the line; secondly, that load must be at a given power-factor; thirdly, there must be a given voltage. These three conditions, at least, must be satisfied before the particular type of resonance which Mr. Thomas has discussed may obtain.

For conditions below full load there is not sufficient current to cause an excessive drop; for conditions above full load the drop will increase very rapidly as the load increases. This increase will be so rapid as to place a practical limit to the line capacity at its full load. Hence we will have a line whose most economical point is at load a; it cannot carry more than load a, for less of a load its regulation will not be so good as for load a. Such a condition must, to say the least, be inconvenient, and would probably result in such a handicap as to render useless the transmission line having such limitations.

Ralph D. Mershon: I have not had an opportunity to give this paper nearly the study I wanted to and that I intend to. There are one or two points in regard to it I would like to mention. If I remember rightly, Dr. Fleming made, at a much earlier date than any of those mentioned, a study of the effects of the distributed inductance and capacity of a transmission circuit.

Mr. Thomas makes mention of a paper by myself, and misinterprets it a little, I think. In that paper it was assumed that with unity power-factor at the middle of the line, or approximately at the middle of the line, the line loss would differ little from that for unity power-factor throughout the line and that such difference as did exist would be negligible in the problem I was considering. Such assumption greatly simplified the problem, and is permissible where large amounts of power are transmitted. It was also assumed that the synchronous motors would be designed to give the requisite voltage regulation.

This brings up a point which, so far as I have been able to study his paper, Mr. Thomas seems to have neglected. A synchronous motor used at the end of a line to regulate the power-factor of the line is not equivalent to a condenser; it behaves quite differently. When the voltage increases on a condenser the current to the condenser increases. When the voltage impressed upon an over-excited synchronous motor increases,

the leading current to the motor *decreases*, and with further increase of impressed voltage the current passes through zero and then becomes a lagging current which *increases* with the impressed voltage. As a result, such a motor may improve or may impair the regulation; meaning by *regulation* the inherent regulation—the maximum change in voltage if full load, say, be thrown off suddenly and no adjustment made in the system. If the inherent regulation of the line, and the synchronous motor at the end of it, happen to be the same, then the regulation of the combination of line and motor will be that of either. If the inherent regulation of the line be worse than that of the synchronous apparatus, then the resultant regulation will be better than that of the inherent regulation of the line, and not so good as that of the synchronous apparatus, and conversely. Synchronous apparatus tends to make the regulation of the system the same as that of itself; better or worse as the case may be.

There would be great mechanical objection to the split conductor Mr. Thomas proposes. There may be another objection in the limitation its use may put upon the permissible line voltage, as governed by the critical point or point at which the air begins to break down, as against the voltage which could be used if a single conductor of larger diameter were employed. The mechanical objections Mr. Thomas has already mentioned, but I think that when it comes to making figures as to the increased cost involved in the line structures and construction generally by the use of three cables, it will be found that, because of the greater wind forces resulting, especially if the cables gather sleet, the increased cost will make the use of the split conductor extremely objectionable, if not absolutely prohibitive.

Henry W. Fisher: About a year ago we had occasion to make some calculations on a proposed 60-mile cable transmission line to operate at 60,000 volts. The power to be delivered was 12,000 kw. at 80 per cent power-factor. On account of the high capacity and low self-induction of the line the calculated power-factor at the generator was practically 100 per cent. In some calculations the load was leading, in others lagging.

Calculations were made with half the capacity concentrated at each end and also, according to Dr. Steinmetz' recommendation, with one sixth of the capacity at each end and two thirds in the middle.

The calculated results based on both of these assumptions were not materially different.

J. B. Whitehead (by letter): The use of approximate formulas should invariably be safeguarded by consideration: first, of the range of values within which the approximation holds true, and the magnitude of variation due to the correcting factors without this range; and, secondly, the degree to which experiment and experience have shown the approximate formulas to be sufficiently correct.

The author of these papers has performed a valuable service in presenting his wave formulas as checks upon the use of the split-capacity formula. Considering his comparison of the two types of formula as given in Table I, one is surprised and rather relieved to find the small differences that are involved. It should be pointed out, however, that the author has considered ohmic resistance, inductance, and capacity only. The losses between lines, over insulators, and in other kindred phenomena have been neglected. These losses would appear as a conductance or parallel connected resistances in the development of a formula taking them into account. The suggestion of the author to subdivide the individual wires of the line into several others would naturally lead to an increase in the insulator loss, and without further modification would also lead to circulating or secondary currents in the several branches of one line, due to inductive action of the others. My feeling after reading these two papers is that a series of accurate measurements on existing lines would be of first importance for answering the questions which are raised.

I find the author's standpoint in the description and explanation of the phenomena as given in the first paper rather confusing, by reason of the apparent separation of the energy into three components. I venture to call attention to several apparent inconsistences which result. Energy and voltage drop or rise appear to have the same physical dimensions. It may also be asked from what quantity involved are the phases of various currents, electromotive forces, and energies to be reckoned; that is, with reference to what quantity are the "leading and lagging" energies leading and lagging? We are asked on one page to note that the leading and lagging energies are equal and opposite when the load power-factor is equal to one, and then on the next page we are cautioned to remember that the leading and lagging energies have no necessary relation to the leading or lagging current in the load. We are also asked to imagine that the energy for charging the line flows against the energy which represents the power transmitted. Is it not the case that in the transmission of power the resultant flow of energy is in one direction? The distinction is also drawn between true power and leading and lagging energies, the inference being that these latter are wattless. My question then is, why call them energies? We are also told that in one instance the synchronous machines at the receiving end supply all of the load lagging current except enough to charge 300 miles of the line.

In calling attention to these few difficulties of my own, I have no wish to depreciate the author's valuable contribution to our knowledge of long-distance transmission problems. I wish only to make a plea for the retention of current and electromotive force as standards of reference where variation with time is involved, and the direction of the resultant flow of energy where variations in space are involved. If the in-

stantaneous energy is of interest, Poynting's theorem based on Maxwell's equations that the flow of energy is in the medium and perpendicular at all points to the electric and magnetic vectors makes it possible to picture the flow of energy at any point in a line in terms of the instantaneous values of current and electromotive force.

C. F. Scott: I think Mr. Thomas deserves special credit for taking what is apparently a simple problem, which most of us have probably regarded as having been satisfactorily solved, and going into it deeply and working it out more fully, to determine what results may be encountered when new conditions may be reached in the future. It is the experience in all power transmission work, that when we go up to a higher voltage, new points, which have sometimes had an unimportant bearing, or which apparently have not been thought of before, force themselves into great prominence; and it is well to go over these problems very carefully, as is done here, so as to determine actually what these conditions are.

A few points have been raised in the discussion which it seems to me are taken care of. Mr. Lincoln, for example, suggested that the nature of the load was such that it was hard to get stable and definite conditions, and that if the conditions were properly established for one load, they would not be right for other loads. The important condition is full load, so that the line will transmit the full-load energy with a high efficiency; it is often of minor consequence what the efficiency is at lesser loads.

Mr. Mershon raised the point that synchronous machines are different from condensers, because when delivering different amounts of current, they were liable to give different voltages. But as Mr. Thomas will use the synchronous machine at the receiving end of the line, it is to be loaded at practically constant potential, and therefore it seems to me that these variables will probably not appear. The synchronous machine at the end of the line is not dependent upon the various voltages and currents coming in over the line, but it of itself is to be made the important voltage regulating element in the system. We may consider, I think, that we have two kinds of current in the system, one, the power current, comes from the generating station, and the other current that has been called by various names this morning. I will, however, use the familiar term wattless current. That has to be supplied from somewhere also, and Mr. Thomas supplies it at the receiving end. Consequently, the power current will be generated at one end of the line, and the wattless current at the other end of the line by synchronous motors, and the latter may control the voltage at the receiving end, which appears an ideal relation.

It occurs to me that possibly two wires as a single conductor would accomplish the same effect as the three proposed by Mr. Thomas.

I have been impressed with the mathematical part of this

paper. I think Mr. Thomas is to be congratulated in not mixing the two things together—the engineering conclusions and the mathematical methods. I was in New York a few days ago, and I met a consulting engineer who said he had been going over these papers and found they could be very much simplified. The first gentleman who opened the discussion this morning said that the matter given in the papers could be presented in a much simpler way. He went to the blackboard to prove it. The next speaker said the matter could be presented in a much simpler way, and he covered the blackboard twice. Another professor said it was a simple thing to buy a book and read all about it, and Dr. Steinmetz then said the purpose of all this mathematical demonstration is to show that it is really not needed at all.

W. A. Del Mar: I wish to call attention to the fact that in neglecting leakage Mr. Thomas has missed an opportunity to make his equations homogeneous and has complicated rather than simplified them by this omission. A set of equations due to Dr. H. Pender* takes leakage into account with the result that the final equations are more simple than those given in the paper under consideration.

Furthermore, Dr. Pender's solution is considerably simplified by the introduction of the power-factor angles of the line impedance and leakage admittance. The equations are put in such form as to give a clear physical conception of the processes, a quality lost by Mr. Thomas' rather lengthy expressions. It is unfortunate that Dr. Pender's article did not appear in the PROCEEDINGS where it could be discussed and freely compared with Mr. Thomas' paper and I hope that this brief contribution will serve to attract the attention of the Institute members who may have read the latter and not the former.

A. E. Kennelly: These papers are of great importance, if only because they show how much labor is involved in the computation of the voltage and current distribution over a long line having distributed resistance, inductance, leakance and capacity, when hyperbolic functions are avoided. In the opening paragraphs of the first paper, the author intimates that the methods which have been suggested hitherto for making such computations are long and unpractical. He proceeds to offer what he considers to be practical working formulas for working out such problems. These practical formulas recommended to us as superior to all preceding formulas are collected in one place.

There can be no doubt that the voltage and current distribution over long lines worked at a single alternating-current frequency in the steady state can be computed by these formulas, and the paper will be useful to engineers if only in the collection of these formulas. It is, however, open to discussion whether these formulas are the best for engineers. These working

***Electrical World*, July 8, 1909.

formulas are so long that the voltage and current equations have first to be split into two parts (V_0 and $V_{\frac{\pi}{2}}$), or (I_0 and $I_{\frac{\pi}{2}}$), the square root of the sum of whose squares is the final voltage or current; while the formula for each component is then so long that it has to be printed lengthwise to the page. It is here contended by the speaker that such methods of computation are deterring and forbidding. They are fearful even to look upon, and the chances of making numerical mistakes in such long unwieldy equations are comparatively great. Fortunately, there are much simpler formulas for reaching the same results.

If the voltage to be supplied at the distant end of a line is e volts and the current to be supplied there is i amperes, at a given power-factor and phase-angle with respect to e; then it has been shown by the speaker* that the voltage E and current I at the sending end of the line are respectively:

$$E = e \cosh \theta + i\, z_0 \sinh \theta \qquad \text{volts} \angle \quad (1)$$

$$I = i \cosh \theta + (e / z_0) \sinh \theta \qquad \text{amperes} \angle \quad (2)$$

where z_0 is the surge impedance of the line:

$$z_0 = \sqrt{\frac{r + j\, l\, \omega}{g + j\, c\, \omega}} \qquad \text{ohms} \angle$$

or as a first approximation

$$z_0 = \sqrt{\frac{l}{c}} \qquad \text{ohms}$$

and $\theta = L\, \alpha$ is the "hyperbolic angle" subtended by the line where L is the length of the line in miles or km. and α is the attenuation constant:

$$\alpha = \sqrt{(r + j\, l\, \omega)\, (g + j\, c\, \omega)} \qquad \text{"hyperbolic radians" per}$$

mile or km.

also r = the linear resistance of the line (ohms per mile or km.)
l = the linear inductance of the line (henrys per mile or km.)
g = the linear leakance of the line (mhos per mile or km.)
c = the linear capacity of the line (farads per mile or km.)
$\omega = 2\, \pi\, n$ = the angular velocity (radians per sec.)
n = frequency impressed on line (cycles per second).
$j = \sqrt{-1}$

Equations (1) and (2) are the equivalent of Mr. Thomas' formulas (40), (41), (42), (43).

If we should attempt to solve plane triangles, in the regular work of surveying, without the use of circular functions, (circular sines, cosines, tangents, etc.,) that is, without the use of trigometrical tables, we could do it by roundabout, long and tedious

* "On the Fall of Pressure in Long-Distance Conductors," by A. E. Kennelly. *Electrical World*, Vol. XXIII, No. 1, p. 17, Jan. 6, 1894. also *The Electrician*, London, Vol. 32, Jan. 5, 1894, pp. 239–240. "Resonance in Alternating-Current Lines," by E. J. Houston and A. E. Kennelly. TRANSACTIONS American Institute Electrical Engineers, Vol. XII, pp. 133–159, April 17, 1895

methods, but the contrast between such methods and those we actually use would be marked. Similarly, hyperbolic trigonometry is as natural and as essential to alternating-current line computations as is ordinary circular trigonometry to the surveyor's work. Hyperbolic trigonometry is just as easy as circular trigonometry, and can be grasped and handled practically with very little extra effort by the engineer who already grasps and handles circular trigonometry. It is true that our tables of hyperbolic sines cosines, tangents, etc. of vector or complex arguments are still very deficient; but even when we have to work out the hyperbolic sines and cosines of formulas (1) and (2) for ourselves, without aid from tables, I contend that these formulas are much more practical, direct, and easy to handle without mistakes, than the long formulas of the paper just read.

Hyperbolic tables are available already, however, in the following cases:

(1) where $\omega = 0$, or direct currents are used.

(2) where $1 = 0$ and $g = 0$*; or in well insulated cables of negligible inductance.

(3) where $r = 0$ and $g = 0$; or where the line has negligible conductor and dielectric losses.

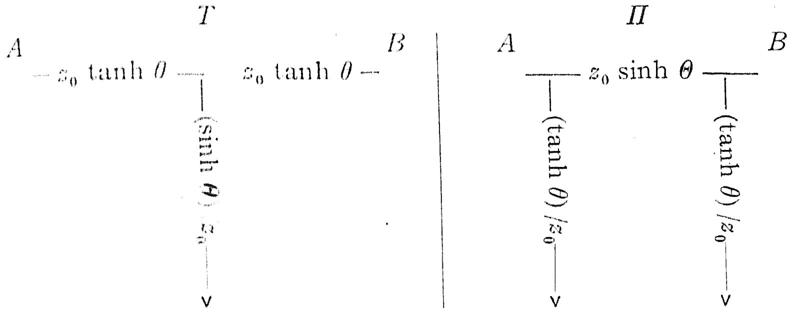
It is to be hoped that, before long, suitable tables of hyperbolic complex quantities may be available for engineers. Meanwhile, however, a hyperbolic sine or cosine of the complex angle θ, subtended by a line, can be computed by regular rules in a few minutes.

As pointed out in the papers before us, it is unnecessary in the great majority of cases to compute alternating-current line voltages and currents by the accurate formulas; whether those formulas be as given in (10), (11), (12), and (13) or those just given at (1) and (2) above. Lines are not long enough and fundamental frequencies are not high enough in power transmission work to make this refinement necessary. It is quite sufficient either to lump all the leakance and capacity in one condenser at the electrical middle point of the line; or, as in the paper, to split it into two equal condensers, one at each end of the line, and then to treat the line as a choking coil with condenser leak or leaks. If, however, higher harmonic frequencies have to be considered in long power-transmission line, it may be necessary to use the correct formulas of distributed capacity. Moreover the correct formulas are nearly always necessary in telephony, even on fairly short lines, owing to the high frequencies there employed.

The speaker has shown that the II — conductor, or split-condenser representation of a line, may be readily corrected for the lumpiness-error; so as to give correct results, just as though the ful-

* The Alternating-Current Theory of Transmission Speed; and Table appended thereto. International Electrical Congress of St. Louis, 1904, Vol. I, pp. 68—105.

distribution formulas had been used.* The correct values of the equivalent T and equivalent Π are given below:



Here A and B are the terminals of the equivalent line, and $\theta = \Theta/2$, or is the hyperbolic-angle subtended by half the line. The total line impedance of the T is $2\,z_0 \tanh \theta$ and of the Π is $z_0 \sinh \Theta$ ohms. The total admittance of the capacity and leakage in the T is $\frac{\sinh \Theta}{z_0}$ mhos and of the Π, $\frac{2 \tanh \theta}{z_0}$ mhos. Since the corresponding impedances and admittances in the uncorrected or nominal T are $2\,z\,\theta$ and Θ/z_0; while for the nominal Π they are respectively the same; viz., $z_0\,\Theta$ and $2\,\theta/z_0$; it follows that the lumpiness-correcting operators are the vector ratios $\frac{\tanh \theta}{\theta}$ and $\frac{\sinh \Theta}{\Theta}$, which are 1 for indefinitely short lines; but which may reach either large or small positive or negative values when θ and Θ are large.

These formulas, for the correct or equivalent-Π, form a fitting addition to the formulas in the paper on the split condenser method, or nominal-Π, method.

It is pointed out in the paper that the linear capacity of a split-conductor circuit is increased as its linear inductance is diminished. It is not generally known, and perhaps may not have been noticed by electrical engineers, that there is a very simple proposition connecting the linear inductance and capacity of any uniform conducting aerial circuit; namely, that the linear capacity in C. G. S. electrostatic units is the reciprocal of the external linear inductance in C. G. S. magnetic units. If l be the linear inductance of a uniform line in abhenrys per cm., counting only the effect of flux external to the conductor, and ignoring whatever additional inductance may be due to flux

* Artificial Lines for Continuous Currents in the Steady State by A. E. Kennelly. *Proceedings of the American Academy of Arts and Sciences.* Vol. 44, No. 4. Nov. 1908, pp. 97—130.

The Influence of Frequency on the Equivalent Circuits of Alternating Current Transmission Lines, by A. E. Kennelly. *Electrical World.* Jan. 21, 1909. Vol. 53, No. 3.

inside the wire; then if c be the linear capacity of the line in statfarads per cm., ignoring the capacity of the insulators, or other accidental condenser loads; then $l = 1/c$ or $c = 1/l$. This proposition depends on the fact that the velocity of electromagnetic wave propagation over any such aerial line is theoretically, $v = 3 \times 10^{10}$ cm./sec. very nearly and that, thus far, experimental observations have confirmed the theory. Moreover, it is known that this velocity of transmission $v = \frac{1}{\sqrt{l\,c}}$ cm/sec. where l and c are the linear inductance and capacity of the line both in magnetic units; *i.e.* in abhenrys per cm. and abfarads per cm. Consequently $c = 1/l\,v^2$ abfarads per cm.; but if we multiply the number of abfarads per cm. by v^2, we obtain, by definition, the same linear capacity expressed in statfarads per cm.; so that $c = 1/l$ statfarads per cm.

This proposition enables us, in any uniform distribution of overhead line conductors; unsplit or split, to choose whether we shall compute the external linear inductance of the system, or the linear capacity, and then the other quantity follows as a simple reciprocal. Correction must subsequently be made for the internal inductance of the conductors, and for any load condensers, or insulator capacities, in the system. In the case of the paper before us, it is the inductance of the new conductor arrangement which has been worked out; so that the capacity follows at once, and clearly increases as the inductance of the line has been diminished by the wire-splitting.

P. H. Thomas: Mr. H. G. Stott has made a valuable suggestion in connection with the starting of non-synchronous generators; namely, that the machines can be built for both synchronous and non-synchronous operation, the revolving field being provided with the ordinary direct-current magnetization, and with the short-circuit squirrel-cage winding in addition. The line could then be started up from the direct-current, independent-field excitation, and when the synchronous machines were started the field circuit could be opened and the short-circuit winding would make the machine operate as a non-synchronous generator.

In the discussion a good deal of time is spent on the terms "leading energy" and "lagging energy." Under the circumstances I do not think it worth while to spend much time in further discussing those points. Briefly, the reason I use the terms is this: the effect of the capacity is a leading current in regard to the generator; the effect of the inductance is a reactance voltage. They cannot be spoken of both as leading or lagging current, leading or lagging voltage; they are different in character but both represent energy, and they balance as energy. A volt cannot be balanced against an ampere, but the energy of one phase can be balanced against the energy of another phase. Both represent real energy, but not of course energy usable in the load circuits.

I wish to call attention to one very excellent equation or formula for the calculation of the line not referred to in the paper, and which takes account of distributed inductance and capacity. This is given by Dr. Kennelly in the *Electrical World* in 1895. This formula makes use of vectorial quantities and involves hyperbolic functions. While extremely simple in form and very advantageous in theoretical analysis, its use requires a knowledge of vectorial algebra and hyperbolic functions and is not nearly so simple for numerical calculations as would seem from its form.

I wish to say again, as stated in the paper, that the derivation here given is not intended as a novel mathematical procedure, but merely as a straightforward derivation by the ordinary mathematical processes in common use among the majority of engineers, of a practical formula easily understood and conveniently used in engineering calculations.

Mr. Mershon has raised the question whether the relatively small sized individual wires of the divided conductor method described in the first paper would mean serious loss directly into the atmosphere. While I have no actual data on which to base an opinion, I should expect a gain rather than a loss in this feature since the divided conductor acts, in other relations, in effect like a larger conductor.

T. Dalemont (by letter): The method set forth by Mr. P. H. Thomas for the calculation of high-tension lines with capacity presents the advantage of doing away with vector diagrams into which errors so easily creep. But in this calculation the exponential and hyperbolic functions are not eliminated. It occurs to me that it might be of interest to call attention to a very simple method for calculating lines where the capacity effects are not negligible, and where they are to be considered as evenly distributed along the line. This method was suggested by Mr. André Blondel,* as far back as 1906. He based it on the fact that at every point of a line fed at the extremities by a sine electromotive force, the oscillations of the electromotive force and the current are both sine, and that each of them is the result of two waves flowing in opposite directions. One of these two waves is produced at the generating point, while the other is a reflex of the first, after this latter has reached the extremity of the line. Passing along the line, they both undergo simultaneously a damping and a phase rotation resulting from the combined effect of the self-induction and the capacity.

The equations of the second degree, differential equations of the tension, and of the current at point x from the end of the line are well known, and their solutions are expressed as follows:

$$V = A_1\, e^{ax} \sin\ (p\, t + b\, x - \alpha) + B_1\, e^{-ax}\ \sin\ (p\, t - b\, x - \beta) \qquad (1)$$

*C. F. A. Blondel, Calcul designes avec self-induction et capacité. *Eclairage Electrique*, 1906, XLIX, p. 121. *Lumiere Electrique* T. I. (2d serie) p. 395.

In this, A and B are constant values and α, β are constant phase angles determined by the conditions at the extremities. On the other hand we have:

$$a = \frac{1}{\sqrt{2}} \sqrt{-p^2 c\, l + r\, g + \sqrt{(p^2\, l\, c - g\, r)^2 + (p\, c\, r + p\, l\, g)^2}}$$

$$b = \frac{1}{\sqrt{2}} \sqrt{p^2 c\, l - r\, g + \sqrt{(p^2\, l\, c - g\, r)^2 + (p\, c\, r + p\, l\, g)^2}}$$

In this r is the resistance per unit of length,
L the inductance, " "
c the capacity " "
g the conductance (corresponding for instance to the losses by ionization)
p the pulsation of the current.

The foregoing may be easily verified by differentiating the equation of V relatively to x and t and introducing the values obtained into the differential equation (1) given by Mr. Thomas, (equation (1)). Another term "$r\, g\, v$" must however be added to the second member if the conductance is negligible.

The developments of the transformation are the same as those which Mr. Thomas mentions, and the equations are not altogether different from his. We see for instance that $b = \frac{p}{v'}$ where v' is one of the values of v deduced from equation 10.

In the same way, we find for the current

$$i = \frac{A_1}{m} e^{ax} \sin (p\, t + b\, x - \alpha + \gamma) \frac{B_1\, e^{-ax}}{m} \sin (p\, t - b\, x - \beta + \gamma) \quad (2)$$

where

$$m = \sqrt{\frac{a^2 + b^2}{g^2 + p^2\, c^2}}$$

$$\gamma = \operatorname{arc} t\, g \frac{p\, c}{g} - \operatorname{arc} t\, g \frac{b}{a}$$

m is therefore a factor of impedance. We may thus express the tension at any point at a distance x from the receiving end (and not from the generator end) by the sum of two vectors:

$$U_x = (A_1\, e^{ax})_{(+bx)} + (B_1\, e^{-ax})_{(-bx)}$$

The vectors $A_1 e^{ax}$ and $B_1 e^{-ax}$ having turned respectively through an angle of $(+b\, x)$ and $(-b\, x)$ from the position which each occupied for $x = 0$.

In the same way, the current at point x will give:

$$I_x = \frac{1}{m} [(A_1\, e^{ax})_{(bx+\gamma)} - (B_1\, e^{-ax})_{(-bx+\gamma)}$$

To find the values of A_1 and B_1 we must make $x = 0$, giving $U_x = U_1$ and $I_x = I_1$ (1); U_1 is the star tension and I_1 the current in each wire at the receiving end.

We thus find:

$$[A_1] = \left[\frac{U_1}{2}\right] + \left[\frac{m\ I_1}{2}\right] \quad (\text{turned} - \gamma)$$

$$[B_1] = \left[\frac{U_1}{2}\right] - \left[\frac{m\ I_1}{2}\right] \quad (\text{turned} - \gamma)$$

These two expressions enable us to determine A_1 and B_1 by a simple diagram.

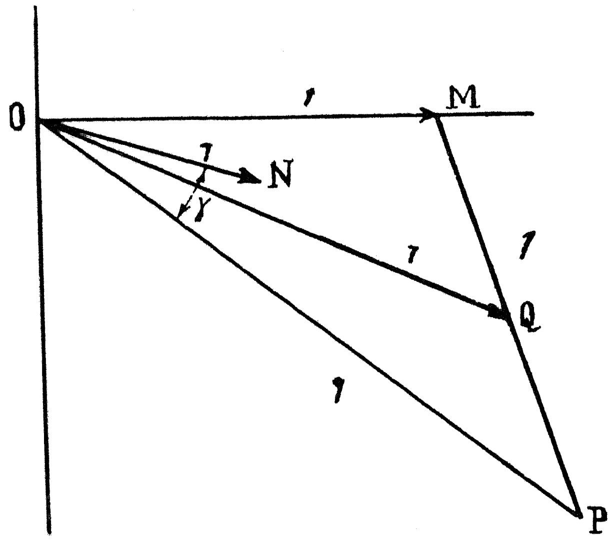


Fig. 1

In Fig. 1 $O\ M = U_1$; $O\ N = I_1$; $O\ P = m\ I_1$ turned through the angle $-\gamma$ and we obtain A_1 and B_1 in the lines $O\ Q$ and $M\ Q$, which are half the sum and half the difference of the vectors U_1 and $m\ I_1$.

First case. Line at no load. $I_1 = 0$ (Fig. 2). The sum of A_1 and B_1 is U_1, and the difference, 0. Then $A_1 = B_1 = \dfrac{U_1}{2}$. Let $O\ M = A_1$, $M\ O = N\ M = B_1$ and $O\ N = U_1$. To find

* I would here call attention to the fact that the equations are established on the supposition of a common return conductor of nil impedance, and that consequently U_1 is not the tension between the lines, but the tension between each wire and the centre of the assumed return.

the values of the tension and the current at point x, we need only recall what has been said above.

In Fig. 2 $O\,p = \frac{U_1}{2}\,e^{ax}$, vector turned through the angle $(+b\,x)$ from 0, is drawn, and in the same way $P\,q = \frac{U_1}{2}\,e^{-ax}$. In carrying $p\,r = p\,q$, we obtain $o\,r = m\,I_x$, and finally $o\,s' = \frac{o\,r}{m}$ turned through an angle of γ, gives the value of I_x.

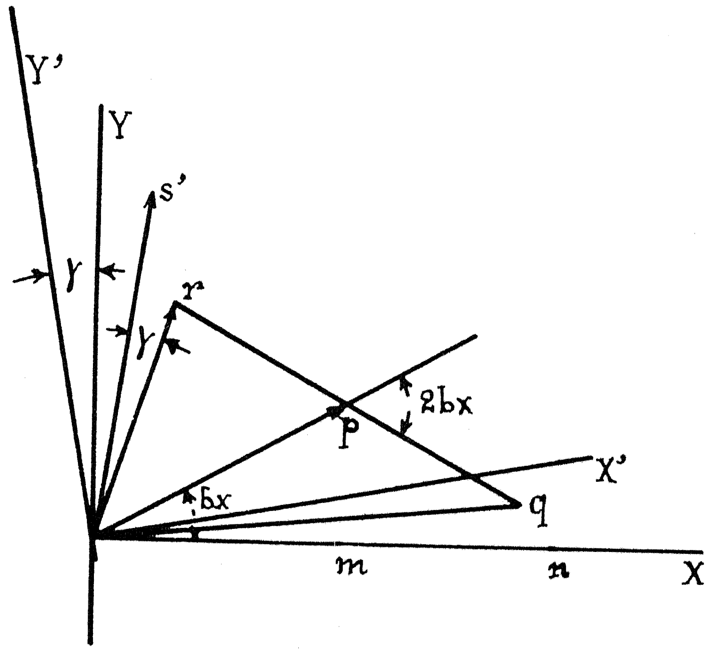


FIG. 2

Second case. Line at short-circuit. $U_1 = 0$ (Fig. 3). Let $O\,N = I_1$ the short-circuit current, and $A_1 = B_1 = O\,M = \frac{I_1}{2}$.

We obtain as above, at distance x from the receiving end,

$$O\,p = \frac{I_1}{2}\,e^{ax} \qquad P\,r = \frac{I_1}{2}\,e^{-ax}$$

The vector $O\,q' = J_x$ gives the short-circuit current in size and phase at point x.

$O\,r = \frac{1}{m}\, V_x$. V_x is the short-circuit tension at point x, but this vector must be multiplied by m and turned through the angle $-\gamma$ to obtain V_x in size and phase.

It will be noticed that the vector A_1 may be considered as the resultant of the two components $\frac{U_1}{2}$ and $\frac{m\,I_1}{2}$, the latter turned through an angle of $-\gamma$.

The same applies to B_1.

Finally U_x may be considered as the resultant of 4 vectors:

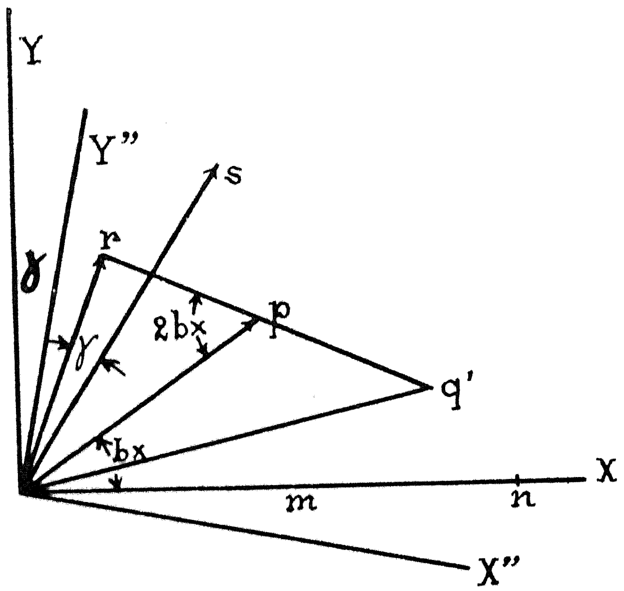


FIG. 3

$\frac{U_1}{2}\, e^{ax}$ turned through the angle $+b\,x$

$\frac{U_1}{2}\, e^{-ax}$ " " " " $-b\,x$

$\frac{m\,I_1}{2}\, e^{ax}$ " " " " $+b\,x$

$\frac{m\,I_1}{2}\, e^{-ax}$ " " " " $-b\,x$

It will be seen in any case, that if we make $A_1 = B_1 = \frac{U_1}{2}$

the two first terms would have for resultant, the tension obtained at the distance x when $I_1 = 0$ (at no load), which would be.

$$(U_x') = (A_1\, e^{ax})\ (b\, x) - (B_1\, e^{-ax})\ (-b\, x).$$

In the same way, if we make $A_1 = B_1 = \frac{I_1}{2}$ that is $U_1 = 0$, (short-circuit), the resultant of the two last terms will be:

$$(U''_x) = (A_1\, e^{ax})_{(bx)} - (B_1\, e^{-ax})_{(-bx)}$$

By a similar method, the current is brought back to its two components at no load and at short-circuit.

We may finally determine the resultant vectors of the current and tension by placing the two Figs. 2 and 3 on each other and giving them a common pole.

The analytical expressions of the vectors may be easily deduced from these diagrams when we use the auxiliary axis, which are drawn in the Figs. 2 and 3. The components of the tension at point x are $o\,q$ in Fig. 2 and $o\,s$ in Fig. 3. where,

$$O\,q = U_1 \sqrt{\frac{\cos h.\ 2\,a\,x - \cos 2\,b\,x}{2}}$$

and

$$o\,s = m\,I_1 \sqrt{\frac{\cos h.\ 2\,a\,x + \cos 2\,b\,x}{2}}$$

and the components of the current are $o\,q'$ in Fig. 3 and $o\,s'$ in Fig. 2.

$$o\,q' = I_1 \sqrt{\frac{\cos h.\ 2\,a\,x - \cos 2\,b\,x}{2}}$$

$$o\,s' = \frac{U_1}{m} \sqrt{\frac{\cos h.\ 2\,a\,x + \cos 2\,b\,x}{2}}$$

The vector angles may also be determined:

(angle between $o\,q$ and $o\,s$) $\phi = \text{arc}\ t\,g\left(\frac{\sin 2\,b\,x}{\sin h\ 2\,a\,x}\right) - (\gamma + \phi_1)$

(angle between $o\,q'$ and $o\,s$) $\Sigma = \text{arc}\ t\,g\left(\frac{\sin 2\,b\,x}{\sin h\ 2\,a\,x}\right) + \gamma + \phi_1$

In these expressions ϕ_1 is the phase angle at the receiving end, or in other words $\cos\phi_1$ is the power-factor of the receiving circuit. All these expressions will be met with in calculating the following line.

. . .

Let us take the line which Mr. Thomas gives as an example:

Transmitted power. . . . . 40,000 kw.
Length of transmission. . . . . 700 miles
Frequency. . . . . 25 cycles
Tension between two wires at receiving end. . . . 150,000 volts
Power-factor. 918
Distance of wires. . . . . 12 ft.

In his calculation, Mr. Thomas replaced the three-phase line by a single-phase line which transmitted half the power with the same tension, as that existing between wires. In the present calculation, the three-phase line was replaced as already mentioned, by a single-phase line, one conductor of which was similar to each of the conductors in the three-phase line, while the other had a nil resistance. We see that this should give the same results.

Let V_1 be the tension between wires of the three-phase line,
I_1 the current in each wire
$\cos \phi_1$ the power-factor of the distributed power.

Half of the power transmitted by the single-phase will be:

$$\frac{\sqrt{3}}{2} V_1 I_1 \cos \phi_1$$

With the tension V_1 between wires, the current will be:

$$I_1' = \frac{\sqrt{3}}{2} I_1$$

and the tension drop:

$$[V'] = [I_1' z'] = \left[\frac{\sqrt{3}}{2} I_1 z'\right]$$

In the case of a single-phase line with an assumed return of nil resistance, the tension here called "star tension" is:

$V_1'' = \dfrac{V_1}{\sqrt{3}}$ and the current in the wire is I_1. The tension drop is $[V''] = I_1 z'' = \frac{1}{2} I_1 z'$, the same therefore as in the preceding case.

We first calculated the current in each wire

$$I_1 = 167.5 \text{ amperes,}$$

and the star tension at the receiving end, $U_1 = 86.700$ volts. We will adopt the same conductor as that employed by Mr. Thomas.

The resistance r per mile of line is 0.085 ohm.
" reactance " " " " 0.309 "
" impedance " " " " 0.321 "

Admittance per mile, $w = \sqrt{g^2 + p^2 c^2}$

If g is neglected, we have: $w = p\,c = 2.386 \quad 10^{-6}$

We next calculated the auxiliary constant:

$$q = p^2\, l\, c - g\, r \quad \text{or } p^2\, l\, c \quad \text{if } g = 0$$

In the present case, we have $q = 0.737 \quad 10^{-6}$
The damping coefficient:

$$a = \sqrt{\frac{w\, z - q}{2}} = \sqrt{\frac{2.386\ 10^{-6} \times 0.321 - 0.737\ 10^{-6}}{2}}$$

$$= 0.12\ 10^{-3}$$

$$b = \sqrt{\frac{w\, z + q}{2}} = 0.865\ 10^{-3}$$

Coefficient of influence of the current on the electromotive force: $m = \sqrt{\frac{z}{w}} = 366.$

Auxiliary factors: (x being here the net length of the transmission, 700 miles).

$$\alpha = \sqrt{\frac{\cos h\ 2\, a\, x + \cos 2\, b\, x}{2}} = 0.825$$

$$\beta = \sqrt{\frac{\cos h\ 2\, a\, x - \cos 2\, b\, x}{2}} = 0.572$$

Auxiliary angles:

$$\gamma = \text{arc}\ t\, g\, \frac{p\, c}{g} - \text{arc}\ t\, g\, \frac{b}{a} = \frac{\pi}{2} - \text{arc}\ t\, g \left(\frac{432}{59}\right) = 8^\circ$$

$$g = \text{perditance} = 0$$

$$\sigma = \text{arc}\ t\, g \left[\frac{\sin 2\, b\, x}{\sin h\ 2\, a\, x}\right] = \text{arc}\ t\, g \left[\frac{0.565}{0.87}\right] = 79^\circ\ 40'$$

We find therefore the following auxiliary angles:

$$\phi = \sigma - \gamma = 71^\circ\ 40' \text{ and } \Sigma = \phi - \phi_1 = 48^\circ\ 10'$$
$$\Sigma = \sigma + \gamma = 87^\circ\ 40' \text{ and } X = \Sigma + \phi_1 = 111^\circ\ 10'$$

and finally,

$$\varepsilon = \text{arc}\ (t\, g\, h\, y\, p\, a\, x\, t\, g\, b\, x) = \text{arc} = 30^\circ\ 20'$$

Resulting vectors at beginning of line, transmitting tension at the end:

$U_1 = 86{,}700$ volts. $\qquad I_1 = 167$ amperes, 5

The star tension at the beginning of the line corresponding to

$$U_1 = U_0 = \alpha\, U_1 = 86.700 \times 0.825 = 71{,}500 \text{ volts}$$

The tension drop produced by the distributing current is

$$V_0 = 366 \times 0.572 \times 167.5 = 35{,}000 \text{ volts}$$

The angle δ of the tension U_0 and of the tension at the beginning of the line is given by:

$$tg\,\delta = \frac{V_0 \sin \Sigma}{V_0 + U_0 \cos \Sigma} = 0.274 \qquad \delta = 15° \, 20'$$

The star tension at the beginning of the line is therefore $U_0' = \dfrac{V_0 \sin \Sigma}{\sin \delta} = 98{,}500$ volts and the tension between the wires is 170,500 volts. The current at the beginning, corresponding to that of the terminal point of the line is likewise:

$$I_0 = \alpha I_1 = 0.825 \times 167.5 = 138.5 \text{ amperes.}$$

The charging current at no load is:

$$J_0 = \frac{\beta}{m} U_1 = \frac{0.572}{366} \times 86{,}700 = 135 \text{ amperes}$$

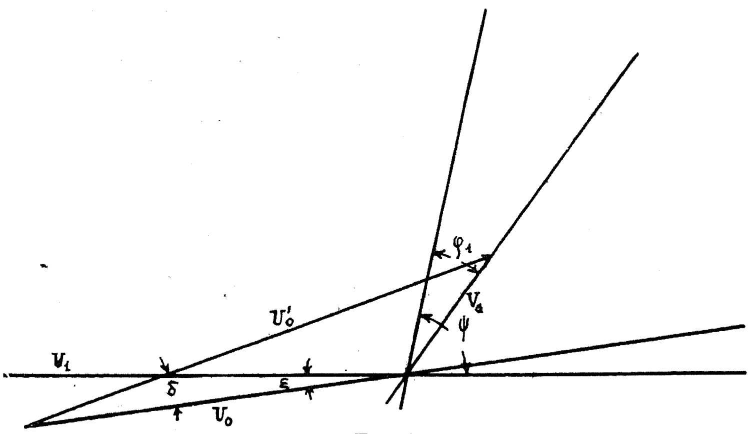


FIG. 4

On the other hand, we have:

$$tg\,\delta' = \frac{J_0 \sin X}{I_0 + J_0 \cos X} = 197 \; \delta' = 54°$$

The total resulting current at the beginning of the line is therefore:

$$J_0' = \frac{J_0 \sin X}{\sin \delta'} = 154.5 \text{ amperes.}$$

The power-factor at the generator end is $\cos \phi_0 = 0.90$.

Fig. 4 and 5 give the diagrams of the tensions and currents. We might note that a similar construction could be made for any value of x, that is to say for any point of the line.

This method would be quite simple, if all the hyperbolic

functions were eliminated. To do this, their development in series might be made use of as has been done. This, however, introduces an approximation in the results.

Instead of maintaining the hyperbolic functions or their development in series, they may be easily transformed as follows:

$$\alpha - \sqrt{\frac{\cos h\, 2\, a\, x + \cos 2\, b\, x}{2}}$$

If we introduce into this, the value of cos hy. $2\, a\, x$ we easily find:

$$\alpha^2 = \frac{e^{4ax} + 1 + 2\, e^{2ax} \cos 2\, b\, x}{4\, e^{2ax}}$$

and finally,

$$\alpha = \sqrt{\frac{(e^{2ax} + \cos 2\, b\, x)^2 + \sin^2 2\, b\, x}{4\, e^{2ax}}}$$

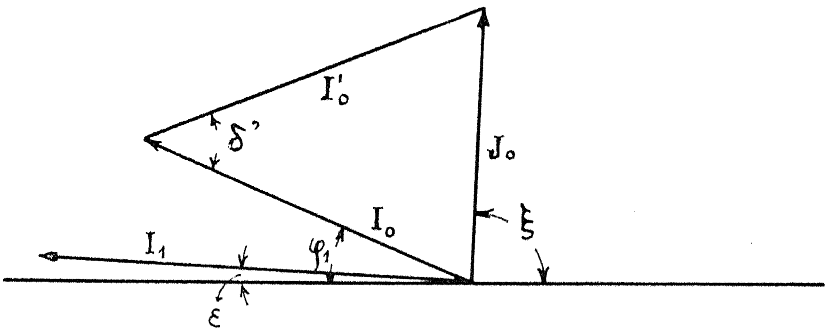


Fig. 5

In this expression, the exponential function e^{2ax} may be calculated without any difficulty. Moreover, this expression suggests a simple graphical method to determine the value of α if the exponential curve $Y = e^x$ has first been drawn in short limits of the value of x.

In the same way, we have:

$$\beta = \sqrt{\frac{(e^{2ax} - \cos 2\, b\, x)^2 + \sin^2 2\, b\, x}{4\, e^{2ax}}}$$

and

$$t\, g\, \sigma = \frac{2\, e^{2ax}}{e^{4ax} - 1} \sin 2\, b\, x$$

In these expressions the hyperbolic functions have disappeared.

Charles P. Steinmetz (by letter): With the very long high-voltage transmission lines that are now being considered, it is not safe to calculate the performance of the line by the approximate

equations that have been used for the lines of moderate voltage and moderate length; but the numerical calculations should be made by using the complete equations of the transmission line, or at least the approximate calculations should be checked in a number of points by the exact equations. Although the complete equations of the transmission line appear complicated on first sight, numerical calculations can be made easily and rapidly by a systematic arrangement in tabulated form. However, even with 60-cycle lines several hundred miles long, satisfactory accuracy can still be derived by the approximation of substituting for the exponential functions of the complete equations the first terms of their series. The calculation is essentially simplified thereby.

I. Investigation

The voltage $\dot{E}$ and current $\dot{I}$ at any point l of the transmission line is given, in the usual representation by complex quantities, by the equations.*

$$\begin{aligned} \dot{I} &= Y\{\dot{C}_1 \varepsilon^{+\alpha l} (\cos \beta l - j \sin \beta l) - \dot{C}_2 \varepsilon^{-\alpha l} (\cos \beta l + j \sin \beta l)\} \\ \dot{E} &= V\{\dot{C}_1 \varepsilon^{+\alpha l} (\cos \beta l - j \sin \beta l) + \dot{C}_2 \varepsilon^{-\alpha l} (\cos \beta l + j \sin \beta l)\} \end{aligned} \qquad (1)$$

where:

$$\begin{aligned} Z &= r - j x = \text{line impedance per unit length,} \\ Y &= g - j b = \text{shunted admittance of the line per unit length,} \\ z &= \sqrt{r^2 + x^2} \\ y &= \sqrt{g^2 + b^2} \\ V &= \alpha - j \beta \\ \alpha &= \sqrt{\tfrac{1}{2} (z y + r g - x b)} \\ \beta &= \sqrt{\tfrac{1}{2} (z y - r g + x b)} \end{aligned} \qquad (2)$$

the distance l is counted towards rising power; that is, from the receiving end towards the generator, and $\dot{C}_1$ and $\dot{C}_2$ are integration constants.

Choosing as unit length the entire line, $l = 0$ gives $\dot{I}_0$ and $\dot{E}_0$ at the receiving end, and $l = 1$ gives $\dot{I}_1$ and $\dot{E}_1$ at the generator end of the line, and Z is the total impedance, Y the total shunted admittance of the entire line.

Neglecting the shunted conductance; that is, assuming the power consumed by leakage and electrostatic losses in the line as negligible compared with that consumed by the effective line resistance r—as is always done—gives:

$$\begin{aligned} \alpha &= \sqrt{\tfrac{1}{2} b (z - x)} \\ \beta &= \sqrt{\tfrac{1}{2} b (z + x)} \\ Y &= - j b \end{aligned} \qquad (3)$$

* *Transient Phenomena.* Section III, Par. 8, equations (21)

Substituting now into equations (1) the values:

$$\begin{aligned} V \underset{\cdot}{C}_1 &= \underset{\cdot}{A}_1 \\ V \underset{\cdot}{C}_2 &= \underset{\cdot}{A}_2 \end{aligned} \qquad (4)$$

$$\frac{V}{Y} = Z_0 = r_0 + j\, x_0$$

where, by (2) and (3):

$$\begin{aligned} r_0 &= \frac{\beta}{b} \\ x_0 &= \frac{\alpha}{b} \end{aligned} \qquad (5)$$

and denoting:

$$Z_0 \underset{\cdot}{I} = \underset{\cdot}{U} \qquad (6)$$

it is:

$$\begin{aligned} \underset{\cdot}{E} &= \underset{\cdot}{A}_1\, \varepsilon^{+\alpha l} (\cos \beta\, l - j \sin \beta\, l) + \underset{\cdot}{A}_2\, \varepsilon^{-\alpha l} (\cos \beta\, l + j \sin \beta\, l) \\ \underset{\cdot}{U} &= \underset{\cdot}{A}_1\, \varepsilon^{+\alpha l} (\cos \beta\, l - j \sin \beta\, l) - \underset{\cdot}{A}_2\, \varepsilon^{-\alpha l} (\cos \beta\, l + j \sin \beta\, l) \end{aligned} \qquad (7)$$

If then $\underset{\cdot}{E}_0$ and $\underset{\cdot}{I}_0$ denote voltage and current at the receiving end, for $l = 0$, and $\underset{\cdot}{E}_1$ and $\underset{\cdot}{I}_1$ denote voltage and current at the generator end, for $l = 1$, and:

$$\begin{aligned} Z_0 \underset{\cdot}{I}_0 &= \underset{\cdot}{U}_0 \\ Z_0 \underset{\cdot}{I}_1 &= \underset{\cdot}{U}_1 \end{aligned} \qquad (8)$$

equations (7) give:

Receiving end: $l = 0$,

$$\begin{aligned} \underset{\cdot}{E}_0 &= \underset{\cdot}{A}_1 + \underset{\cdot}{A}_2 \\ \underset{\cdot}{U}_0 &= \underset{\cdot}{A}_1 - \underset{\cdot}{A}_2 \end{aligned} \qquad (9)$$

Generator end: $l = 1$,

$$\begin{aligned} \underset{\cdot}{E}_1 &= \underset{\cdot}{A}_1\, \varepsilon^{+\alpha} (\cos \beta - j \sin \beta) + \underset{\cdot}{A}_2\, \varepsilon^{-\alpha} (\cos \beta + j \sin \beta) \\ \underset{\cdot}{U}_1 &= \underset{\cdot}{A}_1\, \varepsilon^{+\alpha} (\cos \beta - j \sin \beta) - \underset{\cdot}{A}_2\, \varepsilon^{-\alpha} (\cos \beta + j \sin \beta) \end{aligned} \qquad (10)$$

Since α is a small quantity even in very long lines, because in equation (3), b is small at 25 cycles, while $(z - x)$ is small at 60 cycles, $\varepsilon^{\pm\alpha}$ can be replaced by the first terms of its series:

$$\varepsilon^{\pm\alpha} = 1 \pm \alpha + \frac{\alpha^2}{3} \pm \frac{\alpha^3}{6} + \frac{\alpha^4}{24} \pm \ldots \qquad (11)$$

that is, by:

$$\varepsilon^{\pm \alpha} = 1 \pm \alpha \tag{12}$$

or, if a greater accuracy is desired, by:

$$\varepsilon^{\pm \alpha} = 1 + \frac{\alpha^2}{2} \pm \alpha \tag{13}$$

Substituting (13) into (10), rearranging, and then substituting (9), gives:

$$\begin{aligned} \underset{\cdot}{E}_1 &= \underset{\cdot}{E}_0 \, (a_1 - j\, a_2) + \underset{\cdot}{U}_0 \, (c_1 - j\, c_2) \\ \underset{\cdot}{U}_1 &= \underset{\cdot}{U}_0 \, (a_1 - j\, a_2) + \underset{\cdot}{E}_0 \, (c_1 - j\, c_2) \end{aligned} \tag{14}$$

where:

$$\begin{aligned} a_1 - j\, a_2 &= \left(1 + \frac{\alpha^2}{2}\right) \cos \beta - j\, \alpha \sin \beta \\ c_1 - j\, c_2 &= \alpha \cos \beta - j \left(1 + \frac{\alpha^2}{2} \sin \beta \right) \end{aligned} \tag{15}$$

are line constants, independent of voltage, current, and power-factor.

Substituting (8) into (14) gives:

$$\begin{aligned} \underset{\cdot}{E}_1 &= \underset{\cdot}{E}_0 \, (a_1 - j\, a_2) + \underset{\cdot}{I}_0 \, (b_1 - j\, b_2) \\ \underset{\cdot}{I}_1 &= \underset{\cdot}{I}_0 \, (a_1 - j\, a_2) + \underset{\cdot}{E}_0 \, (d_1 - j\, d_2) \end{aligned} \tag{16}$$

where:

$$\begin{aligned} b_1 - j\, b_2 &= Z_0 \, (c_1 - j\, c_2) \\ d_1 - j\, d_2 &= \frac{c_1 - j\, c_2}{Z_0} \end{aligned} \tag{17}$$

hence, by (4), (5), and (15):

$$\begin{aligned} b_1 - j\, b_2 &= \frac{(\beta + j\, \alpha) \left(\alpha \cos \beta - j \left(1 + \frac{\alpha^2}{2}\right) \sin \beta \right)}{b} \\ &= \frac{\alpha \left(\beta \cos \beta + \left(1 + \frac{\alpha^2}{2}\right) \sin \beta \right) - j \left(\beta \left(1 + \frac{\alpha^2}{2}\right) \sin \beta - \alpha^2 \cos \right)}{b} \end{aligned} \tag{18}$$

$$d_1 - j\,d_2 = \frac{b \left\{ \alpha \cos\beta - j\left(1 + \frac{\alpha^2}{2}\right)\sin\beta \right\}}{\beta + j\,\alpha}$$

$$= \frac{b\,(\beta - j\,\alpha) \left\{ \alpha \cos\beta - j\left(1 + \frac{\alpha^2}{2}\right)\sin\beta \right\}}{\beta^2 + \alpha^2}$$

and as by (3):

$$\beta^2 + \alpha^2 = b\,z$$

it is:

$$d_1 - j\,d_2 = \frac{\alpha\left(\beta\cos\beta - \left(1 + \frac{\alpha^2}{2}\right)\sin\beta\right) - j\left(\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta + \alpha^2\cos\beta\right)}{z} \qquad (19)$$

Thus, if:

e_0 = voltage at the receiving end of line,
i_0 = total current, consisting of the power current i_0' and the reactive current i_0'' (counted positive for lag), that is:

$$\dot{I}_0 = i_0' + j\,i_0''$$

voltage and current at the generator end of the line are given by the equations (16) as:

$$\begin{aligned} \dot{E}_1 &= e_0\,(a_1 - j\,a_2) + (i_0' + j\,i_0'')\,(b_1 - j\,b_2) \\ \dot{I}_1 &= (i_0' + j\,i_0'')\,(a_1 - j\,a_2) + e_0\,(d_1 - j\,d_2) \end{aligned} \qquad (20)$$

If in equations (1) the sign of l is reversed, the distance is counted toward rising power; that is, $l = 0$ gives generator voltage and current, e_1 and i_1, $l = 1$ gives receiving voltage and current, e_0 and i_0. The only change resulting herefrom is a reversal of the sign of α and β in equations (10) and (15), and thus a change of the sign of the second terms of (16) and (20):

$$\begin{aligned} \dot{E}_0 &= e_1\,(a_1 - j\,a_2) - (i_1' + j\,i_1'')\,(b_1 - j\,b_2) \\ \dot{I}_0 &= (i_1' + j\,i_2')\,(a_1 - j\,a_2) - e_1\,(d_1 - j\,d_2) \end{aligned} \qquad (21)$$

II. Conclusions

In a long-distance transmission line, voltage $\dot{E}_1$, current $\dot{I}_1$, and power-factor at the generator end can be accurately calculated from the voltage $\dot{E}_0$, current $\dot{I}_0$ and power-factor at the receiving end, and inversely, by the simple equations:

1. Generator voltage:

$$\left.\begin{array}{l} \underset{\cdot}{E}_1 = e_0\,(a_1 - j\,a_2) + (i_0' + j\,i_0'')\,(b_1 - j\,b_2) \\ \text{Generator current:} \\ \underset{\cdot}{I}_1 = (i_0' + j\,i_0'')\,(a_1 - j\,a_2) + e_0\,(d_1 - j\,d_2) \end{array}\right\} \qquad (22)$$

where:

e_0 = voltage,
i_0' = power current,
i_0'' = reactive current at receiving end of line.

2. Receiver voltage:

$$\left.\begin{array}{l} E_0 = e_1\,(a_1 - j\,a_2) - (i_1' + j\,i_2'\,(b_1 - j\,b_2) \\ \text{Receiver current:} \\ \underset{\cdot}{I}_0 = (i_1' + j\,i_1'')\,(a_1 - j\,a_2) - e_1\,(d_1 - j\,d_2) \end{array}\right\} \qquad (23)$$

where:

e_1 = voltage,
i_1' = power current,
i_1'' = reactive current at generator end of line, and:

$a_1 - j\,a_2$; $b_1 - j\,b_2$; $d_1 - j\,d_2$ are line constants, which are calculated once for all, and then permit the calculation of the performance of the line under all conditions of voltage, load and power-factor, from above equations (22) and (23).

These line constants are:

$$a_1 - j\,a_2 = \left(1 + \frac{\alpha^2}{2}\right) \cos\beta^\circ - j\,\alpha\sin\beta^\circ \qquad (15)$$

$$b_1 - jb_2 = \frac{\alpha\left(\beta\cos\beta^\circ + \left(1 + \frac{\alpha^2}{2}\right)\sin\beta^\circ\right) - j\left(\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta^\circ - \alpha^2\cos\beta^\circ\right)}{b} \qquad (18)$$

$$d_1 - j\,d_2 = \frac{\alpha\left(\beta\cos\beta^\circ - \left(1 + \frac{\alpha^2}{2}\right)\sin\beta^\circ\right) - j\left(\beta\left(1 + \frac{\alpha^2}{2}\right)\sin\beta^\circ + \alpha^2\cos\beta^\circ\right)}{z} \qquad (19)$$

where:

$$\left.\begin{array}{l} \alpha = \frac{1}{2}\sqrt{b\,(z - x)} \\ \beta = \frac{1}{2}\sqrt{(z + x)} \end{array}\right. \qquad (24)$$

and in degrees:

$\beta^\circ = 57.3\,\beta$

and:

$$\left.\begin{aligned} r &= \text{resistance,} \\ L &= \text{inductance,} \\ C &= \text{capacity of total line; thus if:} \\ f &= \text{frequency,} \end{aligned}\right\} \quad (25)$$

$$\left.\begin{aligned} x &= 2\pi f L = \text{line reactance,} \\ b &= 2\pi f C = \text{shunted line susceptance,} \\ z &= r^2+x^2 \ \ = \text{line impedance.} \end{aligned}\right\} \quad (26)$$

The possible error of the calculation is of the magnitude $\frac{\alpha^3}{6}$, hence, very small.

III. Instance

As instance may be considered a 180-mile, three-phase transmission line, consisting of three wires No. 000 B. & S. G., 10 ft. between wires, operated at 60 cycles.

The line constants are:

$$r = 180 \times 0.33 = 60 \text{ ohms per line.}$$

$$L = 180\left(0.74 \log \frac{D}{R} + 0.08\right) = 383 \text{ mh.}$$

(where $D = 10$ ft. $= 120$ ins. $=$ distance between conductors, $R = 0.205$ ins. $=$ radius of conductor.)

$$\begin{aligned} f &= 60 \text{ cycles, hence:} \\ x &= 2\pi f L = 145 \text{ ohms,} \\ z &= r^2+x^2 = 157 \text{ ohms.} \end{aligned}$$

$$C = 180 \times \frac{0.041}{\log \frac{D}{R}} \text{ mf., hence:}$$

$$b = 2\pi f C = 10^{-3} \text{ mhos.}$$

This gives:

$$\begin{aligned} \alpha &= 0.074 \\ \beta &= 0.389 = 22.3° \end{aligned}$$

and:

$$\begin{aligned} a_1 - j\,a_2 &= 0.928 - 0.028\,j \\ b_1 - j\,b_2 &= (0.055 - 0.143\,j)\ 10^{+3} \\ d_1 - j\,d_2 &= (-0.010 - 0.974\,j)\ 10^{-3} \end{aligned}$$

A. Determine generator voltage, current, and power-factor required to deliver 95,000 volts at non-inductive load.

95 kv. between lines gives $95 \div \sqrt{3} = 55$ kv. per line. Non-inductive load means $i_0'' = 0$.

Thus substituting: $e_0 = 55$, $i_0'' = 0$ into equations (22) gives:

$$\dot{E}_1 = 55\ (0.928 - 0.028j) + i_0'\ (0.055 - 0.143j).$$
$$= (50.9 - 1.5j) + i_0'\ (0.055 - 0.143j) \text{ kv.}$$
$$\dot{I}_1 = i_0'\ (0.928 - 0.028j) - 55\ (0.010 + 0.974j).$$
$$= i_0\ (.0928 - 0.028j) - (0.5 + 53.5j) \text{ amperes.}$$

Hence, for: i_0 =	0	40	80	120	160 amperes
$E_1 = e_1' - j\,e_1''$ =	$50.9 - 1.5j$	$53.1 - 7.2j$	$55.3 - 12.9j$	$57.5 - 18.7j$	$59.7 - 24.4j$
$I_1 = i_1' - j\,i_1''$ =	$0.5 - 53.5j$	$36.6 - 54.6j$	$73.7 - 55.7j$	$110.9 - 56.9j$	$140. - 58.j$
hence, absolute:					
$e_1 = \sqrt{e_1'^2 + e_1''^2}$ =	50.9	53.6	56.8	60.5	64.5 kv.
$e_1\sqrt{3}$ =	88.3	92.8	98.5	104.8	112.0 kv.
$i_1 = \sqrt{i_1'^2 + i_1''^2}$	53.5	65.6	92.4	125.8	159.0 amp.
Input: $P_1 = e_1'\,i_1' + e_1''\,i_1''$ =	55	2330	4790	7430	10,250 kw.
Apparent input: $Q_1 = e_1\,i_1$ =	2720	3520	5260	7610	10,270 kva.
Output: $P_0 = e_0\,i_0$ =	0	2200	4400	6600	8,800 kw.
Power-factor: $\frac{P_1}{Q_1}$ =	2.0	66.3	91.1	97.6 lag	99.8% lead
Efficiency: $\frac{P_0}{P_1}$ =	0	94.4	91.8	85.8	88.8%

The possible error of the calculation is of the magnitude:

$$\frac{\alpha^3}{6} = 0.00007, \text{ or } 0.007\%,$$

thus far beyond the accuracy of the numerical calculation.

Generator volts and amperes are plotted in Fig. 1.

B. Determine generator voltage, current and power-factor, required to deliver 95,000 volts at 90 per cent power-factor lag, and at 90 per cent power factor lead.

$$e_0 = 55$$
$$i_0' = 0.90\ i_0$$
$$i_0'' = \pm 0.44\ i_0$$

Thus:

$$\dot{E}_1 = 55\ (0.928 - 0..28j) + i_0\ (0.90 \pm 0.44j)\ (0.055 - 0.143j)$$

$$= (50.9 - 1.5j) + i_0 \begin{cases} 0.112 - 0.126j & \text{lag.} \\ 0.013 - 0.131j & \text{lead.} \end{cases}$$

$$\dot{I}_1 = i_0\ (0.90 \pm 0.44j)\ (0.928 - 0.028j) - 55\ (0.10 + 0.974j)$$

$$= i_0 \begin{Bmatrix} 0.847 + 0.383j \\ 0.823 - 0.433j \end{Bmatrix} - (0.5 + 53.5j) \begin{matrix} \text{lag.} \\ \text{lead.} \end{matrix}$$

Hence, for: *Lag*:					
$i_0 =$	0	40	80	120	160 amperes
$I_0 = i_0' + j\, i_0'' =$	0	$36+17.6j$	$72+35.2j$	$108+52.8j$	$144+70.4j$
$E_1 = e_1' - j\, e_1'' =$	$50.9-1.5j$	$55.4-6.5j$	$59.9-11.6j$	$64.3-16.6j$	$68.8-21.7j$
$I_1 = i_1' - j\, e_1'' =$	$-0.5-53.5j$	$33.4-38.2j$	$67.3-22.9j$	$101.1-7.5j$	$134+7.8j$
$e_1 = \sqrt{e_1'^2+e_1''^2} =$	50.9	55.8	61.0	66.4	72.1 kv.
$e_1 \sqrt{3} =$	88.3	96.8	105.6	115.3	125.0
$i_1 = \sqrt{i_1'^2+i_1''^2} =$	53.5	50.7	71.1	101.4	134.3 amp.
$P_1 = e_1'\, i_1' + e_1''\, i_1'' =$	55	2100	4310	6620	9060 kw.
$Q_1 = e_1\, i_1 =$	2720	2830	4340	6730	9690 kva.
$P_0 = e_0\, i_0' =$	0	1980	3960	5940	7920 kw.
Power-factor: $\frac{P_1}{Q_1} =$	2.0	74.3	99.3	98.4	93.5%
Efficiency: $\frac{P_0}{P_1} =$	0	94.3	91.9	89.7	87.4%
			lead<———	———>lag	

Lead:					
$i_0 =$	0	40	80	120	160 amperes
$I_0 = i_1' - j\, i_0'' =$	0	$36-17.6j$	$72-35.2j$	$108-52.8j$	$144-70.4j$
$E_1 = e_1' - j\, e_1'' =$	$50.9-1.5j$	$50.4-6.7j$	$49.9-12j$	$49.3-17.2j$	$48.8-22.5j$
$I_1 = i_1' - j\, i_1'' =$	$-0.5-53.5j$	$32.4-70.8j$	$65.3-88.1j$	$98.3-105.5j$	$131.2-122.8j$
$e_1 = \sqrt{e_1'^2+e_1''^2} =$	50.9	50.9	51.3	52.2	53.7 kv.
$e_1 \sqrt{3} =$	88.3	88.3	88.9	90.5	93.1 kv.
$i_1 = \sqrt{i_1'^2+i_1''^2} =$	53.5	77.9	109.7	144.2	179.7 amp
$P_1 = e_1'\, i_1 + e_1'\, i_1'' =$	55	2110	4320	6650	9180 kw.
$Q_1 = e_1\, i_1 =$	2720	3960	5610	7520	9660 kva.
$P_0 = e_0\, i_0' =$	0	1980	3960	5940	7920 kw.
Power-factor: $\frac{P_1}{Q_1} =$	2.0	53.3	77.0	88.4	95.0%
Efficiency: $\frac{P_0}{P_1} =$	0	93.8	91.6	89.3	86.3%

Generator volts and amperes are plotted in Fig. 1.

C. Determine voltage, current, and power-factor of receiving circuit, at which maximum power is delivered from a generating station giving 120 amperes at 110,000 volts three phase.

110 kv. between lines gives $e_1 = 110 \div 3 = 63.6$ kv. per line. Let:

p_1 = power-factor,
g_1 = reactive-factor at generator,

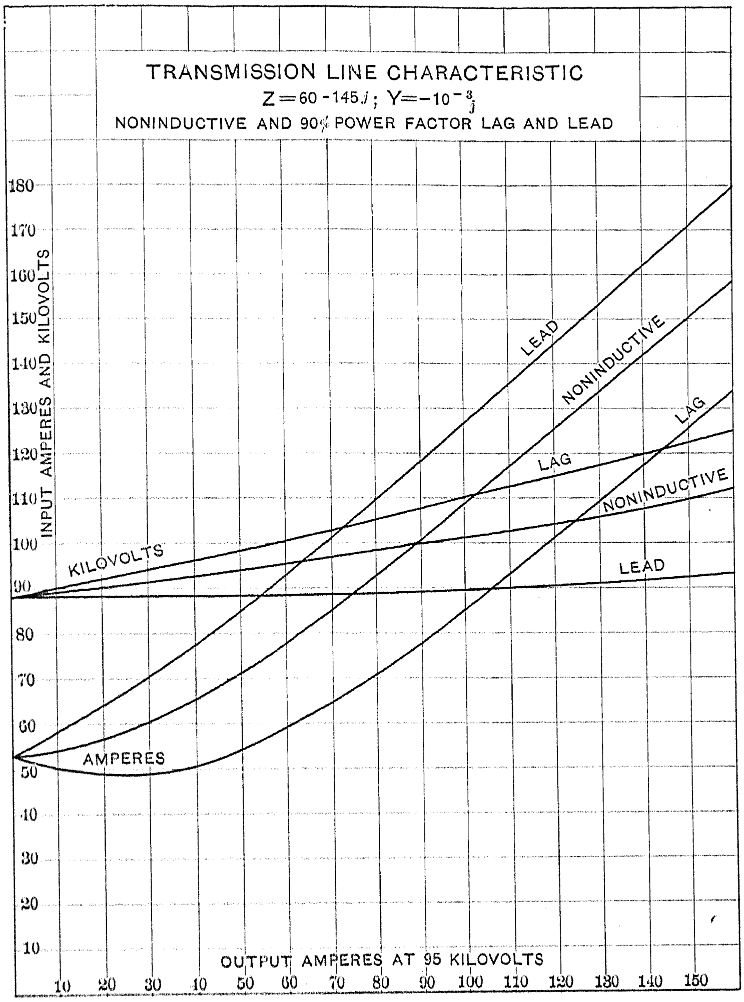


Fig. 1

it is:

$$I_1 = i_1' + j\,i_1'' = i_1\,(p_1 + j\,g_1) = 120\,(p_1 + j\,g_1)$$

Hence, by equations (23):

$$E_0 = 64.6\,(0.028\,j) - 120\,(p_1 + j\,g_1)\,(0.055 - 0.143j)$$
$$= (59.9 - 1.9j) - (6.6 - 17.2j)\,(p_1 + j\,g_1)$$
$$I_0 = 120\,(p_1 + j\,g_1)\,(0.928 - 0.028j) - 64.6\,(-0.010 - 0.974j)$$
$$= (111.4 - 3.4j)\,(p_1 + j\,g_1) + (0.6 + 62.9j)$$

Hence, for									
$g_1 =$	+0.5	+0.3	+0.2	+0.1	0	−0.1	−0.2	−0.3	−0.5
$p_1 =$	0.87	0.95	0.98	0.99	1.00	0.99	0.98	0.95	0.87
$E_0 = e_0' + j e_0'' =$	$45.6+9.8j$	$48.4+12.4j$	$50+13.7j$	$51.7+14.4j$	$53.3+15.3j$	$55.1+15.8j$	$56.8+16.3j$	$56.8+16.4j$	$62.8+16.4j$
$I_0 = i_0' + j i_0'' =$	$99.4+116.6j$	$107.7+93.1$	$110.6+81.9j$	$111.7+70.6j$	$112+59.5j$	$111.1+48.4j$	$109.2+37.3j$	$105.7+26.3j$	$96.0+3.2j$
hence, absolute: $e_0 = \sqrt{e_0'^2 + e_0''^2} =$	46.6	50.0	51.8	53.7	55.5	57.3	59.1	61.0	64.9
$e_0 \sqrt{3} =$	80.7	86.6	89.7	93.0	96.3	99.2	102.3	105.5	112.3
$i_0 = \sqrt{i_0'^2 + i_0''^2} =$	153.3	142.4	137.6	132.2	126.8	121.2	115.4	109.0	96.1
$P_0 = e_0' i_0' + e_0'' i_0''$	5670	6380	6650	6800	6880	6900	6810	6640	6080
$P_1 = e_1 i_1 p_1 =$	6740	7360	7600	6700	7750	7700	7600	7360	6740
$Q_1 = e_1 i_1 =$	7750								
hence, efficiency $\frac{P_0}{P_1} =$	84.1	86.7	87.5	88.3	88.8	89.6	90.1	90.7	90.2
$\frac{e_0''}{e_0'} = \tan E_0 =$	.215	.256	.274	.278	.287	.287	.287	.278	.261
$\frac{i_0''}{i_0'} = \tan I_0 =$	1.17	.865	.741	.633	.532	.436	.342	.249	.033
$\measuredangle E_0 =$	12.2	14.4	15.3	15.8	16.0	16.0	16.0	15.8	14.8
$\measuredangle I_0 =$	49.5	41.0	36.6	32.3	28.0	23.6	18.9	14.0	2.0
$\measuredangle \theta_0 = \measuredangle I_0 - \measuredangle E_0$	37.3	26.6	21.3	16.5	12.0	7.6	+2.9	−1.8	−12.8
$p_0 = \cos\theta_0 =$	79.2	89.4	93.1	95.9	97.8	99.1	99.9	99.9	97.6
$q_0 = \sin\theta_0 =$	61.5	44.8	36.3	28.4	20.8	13.2	+5.1	−3.1	−22.2

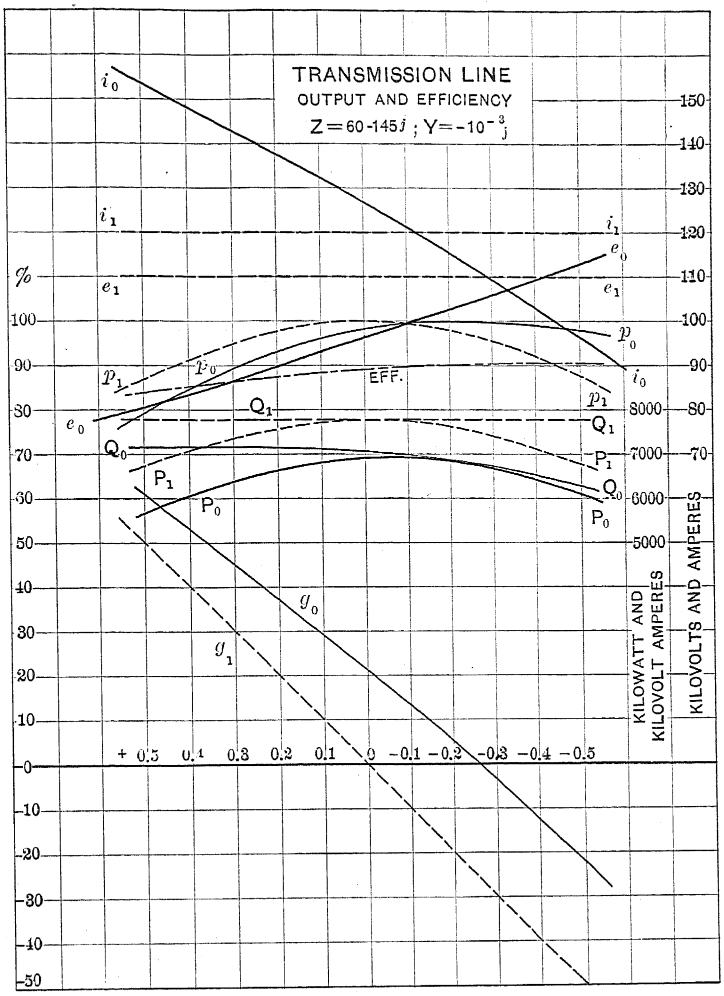


FIG. 2

Volts, amperes, efficiency, power-factor, and reactive-factor are plotted in Fig. 2, with the values at the receiving end in full lines, at the generator end in dotted lines, and the reactive-factors at the generator end of the line as abscissas.

A paper presented at the 26th annual convention of the American Institute of Electrical Engineers, Frontenac, N. Y., June 29, 1909.

EVEN HARMONICS IN ALTERNATING-CURRENT CIRCUITS

BY JOHN B. TAYLOR

In the usual presentation and consideration of wave-forms in alternating-current circuits we are so used to the mathematical expressions which omit the second, fourth, sixth, etc. harmonics and so generally assume that these even harmonics can be neglected at the outset, that a short consideration of usual and special conditions giving rise to even harmonics in alternating-current circuits, seems opportune.

From discussion with many engineers, I find that the following misconceptions are quite popular:

a. There can be no second harmonics in alternating-current circuits, because this would result in a direct-current component in the circuit, obtained without commutator or other rectifying device.

b. Even harmonics cannot be handled by typical transformers, so that should there be any even harmonics at one part of the circuit these will be in some way suppressed or balanced at the first transformation.

c. Wave-forms of electromotive force from commercial generators do not contain even harmonics, hence nothing can give rise to them later in the system

No explanation is offered as to why these misconceptions should be so common, and a slight endeavor to get at the root of the notions shows that the average text-book on alternating currents, at some point usually near the introduction, states that windings and poles of commercial generators are symmetrical, so that alternate waves will be of the same shape, and

hence have no even harmonics. Little or no reference to even harmonics is made later in such books.

As a matter of fact, the number of transmission systems at this

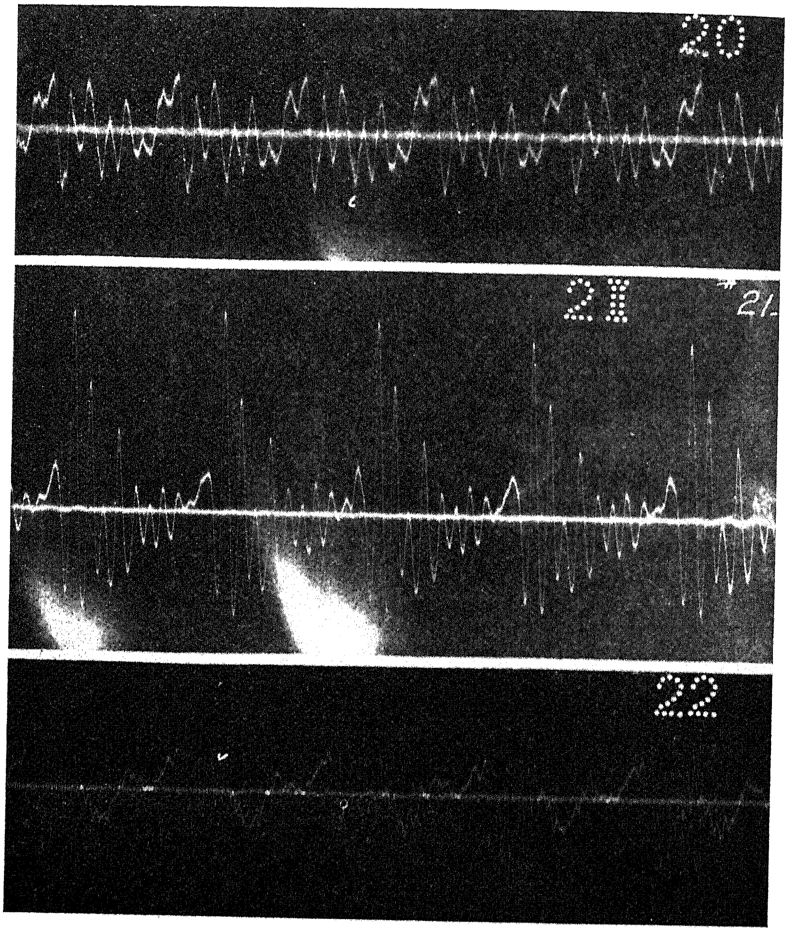
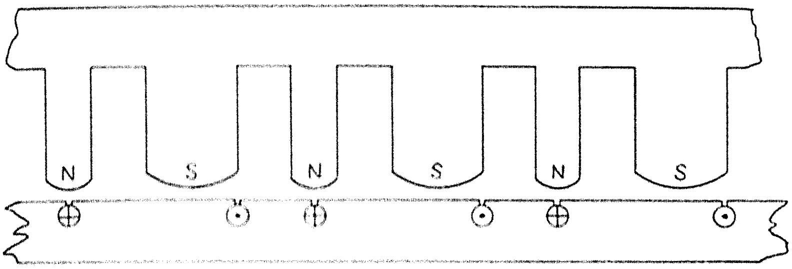


FIG. 1.—Record of telephone currents with sustained vowel sounds in transmitter. Upper curve, *u* (as in rude). Middle curve, *a* (as in far). Lower curve, *i* (as in machine). All pitched at *A* 110; that is, the *A* above 8 foot or cello *C*. J. B. Taylor's voice, April 18, 1907. (Time proceeds from left to right.)

very moment transmitting, with transformers transforming, and receiving apparatus receiving, alternating currents in which the even harmonics are quite as prominent as the odd and al-

together essential to the proper operation of the system—the number of these systems is so much greater than all the lighting, power, and railway plants added together, as to leave no comparison between the figures. By this, reference is made to the number of telephone conversations being carried on at any one time. The wave-form corresponding to speech is very complex if analyzed in the usual method by Fourier's Series, and the accompanying oscillograph records in Fig. 1, which shows current corresponding to simple sustained vowel sounds, will give an idea of the even, as well as odd, harmonics that telephone lines, transformers and receiving apparatus have to deal with.

There is no difficulty in constructing an alternating-current



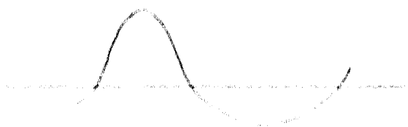


Fig. 3

generator to have a wave-form containing considerable even harmonics, when such are desired. There is also no difficulty in connecting standard generators in series or in parallel for example, a 25-cycle generator and a 50-cycle generator, when the resulting wave-form will have pronounced even harmonics.

If an alternating-current generator has a *symmetrical* armature winding and *unsymmetrical* north and south poles, the even harmonics will not appear. If the winding is *unsymmetrical* with the north and south poles *symmetrical*, again the even harmonics will be absent. The combination, however, of an *unsymmetrical* winding with *unsymmetrical* poles, will in general bring out even harmonics. Fig. 2 shows diagrammatically such

an alternating-current generator with approximate wave-form due to the special arrangement of winding and the differently shaped north and south poles.

Neglecting the special case of the telephone, of more practical consideration than what has preceded are typical commercial systems in which even harmonics are likely to be introduced and the effect of these harmonics on transformers. The simplest way in which even harmonics may be brought into an alternating-current system is by grounding at more than one point, and these points may be neutral or not. For example, Fig. 3 shows a typical three-phase, high-tension transmission system with transformer neutrals grounded at both ends. Fig. 4 shows the typical single-phase railway with ground at sub-station and also at various cars along the line. A neighboring direct-current traction system causes direct current, in a greater or less

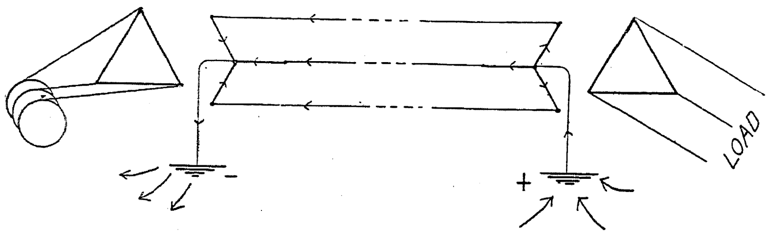


Fig. 3

amount depending on bonding and other features, to be shunted into the alternating-current system. This flows through the transformer windings giving what I term a "magnetic biased" condition. In other words, there is magnetic flux in the iron of the transformer, apart from excitation derived from alternating-current mains.

This shunted or leakage direct current may easily be as much as, or greater than, the exciting current of the transformer, which will give normal or greater magnetic density in the iron, and since many transformers are worked well up toward the saturation point, the iron in the transformer may be nearly saturated before any connection is made with alternating-current supply.

This combination in which transformer iron is initially at a greater magnetic density than is reached in normal alternating-current service, gives a condition which some persons, on superficial consideration, have stated to be equivalent to a heavy

overload or short-circuit on the transformer. The argument is, that since the iron is already practically saturated, the normal flux must be added to this saturated condition in order to give the normal counter electromotive force. While such a condition may exist for a very short interval of time when closing the circuit the stable condition is not at all of this nature. The magnetizing current under the conditions described, that is, with direct current in windings of the same order of magnitude as the normal exciting current, does not cause the exciting current to become of the order of a short-circuit current nor even full-load current. This is because the iron, instead of working from the saturated condition as a mean point, works about a mean point not far from zero, and in that portion of a cycle when the flux is in the opposite direction to the initial saturated condition due to direct current, enough extra current from the alternating

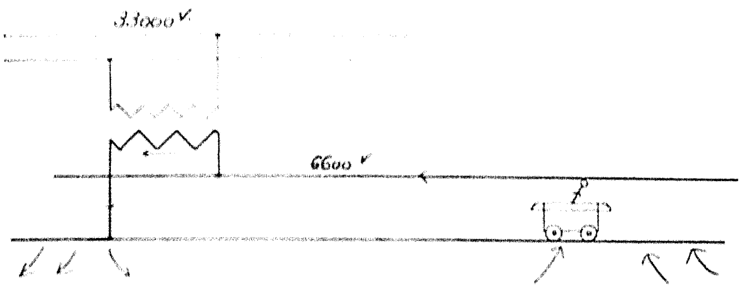


FIG. 4

circuit is supplied practically to neutralize the direct-current ampere-turns. The general effect of this is to increase the exciting current, accompanied by a distortion of the current wave-shape, as shown in the oscillograph record in Fig. 5.

Roughly, the value of this exciting current, when transformers are magnetically biased, is the value of the normal exciting current added to the direct current. In other words, it may be said that there is a component of the alternating current tending to neutralize the direct-current ampere-turns, and, as another component, the normal exciting current.

If the direct current is in the primary winding and the excitation is from the secondary, or vice versa, the direct current must be multiplied by the ratio of transformation in order to obtain the equivalent direct current for this addition. For example, referring to Fig. 4, the 6600-volt, single-phase trolley

wire with sub-station transformer and car as indicated, may have shunted into the winding a current of 5 amperes. The normal exciting current of the sub-station transformer from the 33,000 volt transmission line is, say, 0.5 ampere, but owing to the 5 amperes in the secondary winding, the primary exciting current will become $0.5 + \frac{5 \times 6600}{33000} = 1.5$ amperes. There will be some additional losses in transformers carrying direct current, due mainly to increased current density in the winding, and probably

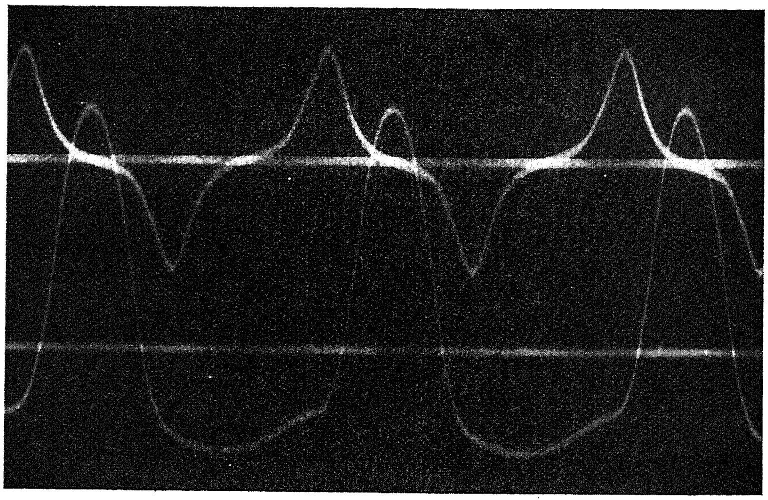


FIG. 5.—Record showing pronounced second harmonic in transformer magnetizing current due to direct current in windings. Upper curve: electromotive force at terminals from 60-cycle generator. Lower curve: exciting current.

a small increase in iron losses due to the unsymmetrical magnetic cycle with maximum flux of one polarity greater than normal.

The arrangement of apparatus as shown in Fig. 6, gives a convenient way of introducing direct current into a transformer winding without at the same time allowing the direct-current circuit to become a load or short-circuit on the transformer. This test requires two transformers, preferably identical, with primaries connected in parallel, and secondaries (with alternating potential balanced) in series with any convenient source of direct current.

In case it is necessary to make such test on a single transformer, a reactance can be introduced in the direct-current circuit to prevent appreciable alternating current from circulating in the direct-current system, or a low-voltage winding on the transformer can be connected through resistance to a high-voltage, direct-current source, such as a 600-volt railway system. If the voltage of the transformer winding is small compared with the direct voltage, the alternating current flowing into the direct-current system will be negligible.

Another combination of apparatus which may introduce direct current in transformer windings, is a synchronous converter connected for Edison three-wire service, the neutral for the

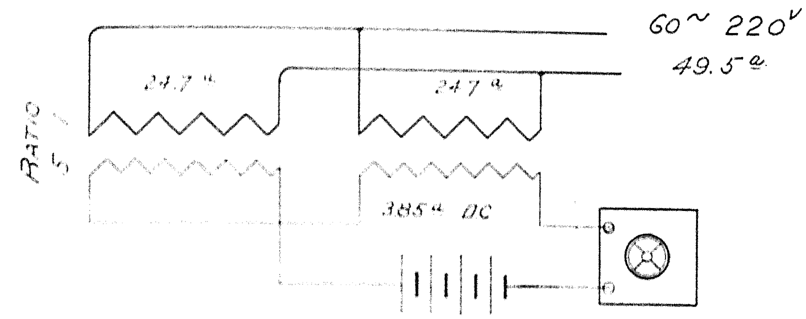


FIG. 6. Connections for test. 75-kw. transformers. Connected for 50-230 volts. Excited from 220 volt, 60-cycle circuit. Direct current, 3.85 amperes in 1150-volt winding. Normal exciting current at 220 volts, 60-cycles, approximately 5 amperes. Calculated exciting current with direct-current magnetization. (5 x 3.85) + 5 = 24.25 amperes. (Observed value was 24.7 amperes.)

three-wire service being derived from connection with transformer neutrals.

With this combination, the difference between the currents on the two sides of the lighting system passes through transformer windings. If the transformers are "diametrically connected" to the converter, this direct current will divide; and if the resistance of transformer and converter connections are symmetrical from the neutral point, this divided direct current will cause no magnetic flux.

However, with the direct-current neutral derived from star-connected transformers, direct current from the neutral does give a biased magnetic condition in the transformer iron with accompanying increase of transformer exciting current and

change in wave-shape, as explained above. Since this current divides among three transformers and its magnitude is not great, unless the lighting system is badly unbalanced, little additional transformer heating is likely to be noticed. Star-connected transformers have been so used with converters for a number of years without serious results, as far as I can learn. However, a diametrical connection seems preferable when laying out a new system.

Conclusion. The practical feature of this discussion is that leakage of direct current into transformer windings—unless these currents are of themselves sufficient to cause considerable additional heating in the windings—will not indirectly cause sufficient increase in the exciting current to cause serious over-heating or burn-out. I have already proposed the use of counter electromotive force cells, similar to storage-batteries but with plates unformed, in order to check leakage direct currents, but have not yet seen a case where the magnitude of these currents was sufficient to warrant such additional and undesirable complication of apparatus.

DISCUSSION ON "EVEN HARMONICS IN ALTERNATING-CURRENT CIRCUITS." FRONTENAC, N. Y., JUNE 29, 1909

Frederick Bedell: Confirming the author's conclusions, I have found that a direct current in a transformer increases the iron losses very much less than might be supposed, and also increases the copper losses very much less than might be supposed. The increase in the copper losses is due to two causes: 1. The increase of the exciting component of the alternating current, which is in quadrature with the power component, and so increases the total alternating current by a very small amount. 2. The direct current itself. So far as heating effect is concerned, the direct current behaves as if it were in quadrature with both the power component and the exciting component of the alternating current—the three quantities being mutually in quadrature as though in three dimensions—so that the direct current increases the effective value of the whole current very little. For example, 20 per cent direct current superposed upon an alternating current increases its effective value only 2 per cent.

V. Karapetoff: On the first page of his paper Mr. Taylor refers to certain statements usually made in text-books. He says that there is a common misconception to the effect that even harmonics could not be present in commercial circuits, because otherwise the alternating-current wave would have a direct-current component; this component could in some way be utilized, and thus we would have direct current without the use of commutating devices. I think the whole subject of even harmonics can be understood with reference to the following two simple statements:

1. Let a conductor revolve in an unsymmetrical field, such as is represented in Fig 2, the poles being of different size.

The electromotive force induced at any instant is proportional to the rate at which the flux is cut, or

$$e = \frac{d\,\phi}{d\,t}.$$

Integrating this, gives

$$\phi_1 = \int_0^{t_1} e\,d\,t,$$

where ϕ_1 is the total flux issuing, say, from the north pole, and t_1 the interval of time during which the conductor cuts the lines of force under this pole. Similarly, for the south pole

$$\phi_2 = \int_{t_1}^{t_2} e\,d\,t.$$

But ϕ_1 is always equal to ϕ_2; this is a fundamental law of nature: It is impossible to have a north flux larger or smaller than the corresponding south flux. Hence

$$\int_0^{t_1} e\,d\,t = \int_{t_1}^{t_2} e\,d\,t.$$

With reference to the curve shown in Fig. 2, the above equation means that the area of the curve above the axis of abscissas is always equal to that below the axis, however unsymmetrical the curve itself might be. In practical terms, a direct-current measuring instrument, consisting of a permanent magnet and a moving coil, always shows zero in this case. Therefore, in whichever way the machine is constructed, direct current cannot be obtained from it without the use of a commutator or an electrical valve. (Who of us in our younger years did not attempt to invent a direct-current generator without a commutator!) The above simple equation or rather the fundamental law $\phi_1 = \phi_2$ shows why this is impossible.

2. When more than one conductor is connected in series, the conductors must be spaced unsymmetrically in order to preserve even harmonics, provided such harmonics are induced in each individual conductor. Indeed, with reference to the curve in Fig. 2, let two conductors, in which such electromotive forces are induced, be spaced by 180 electrical degrees and connected in series. To find the resultant electromotive force, a second curve must be drawn, inverted, and shifted through 180°. It will be easily seen that the resultant curve has identical half-waves, and all the even harmonics are cancelled. But if the shift is different from 180°, some of the even harmonics can be preserved.

Since in practice armature conductors are spaced symmetrically, the statement remains true that no even harmonics are present in ordinary commercial circuits.

By suitably spacing the conductors, odd harmonics may be cancelled, instead of even ones. Thus, by shifting conductors through 120° and connecting them in series, the third harmonic is abolished, while the second, fourth, etc., are preserved to some extent, provided they are induced by an unsymmetrical field in each individual conductor.

Chas. F. Scott: Some years ago when synchronous converters were first used with a middle wire, in the way familiar to all, the two-phase converter was employed and the middle point was brought out from the middle of the transformer winding, which served as a balance coil. That was a simple matter. When the problem came to use star-connected coils on three-phase circuits, it seemed at first that a peculiar and unbalanced condition would result and the direct current would magnetize the cores independently, and the neutral point might go floating

about in some peculiar way, the transformers probably behaving quite badly. We were surprised on making a test to find how well the apparatus did behave, although the alternating-current transformers were treated with large amounts of the direct current. In such cases, however, the magnetizing current is in general small compared with the normal current, hence a large magnetizing current may not greatly increase the total current. In other cases, however, this direct current may lead to disturbances which are considerable. In one case with which I had to do some time ago, involving transformers which had little or no output but were depended on for their voltage relations, it happened that there was considerable resistance in the primary circuit, but this ordinarily had but little effect as the flow of magnetizing current was very small. When, however, direct current flowed in the secondary circuit, then the magnetizing current was increased many times, which caused an undue drop in the primary resistance, and did affect the operation of the apparatus.

The curve shown in Fig. 5 is of interest, as it shows the peculiar form of the magnetizing current which has to do double the duty of counteracting the direct current and of producing its own magnetization.

Charles P. Steinmetz: Regarding the statement of the author, confirmed by Dr. Bedell, that the iron loss in the unsymmetrical cycle produced by the existence of second harmonics resulting from direct current, is not seriously increased. I made some investigations on the subject 18 years ago, which were published in the A.I.E.E. PROCEEDINGS of 1892, in the second paper on the law of hysteresis. There the conclusions are that at the same difference between the limits of flux density the hysteresis loss is the same regardless as to whether the cycles are symmetrical or between unsymmetrical limits. There would thus be no increase in iron losses. The unsymmetrical cycles, which I investigated, were relatively small, and the accuracy was not so great. There may have been smaller differences which would have escaped notice, but they certainly prove that there is no serious increase of hysteresis loss in the unsymmetrical cycle over the symmetrical cycle, with the same distance between the limits of flux density. The eddy-current loss, however, is probably increased by the greater abruptness of the change.

It is also interesting to realize that even harmonics are practically as frequent as odd harmonics in the transient phenomena of alternating-current circuits.

As to the discussion of the origin of even harmonics, the usual statement that they cannot exist is misleading. The correct statement is that the *constant term*, which is the first term of even harmonics, cannot exist in a machine without commutating or rectifying devices, whether symmetrical or unsymmetrical arrangements are used; but the further terms of the even har-

monics, from the second upward, can exist; there is no physical reason why they should not exist, but owing to constructive conditions, they are practically always absent.

J. B. Taylor: I do not think there is anything to rebut in the discussion. Professor Karapetoff possibly misunderstood a reference to the text-books. The books themselves are not in error, but hazy ideas of even harmonics exist in the minds of many engineers, and the treatment in the text-books has been rather meagre, which possibly accounts for the misconceptions.

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